Practical Evaluation of Phase Uncertainty in Scattering Parameter Measurements

Yu Song Meng* and Yueyan Shan†
National Metrology Centre, Agency for Science, Technology and Research (A*STAR), Singapore 118221
*ysmeng@ieee.org, meng_yusong@nmca-star.edu.sg; †shan_yueyan@nmca-star.edu.sg

Abstract—This paper reports an evaluation of phase uncertainty in scattering parameter (S-parameter) measurements. Methods of Law of Propagation of Uncertainty and Coordinate Rotation have been used to transform uncertainty information of the real and imaginary components in a complex-valued S-parameter to its phase. Practical evaluations with measurements were carried out to check the consistency between the two methods. Good agreements have been observed except for the situation where the uncertainty region encompasses the origin of a rectangular coordinate.

Index Terms—Measurement uncertainty, phase uncertainty, S-parameter, uncertainty propagation.

I. INTRODUCTION

Uncertainty evaluation for S-parameters is always important to the RF and microwave metrological community. Previous work [1] has strongly recommended that for such a complex-valued parameter \( S = R + j I \), its uncertainty evaluation should be performed in a real–imaginary rectangular coordinate, following the Guide to the Expression of Uncertainty in Measurement [2]. However, the magnitude \( |S| = \sqrt{R^2 + I^2} \) and phase \( \phi = \tan^{-1}(I/R) \) of a S-parameter is sometimes preferred [3], [4], but their uncertainty evaluations are less discussed.

As reported in [5], [6], uncertainty evaluated in a rectangular coordinate could be transformed to \( |S| \) and \( \phi \) in a polar coordinate. Law of Propagation of Uncertainty in [2] and Coordinate Rotation [5] have been used to transform uncertainty information in [1], [7] where only the magnitudes of S-parameters were under evaluation for comparison. A 4-port Agilent PNA-X N5242A with its controller N5262A was used with two OML extenders, V15VNA2-T/R and V10VNA2-T/R for S-parameter measurements. The measurements were carried out at a temperature of \((23 \pm 1)°C\) and a relative humidity of \((55 \pm 5)\%\), and were repeated 6 times.

In the following, we will focus on analyzing the phase uncertainty of S-parameters under two circumstances; 1) the uncertainty region encompasses the origin, \( u(\phi) = \pi/2 \) (happened when the uncertainty region encompasses the origin), \( u(\phi) = \pi/2 \).

B. Coordinate Rotation

When rotating a real–imaginary rectangular coordinate with an angle of \(-\phi\) to align with the radial and tangential directions of \((|S|, \phi)\), a ‘radial–tangential’ rectangular coordinate system can be formed with a covariance matrix \( V_{rt} \) as [5],

\[
V_{rt} = M^TV_{rec}M,
\]

where \( M \) is a matrix for rotation and defined as,

\[
M = \begin{bmatrix}
\cos(\phi) & -\sin(\phi) \\
\sin(\phi) & \cos(\phi)
\end{bmatrix}.
\]

For this method, \( u(\phi) = \tan^{-1}\left(\sqrt{V_{rt}(2,2)/|S|}\right) \).

III. RESULTS AND ANALYSIS

A. Measurements

The measurement data analyzed is a part of a recent bilateral comparison [4] where only the magnitudes of S-parameters were under evaluation for comparison. A 4-port Agilent PNA-X N5242A with its controller N5262A was used with two OML extenders, V15VNA2-T/R and V10VNA2-T/R for S-parameter measurements. The measurements were carried out at a temperature of \((23 \pm 1)°C\) and a relative humidity of \((55 \pm 5)\%\), and were repeated 6 times.

In the following, we will focus on analyzing the phase uncertainty of S-parameters under two circumstances; 1) the size of \( u(|S|) \) is significant with respect to the size of \(|S|\), 2) \( u(|S|) \) is very small comparing to \(|S|\). The measured \( S_{11} \) and \( S_{21} \) of a well-matched Line are then used for evaluations. Fig. 1 and Fig. 2 show some typical results of \( S_{11} \) and \( S_{21} \) measurements, and the estimated uncertainty region (95% level of confidence) with mean value in a rectangular coordinate.
B. Analysis and Discussions

For simplicity, only Type A phase uncertainties of $S_{11}$ and $S_{21}$ are evaluated and presented in Fig. 3 and Fig. 4 using the two methods mentioned above. It can be observed that very good agreements have been achieved except for $S_{11}$ at 110 GHz. With analysis of all the data, it is found that for such a situation, its uncertainty region encompasses the origin of rectangular coordinate as shown in Fig. 5.

IV. Conclusion

Evaluation of phase uncertainty for $S$-parameters has been discussed using the methods of Law of Propagation of Uncertainty and Coordinate Rotation. It is found that both the methods produce very close results when transforming the uncertainty information of real and imaginary components to its phase. However, if uncertainty region encompasses the origin of rectangular coordinate, consistency of the two methods becomes poor, and the phase uncertainty approaches $\pi/2$.

REFERENCES


