Tradeoff of Spectral and Energy Efficiencies: Impact of Power Amplifier on OFDM Systems

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Abstract—Spectral efficiency (SE) and energy efficiency (EE) are key measures for wireless communication systems. In OFDM systems, the non-linear effects and inefficiencies of power amplifiers (PAs) have posed practical challenges for system design. In this paper, we analyze the impact of the PA on the SE and EE tradeoff of OFDM systems. We also propose a PA switching technique which achieves a better SE-EE tradeoff, e.g., the EE can be improved by 323% with 15% SE reduction.

Index Terms—Energy efficiency, spectral efficiency, OFDM, power amplifier, power amplifier switching.

I. INTRODUCTION

Wireless access communication networks consume significant amount of energy to overcome fading and interference, compared to fixed line networks [1]. In wireless networks, energy is mostly consumed at the base station (BS) [2], of which 50%–80% of power consumption is consumed at power amplifiers (PAs) [3]. Fig. 1 plots PA output power $P_{\text{out}}$ versus PA power consumption $P_{\text{PA}}$, based on our survey of commercially available PAs for which we give a summary of the key parameters in Table I in the Appendix. A measure of the PA efficiency is given by the drain efficiency $\eta = P_{\text{out}}/P_{\text{PA}}$. From Fig. 1, we see that $\eta$ is typically between 20% and 30%, which confirms that the overhead incurred at PA is substantial. To ensure high energy efficiency (EE), the PA characteristics have to be carefully considered in system designs.

On the other hand, high spectral efficiency (SE) is needed to support the growing demands of high data traffic. To support high SE in wireless fading channels, orthogonal frequency division multiplex (OFDM) and orthogonal frequency division multiple access (OFDMA) systems remain as popular choices. However, OFDM and OFDMA modulated signals exhibit high peak-to-average power ratio (PAPR). To circumvent the resulting performance degradation, input backoff (IBO) is implemented by reducing the input power. In order to retain high SE, this popular technique of IBO in turn results in a reduction of EE, because PAs are operate at high efficiency when it is near its power saturation point [4]–[6]. Hence, a SE-EE tradeoff exists when the optimization is performed with respect to the PA. It is thus important to jointly characterize the role that a PA plays in both SE and EE of wireless communication systems.

In this paper, we quantify the impact of PA on the degradation of both SE and EE in practice. To do this, it is essential to have a sufficiently accurate, yet tractable, model for the PA (i) on its non-linearity behavior, and (ii) on its imperfect efficiency to specify the relationship of $P_{\text{PA}}$ and the transmission power $P_{\text{out}}$, i.e., $P_{\text{PA}} > P_{\text{out}}$ and $\eta < 100\%$. To provide tractable results, we assume that the non-linearity of the PA is modeled by a soft limiter [7], i.e., the output is clipped to a constant if the input signal exceeds a threshold value, and experiences a linear scaling of its input otherwise. As a consequence, we obtain a closed-form expression of SE which allows numerical evaluation. We also establish a PA-dependent (class-A, class-B, and Doherty Types) nonlinear power consumption model from various recent studies on empirical power measurement and parameters for cellular and wireless local area networks [8]–[12] and derive the EE. From the analysis and numerical evaluation, we observe that the practical PA can support a narrow SE-EE tradeoff with only a limited range of SE.

In cellular communications, however, a wide range of SE-EE tradeoff is desired because the BSs need high data rates intermittently, yet need to save energy whenever possible to save operation costs. To achieve a wide Pareto-optimal SE-EE tradeoff region, we propose a PA switching (selection or time sharing) technique, in which one or more PAs are switched on at any time between frames to maximize the EE while satisfying the required SE. As a result, the degree of freedom offered from the multiple PAs yields a high EE over a wide range of SE. Numerical results show that the SE-EE tradeoff improvement is significant even though practical losses are considered, such as switch insertion loss and switching time overhead.
Fig. 2. An OFDM system with a non-linear memoryless power amplifier (PA) represented by function $L_{\text{PA}}(\cdot)$.

II. SIGNAL MODEL

Consider the OFDM system with a non-linear memoryless PA shown in Fig. 2. Without loss of generality (w.l.o.g.), we consider the transmission of one OFDM symbol consisting of $N$ complex-valued data symbols $\mathbf{x} = [x_0, \cdots, x_{N-1}]^T$, over a total frequency band of $\Omega$ Hz. The data symbol $x_n$ is sent on the $n$th orthogonal subcarrier. The data symbols are assumed to be identical and independently distributed (i.i.d.) subject to the power constraint $E[|x|^2] \leq P_{\text{in}}$; hence and subsequently, we drop the subcarrier or time index if there is no dependence or when there is no ambiguity. We transform $\mathbf{x}$ to the time domain signal vector $\mathbf{x} = [x_0, \cdots, x_{N-1}]^T$ according to $\mathbf{x} = \mathbf{F}\tilde{\mathbf{x}}$, where $\mathbf{F}$ is the $N$-by-$N$ unitary inverse discrete Fourier transform (IDFT) matrix. Thus, $E[|\tilde{x}|^2] \leq P_{\text{in}}$. Then a cyclic prefix (CP) of length $N_{\text{CP}}$ is added to $\mathbf{x}$ and passed to a parallel-to-serial converter (P/S), followed by a digital-to-analogue converter (DAC). We assume the DAC, analogue-to-digital converter (ADC) and subsequent processing (such as timing and frequency synchronization) are ideal such that we may use $\mathbf{x}_t$ to represent the output of the DAC at discrete time index $t$. We write $x_t = a_t e^{j\theta_t}$, where $a_t \triangleq |x_t|$ is its amplitude and $0 \leq \theta_t < 2\pi$ is its phase.

Next, the DAC output $x_t$ is amplified through a memoryless PA described by the non-linear function $L_{\text{PA}}(\cdot)$ to give the output $\mathbf{w}_t = L_{\text{PA}}(x_t)$ at time $t$, denoted collectively by the vector $\mathbf{w} = [w_0, \cdots, w_{N+N_{\text{CP}}-1}]^T$. We assume there is no phase distortion, since typically its effect is small compared to the amplitude distortion [4]. To ease analysis, assuming the perfect linearization in low power regime, we can use the soft limiter model [7], [13], which is the simplest model that can capture the amplitude distortion (clipping effect) as follows:

$$ L_{\text{PA}}(a_t) = \begin{cases} \sqrt{\gamma}a_t, & \text{if } a_t < a_{\text{max}}, \\ b_{\text{max}}, & \text{if } a_t \geq a_{\text{max}}, \end{cases} $$

(1)

where $\sqrt{\gamma} \geq 1$ is a parameter interpreted as the desired linear gain, $a_{\text{max}} \triangleq \sqrt{P_{\text{in}}^{\text{max}}}$, and $b_{\text{max}} \triangleq \sqrt{P_{\text{out}}^{\text{max}}}$. Under this model, we can write $w_t = b_te^{j\theta_t}$, where $b_t \triangleq |w_t|$ while the phase remains the same as that of $x_t$.

Finally, the PA output is transmitted through a $L$-tap multipath channel $\{h_0, h_1, \cdots, h_{L-1}\}$, where we assume $L \leq N_{\text{CP}}$. Assuming perfect timing synchronization, the CP is removed and the received signal is then

$$ y_t = h_t \otimes w_t + z_t \triangleq r_t e^{j\phi} $$

(2)

for $t = 0, \cdots, N-1$. Here, $\otimes$ is the circular convolution operator, $z_t \sim \mathcal{CN}(0, \sigma_z^2)$ is an additive white Gaussian noise (AWGN), and $r_t$ and $\phi_t$ represent the amplitude and phase of $y_t$. The received signal vector $\mathbf{y} = [y_0, \cdots, y_{N-1}]^T$ (for convenience, we shift the time indices to start from zero) is then transformed via a DFT to give $\tilde{\mathbf{y}} = [\tilde{y}_0, \cdots, \tilde{y}_{N-1}]^T = \mathbf{F}^H\mathbf{y}$.

Based on the signal model, we shall obtain tractable results which offer insights on how the PA affects the SE and EE in Sections III and IV, respectively. Then we study how this leads to the analysis of a new transmitter architecture in Section IV-C, which offers a better tradeoff in terms of SE and EE.

III. SPECTRAL EFFICIENCY

In this section, we determine the SE of the system in Fig. 2. We ignore the CP overhead for simplicity. We assume the data symbols are i.i.d. with distribution $X \sim \mathcal{CN}(0, P_{\text{in}})$. Hence the time-domain signals are also i.i.d. with distribution $X \sim \mathcal{CN}(0, P_{\text{in}})$. The time domain signals have high PAPR and thus they are representative of the scenario when a high-order modulation is used or when $N$ is large.

We fix the following PA-related parameters: the maximum power input $P_{\text{in}}^{\text{max}}$; the power loading factor $\xi \triangleq \frac{P_{\text{in}}}{P_{\text{out}}^{\text{max}}}$, $\xi \geq 0$, which is related to the IBO as $\text{IBO} \triangleq 10 \log_{10}(\xi^{-1})$ dB; and the gain $g$ in the linearity region. Thus the maximum power output $P_{\text{out}}^{\text{max}} = gP_{\text{in}}^{\text{max}}$ is also fixed; for convenience, let $\gamma \triangleq P_{\text{out}}^{\text{max}}/\sigma_z^2 > 0$ be the maximum power output normalized by the noise variance $\sigma_z^2$.

We use upper case letters to represent random variables, such as $X$, $W$, and $Y$; we use lower case letters to represent their realizations, such as $x$, $w$, and $y$. The probability density function (pdf) of random variable $X$ is denoted by $f_X(\cdot)$. We recall that we also write the signals in terms of their amplitudes and phases as $x = ae^{j\theta}$, $w = be^{j\theta}$, and $y = re^{j\phi}$.

Specifically, we define SE, in $\text{b/s/Hz}$, as the amount of bits that are reliably decoded per channel use (i.e., per unit bandwidth and per unit time). We define EE, in $\text{b/J}$, as the total amount of bits that are reliably decoded bits normalized by the energy. Thus, SE and EE are given by [14], [15]

$$ \text{SE} = \frac{I(\tilde{X};\tilde{Y})}{N} \quad (3a) $$

$$ \text{EE} = \frac{T \Omega \text{SE}}{P_{\text{c}}} = \frac{\Omega \text{SE}}{P_{\text{c}}} \quad (3b) $$

where $I(\tilde{X};\tilde{Y})$ is the mutual information in $\text{b/s/Hz}$ given the length-$N$ transmitted and received vectors $\tilde{X}$ and $\tilde{Y}$, representing an achievable sum rate over $N$ channel uses [16]; $T$ is the total time used; and $P_{\text{c}}$ is the total power consumption including $P_{\text{PA}}$.

A. Mutual Information in Flat Fading Channel

Consider a flat fading channel where $L = 1$. Let $h_0 = 1$, w.l.o.g., as the actual channel attenuation and any fixed energy losses incurred can be reflected by changing the noise variance.
such that the channel-to-noise ratio (SNR) is maintained. Hence the channel model at time index $t$ is

$$ Y_t = W_t + Z_t, \quad \text{where } W_t = L_{PA}(A_t)e^{ih}. $$

(4)

The achievable rate averaged over $N$ transmissions is given by

$$ I(\tilde{X}; \tilde{Y})/N \overset{(a)}{=} I(X; Y)/N \overset{(b)}{=} \sum_{i=1}^{N} I(X_i; Y_i)/N \overset{(c)}{=} I(X; Y) \overset{(d)}{=} H(Y) - \log_2 \pi \sigma^2. $$

(5)

Here, $(a)$ follows since the time-domain and frequency-domain signals are related by a unitary transform, $(b)$ follows from the independence of the signals in the time domain (due to the memoryless PA and the i.i.d. transmitted signals and noise), $(c)$ follows since the mutual information is identical over time and so we can drop the time index, and $(d)$ follows from $I(X; Y) = H(Y) - H(Y|X)$ and the conditional entropy $H(Y|X) = H(N) = \log_2 \pi \sigma^2$. Denote $S$ as a binary random variable that denotes if clipping at the PA occurs, i.e., $S = 1$ if $A \geq a_{\text{max}}$ and $S = 0$ otherwise. Since $X = Ae^{j\theta} \sim \mathcal{CN}(0, P_n)$, the random variable $A$ follows the Rayleigh distribution. Thus, the probability of $S$ is

$$ \Pr(S = 0) = \Pr(A \leq a_{\text{max}}) = 1 - \exp(-\gamma^{-1}) $$

$$ \Pr(S = 1) = 1 - \Pr(S = 0) = \exp(-\gamma^{-1}) $$

(6)

The differential entropy of $Y$ is given by [16]

$$ H(Y) = - \int_y f_Y(y) \log_2 f_Y(y) dy, $$

(7)

where the pdf of $y$ can be written as $f_Y(y) = \sum_{i=0,1} f_Y(y, S = i)$. The numerical computation of the entropy (7) is straightforward given a closed-form expression of $f_Y(y, S = 0)$ and $f_Y(y, S = 1)$, which are obtained as follows (refer to [17] for the proofs):

$$ f_Y(y, S = 0) = N_0(y)(1 - Q_1(\sqrt{\mu \sigma^2})) $$

$$ f_Y(y, S = 1) = N_1(y) \left( \Pr(S = 1) \exp\left(\frac{-2b_{\text{max}}|y|}{\sigma^2}\right) \right) $$

(8)

where $N_0(y)$ denotes the pdf of $\mathcal{CN}(0, g P_{\text{in}} + \sigma^2)$; $Q_1(\cdot)$ is the Marcum-Q-function [18] with parameters $\mu = 2gP_{\text{in}}(gP_{\text{in}} + \sigma^2) \gamma^2$; $b_{\text{max}} = \frac{g^2 P_{\text{in}}(gP_{\text{in}} + \sigma^2)}{\sigma^2} |y|^2$; $N_1(y)$ is the pdf of $\mathcal{CN}(b_{\text{max}}, \sigma^2)$; $y_{\text{re}}$ is the real part of $y$; and $I_0(\cdot)$ is the modified Bessel function of the first kind [18].

**B. Mutual Information in Multipath Channel**

We now consider the general case of a $L$-tap multipath channel, where $L \leq N_{CP}$. The received signal in the time domain is given by (2). Because the PA non-linearity resulted in a correlated interference in the frequency domain which does not appear to evaluate as a closed-form expression, exact analysis of the mutual information in multipath case is intractable. Instead, we obtain a lower bound for the mutual information, denoted by $I_{LB}^{'\text{B}}$, which is given by the mutual information of the following channel (proof is shown in [17])

$$ Y'_t = h'_t L_{PA}(A_t)e^{ih} + Z'_t $$

(9)

where $h'_t = \sqrt{|h_0|^2 + \sum_{i=1}^{L-1} \frac{|h_i|^2}{\sigma_i^2 + gP_{\text{in}} \sum_{j=1}^{L} |h_j|^2}}$ is the equivalent channel and $Z'_t \sim \mathcal{CN}(0, \sigma_z^2)$. The channel (9) is simply the flat fading channel (4) with the noise variance modified accordingly. Hence we can obtain $I_{LB}^{'\text{B}}$ from (5) directly. Although the lower bound always holds, we can intuitively treat (9) as the channel where the power of all the multipaths are collected and any interference is treated as Gaussian distribution. For the details, refer to [17].

**C. Analytical Results on SE**

Using (8) into (7), we find $H(Y)$ and get $I(X; Y)$ from (5) in flat fading channel. Similarly, from (9), we can obtain the mutual information of the signal in multipath channel. Using the lower bound of the mutual information, eventually, we can derive the spectral efficiency from (3a). Generally, the SE in (3a) can be represented by the function of $\xi$ as

$$ \text{SE}(\xi) = H(Y) - \log_2 \pi \sigma^2 $$

(10)

since the conditional probabilities in (8) are functions of $P_{\text{in}} = \xi P_{\text{in}}^{\text{max}}$ and $b_{\text{max}} = \sqrt{g P_{\text{in}} \xi^{-1}}$, i.e., $H(Y)$ is a function of $\xi$.

By setting $b_{\text{max}} = \sqrt{P_{\text{in}}^{\text{max}}} \rightarrow \infty$, we can recover perfectly linear PA as

$$ \text{SE}_{\text{ideal}}(\xi) = \log_2 (1 + \gamma \xi) $$

(11)

which is the well-known log-shape function.

An approximation of the SE can be obtained if $P_{\text{in}} \ll P_{\text{in}}^{\text{max}}$ or $\xi \ll 1$, which gives $f_Y(y, S = i) \approx N_i(y)$, $i = 1, 2$, in (8). We note that typically this holds due to the IBO. We call the resulting SE as SE$^{\text{IBO}}$. We can also show that SE$^{\text{IBO}}$ is a concave function over $0 < \xi \leq \frac{1}{2}$. From this fact, we can derive the following theorem (refer to [17] for the proofs):

**Theorem 1. The optimal power loading (or equivalently optimal IBO) factor $\xi_{\text{SE}}^*$ which maximizes SE$^{\text{IBO}}$ is**

$$ \xi_{\text{SE}}^* \approx -W^{-1}(\ln^{-1}(\pi \sigma_z^2)) $$

(12)

where $W(\cdot) \leq -1$ is a Lambert W function [19].

Interestingly, the approximated $\xi_{\text{SE}}^*$ depends only on $\sigma_z^2$ as we assume $\xi \ll 1$ in derivation. This makes $\xi_{\text{SE}}^*$ independent of other PA parameters. The typical value of IBO is around 8 dB in practice, which includes an additional margin for fading channels [10]. Numerical results verify that $\xi_{\text{SE}}^*$ maximizes the true SE(\xi) effectively as well regardless of the IBO.

**IV. ENERGY EFFICIENCY**

To derive the EE of the system in Fig. 2, we use a power consumption model in the literature [8], [9]:

$$ P_t = (1 + C_{PS})(1 + C_{CB})(P_{BB} + P_{RF} + P_{PA}) $$

(13)

where $C_{PS}$ is a power supply coefficient (typically 0.1 $\leq C_{PS} \leq 0.15$) and $C_{CB}$ is an active cooler and battery coefficient (typically less than 0.4); $P_{BB}$, $P_{RF}$, and $P_{PA}$ are the measurements of power consumption at a base band module, a radio frequency module, a PA module, respectively. The advantage of this model in (13) is that the power consumption...
of the PA is separated from the power consumption of other components; therefore, we can modify (13) according to the PA types and the power loading factor. Here, though $P_{PA}$ actually depends on many factors including the specific hardware implementation, direct current (DC) bias condition, load characteristics, operating frequency and PA output power, it is dominated by DC power fed to the PA. Hence, using the drain efficiency $\eta$ depending on the DC power and PA types [5], [6], we can express $P_{PA}$ as a function of $\xi$ as $P_{PA} = \frac{1}{2} \eta \xi P_{in}^{max}$. Substituting the $P_{PA}$ to (13), we get a $PA$-dependant nonlinear power consumption model as

$$P_c(\xi) = P_{fix} + \frac{P_{out}^{max}}{c_1 + c_2 \sqrt{\xi}}, \quad 0 < \xi \leq 1,$$  \tag{14}$$

where $P_{fix} = (1 + C_{PS})(1 + C_{CB}) (P_{BB} + P_{RF})$, and the parameters $(c_1, c_2)$ are: $(\frac{1}{2} \xi c, 0)$ for class A; $(0, c)$ for class B; $(0, \frac{1}{2})$ if $0 < \xi \leq \frac{1}{2}$ or $(-\frac{1}{2}, \frac{c_1}{c_2})$ if $\frac{1}{2} < \xi \leq 1$ for $\ell$-way Doherty type. Here, $P_{fix}$ and $c$ can be obtained from empirical results in [10]–[12].

A. Energy Efficiency

Using the SE in (10) and the total power consumption $P_c(\xi)$ in (14), we can rewrite the EE in (3b) as a function of $\xi$ as

$$EE(\xi) = \frac{\Omega \Delta SE(\xi)}{P_c(\xi)}.$$ \tag{15}$$

For comparison and further analysis, the upper bound of $EE(\xi)$ is given for a perfectly linear PA as

$$EE(\xi) \leq EE^{linear}(\xi) = \frac{\Omega \Delta SE^{ideal}(\xi)}{P_c(\xi)}.$$ \tag{16}$$

B. Analytical Results on EE

From the fact that $EE^{linear}(\xi)$ is a piecewise quasi-concave function over $\xi$, we can derive the following theorem (proofs are in [17]).

Theorem 2. The optimal $\xi_{\text{EE}}^{*}$ maximizing $EE^{linear}(\xi)$ is as follows: for class A $\xi_{\text{EE}}^{*} = 1$; for class B $\xi_{\text{EE}}^{*} = \frac{1}{2} \exp \left(2 + 2W_0 \left(\frac{\sqrt{\xi} P_{in}^{max}}{c_1 + c_2 \sqrt{\xi}}\right)^{-1}\right)$; for $\ell$-way Doherty $\xi_{\text{EE}}^{*} = \frac{1}{\gamma} \left(\frac{W_0}{\sqrt{\xi}(P_{out}^{max} + \xi P_{in}^{max}) c_1 c_2}\right)^{-1}$ if $\frac{\partial EE^{linear}(\xi)}{\partial \xi} = 0$ at $\xi \neq 1$, or $\xi_{\text{EE}}^{*} = 1$ if $\frac{\partial EE^{linear}(\xi)}{\partial \xi} \neq 0$.

The numerical results verify that $\xi_{\text{EE}}^{*}$ maximizes the true EE effectively as well.

C. SE-EE Tradeoff and PA Switching Methods

In contrast to the ideal SE-EE tradeoff in (3) which is a decreasing convex function, the practical SE-EE in (15) has a concave shape as shown in Fig. 5 later. We observe that typically the EE drops rapidly when the SE exceeds beyond some threshold (which corresponds to the SE when the power of the PA approaches saturation). Thus, the PA can support a narrow SE-EE tradeoff with only a limited range of SE. In cellular communications, a wide range of SE-EE tradeoff is required because the BSs need high data rates intermittently or low data rates with low energy consumption. This motivates the use of multiple PAs, where any PA is switched on at any time between frames by using a switch. We call this technique PA switching. The switch insertion loss $G_S$ causes EE and SE degradation, and the switching time consumption $\epsilon$ causes further degradation. Numerical results in Section V supports the use of PA switching to achieve significant improvement of the SE-EE tradeoff even after considering the losses incurred in the switching.

V. NUMERICAL EVALUATION AND DISCUSSION

Numerical results for SE-EE tradeoff are obtained by varying the power loading factor $\xi$. We also evaluate the PA switching method based on the two PAs in Table I: a low power PA SM2122-44L (denoted by PA$^{low}$) with $P_{out}^{max} = 25$ W and a high power PA SM1720-50 (denoted by PA$^{high}$) with $P_{out}^{max} = 100$ W. Both PAs operate in the 2 GHz band and can be used for LTE BSs. Bandwidth is set to 10 MHz. The channel attenuation is modeled as follows [20]: $G + G_S = -128 + 10 \log_{10}(d^{-\alpha})$ dB where $G$ includes the transceiver feeder loss and antenna gains; and $d^{-\alpha}$ is the path loss where $d$ is the distance in kilometers between a transmitter and a receiver and $\alpha$ is a path loss exponent. In simulations, we set $G = 5$ dB, $\alpha = 3.76$, $d = 200$ m, and $\sigma^2 = -174$ dBm/Hz. For the power consumption model, we set $c = 4.7$ and $P_{fix} = 130$ W [10] to model the marco cell communication systems.

Fig. 3 shows the numerical evaluation of SE. As expected, $SE^{ideal}(\xi)$ in (11) is an increasing concave (log-shape) function, while $SE(\xi)$ in (10), the SE modeled by a soft limiter at the PA, is a concave function when $\xi \leq \frac{1}{2}$. The SE $SE^{BDO}$ is illustrated as a dashed line. The optimal $\xi_{\text{EE}}^{*}$ in (12) yields the highest SE as marked by ‘◦’ (Theorem 1), which closely approaches to the true SE $SE^{\xi}$.

Fig. 4 shows EE for various types of PAs: Class-A, class-B, and 2-way Doherty PAs. The dotted lines represent $EE^{linear}(\xi)$. EE obtained with $\xi_{\text{EE}}^{*}$ in Theorem 2 is illustrated by ‘×’ and ‘□’ for $EE(\xi)$ and $EE^{linear}(\xi)$, respectively. The results show that $\xi_{\text{EE}}^{*}$ yields the maximum EE and that $EE^{linear}(\xi_{\text{EE}}^{*}) = EE(\xi_{\text{EE}}^{*})$ (which are overlapped in the figure), except class-A PA. This is because the practical EE is maximized in the linear region. From the results, we can surmise that the optimal $\xi_{\text{EE}}^{*}$ derived under the ideal PA assumption is applicable to maximize the practical EE.

Fig. 5 shows the SE-EE tradeoff with and without PA switching. We assume that PA can be switched once every 20 frames. Each frame length is set to be 10 ms. The losses incurred in the switching are set as $G_S = 1$ dB and $\epsilon = 1$ ms. For comparison, ideal switching with $G_S = 0$ dB and $\epsilon = 0$ is evaluated. The EE can be improved by around 210% (323%) if we reduce SE by 12% (15%) from A to B (C), respectively. In contrast, if a single PA PA$^{high}$ is used instead, the EE is improved by around 41% with a 12% reduction of SE from A to D. For simplicity, we assume the fraction of time when any PA is switched on is a continuous value from 0 to 1.
We have given a theoretical analysis of the spectral and energy efficiency (SE-EE) tradeoff, by taking into account the practical non-ideal effects of the power amplifier (PA). This analysis motivates PA switching to achieve a better SE-EE tradeoff, which is verified by numerical results.

VI. CONCLUSION

We have given a theoretical analysis of the spectral and energy efficiency (SE-EE) tradeoff, by taking into account the practical non-ideal effects of the power amplifier (PA). This analysis motivates PA switching to achieve a better SE-EE tradeoff, which is verified by numerical results.

REFERENCES


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<td>10</td>
<td>2.4 – 2.5</td>
<td>Maxim</td>
</tr>
</tbody>
</table>

**APPENDIX: TABLE I**

**POWER AMPLIFIER CHARACTERISTICS (ASCENDING ORDER OF $P_{\text{out}}^{\text{max}}$).**