Abstract—This paper presents a performance evaluation of the RF–DC transfer difference for a calorimetric thermal voltage converter (CTVC) designed by the National Research Council of Canada (NRC) at frequencies up to 1 GHz. The first part of this study describes a bilateral comparison of the RF–DC difference standards between the NRC and the National Metrology Centre, Agency for Science, Technology and Research of Singapore (NMC, A*STAR) in the frequency band from 1 kHz to 100 MHz. A good agreement has been observed between the two laboratories using the CTVC as a traveling standard. The second part of this study evaluates the performance of the CTVC at higher frequencies up to 1 GHz. In this part, RF–DC difference of the CTVC has been mathematically modeled and experimentally evaluated in terms of the calibration factor of a thermistor mount and the reflection coefficients at its type-N input connector.

Index Terms—AC–DC difference, calorimetric thermal voltage converter (CTVC), mathematical modeling, RF–DC difference.

I. INTRODUCTION

Development of a reference standard for high-frequency (HF) AC–DC transfer difference up to 100 MHz with relatively low uncertainties has been of interest to many National Metrology Institutes (NMIs) [1-7]. At the National Research Council of Canada (NRC), the realization of such a standard is a calorimetric thermal voltage converter (CTVC) [4-5] which is different from other commonly used standards such as an assembly of a calculable coaxial resistor and a single-junction thermal converter (SJTC) [1, 3], a coaxial calorimeter and a bolometric bridge [2], or a planar multi-junction thermal converter (MJTC) on a quartz crystal chip [6-7].

The CTVC designed at the NRC has several advantages in addition to its calculable frequency characteristic; it is small, physically robust, not sensitive to minor accidental overloading, and very well suited for transport. It has been used as a traveling standard in an international comparison [8] where its design as a reference standard of AC–DC difference at frequencies up to 100 MHz has been validated. Moreover, NRC participated in the CCEM-K6.c key comparison [9] of AC–DC voltage transfer standards at selected frequencies between 1 MHz and 100 MHz. This key comparison has fully confirmed the CTVC frequency characteristic and its uncertainty budget.

Since the CTVC is based on a design of a microwave calorimeter, its applicable frequency range as a wideband calculable standard could be extended up to 1 GHz. The CTVC then becomes a good candidate for calibrating RF power sensors in term of its related AC–DC/RF–DC transfer difference instead of the effective efficiency/calibration factor. Therefore, the National Metrology Centre, Agency for Science, Technology and Research of Singapore (NMC, A*STAR, formerly SPRING)), a participant in the CCEM.RF-K4.CL key comparison [10] of RF Voltage measurements up to 1 GHz, worked together with the NRC to evaluate the performance of the CTVC up to this frequency. To indicate that in this study the operating frequency is up to 1 GHz, we use RF–DC acronym rather than AC–DC (< 100 MHz normally) in the rest of the paper.

In this work, firstly both NMIs (NRC of Canada and NMC, A*STAR of Singapore) performed a bilateral comparison of the HF (1 kHz – 100 MHz) RF–DC voltage transfer difference to demonstrate their measurement capabilities and degrees of equivalence, and to further validate the design of the NRC CTVC as a traveling standard. Secondly, the RF–DC transfer difference of the CTVC at the frequency band between 100 MHz and 1 GHz is mathematically modeled and experimentally evaluated with the traceability to the NMC, A*STAR micro-calorimeter. The CTVC with a built-in Tee used in this study is shown in Fig.1.
II. BILATERAL COMPARISON OF RF–DC DIFFERENCE
(1 kHz–100 MHz)

The aim of this exercise is to compare the capabilities of both NMIs in providing calibration services to industries of two different metrological regions and also to validate the design of the NRC CTVC as a traveling standard of RF–DC difference at frequencies up to 100 MHz. Both laboratories used their regular procedures normally applied to the calibration of a voltage standard.

The RF–DC difference $\delta$ of a voltage standard (e.g. a thermal voltage converter (TVC)) is defined by the relative difference of the input RF and DC voltages, $V_{RF}$ and $V_{DC}$:

$$
\delta = \frac{V_{RF} - V_{DC}}{V_{RF}}
$$

where $V_{RF}$ is the rms value of the applied RF voltage, and $V_{DC}$ is the value of the direct and reversed DC voltage which produces the same mean output voltage $E$ of the standard as $V_{RF}$.

A. References and Test Methods at NRC

The RF–DC transfer difference of the CTVC can be presented as a sum of a frequency independent thermoelectric component and a frequency dependent component. The thermoelectric RF–DC difference of the CTVC was measured by comparing it at 1 kHz to the NRC primary standard MJTC. The frequency dependent RF–DC difference was determined from theoretical calculations taking into account mechanical and electrical parameters of the design. The uncertainty of this theoretical characterization was derived from uncertainties of parameters used in calculations. Over the years, several CTVCs differing in materials used and design parameters were fabricated and compared. Disagreements between their experimental characteristics were also incorporated in the uncertainty budget. References [4-5] contain detailed description of the procedure used, corrections applied, and the NRC Comparator System.

B. References and Test Methods at NMC, A*STAR

Three calculable coaxial single-junction TVCs (EUR-60, EUR-65B, and EUR-74) manufactured by the VSL Dutch Metrology Institute [3] are used as the reference standards at NMC, A*STAR for voltage transfer at frequencies below 100 MHz. The RF–DC difference of a TVC under test is calculated from [11]:

$$
\delta = \frac{V_{RF} - V_{DC}}{V_{RF}}
$$

where $\delta$ is the RF–DC difference of the reference standard, and $V_{RF}$ and $V_{DC}$ are the output of the reference standard at RF and DC input voltages respectively, and $V_{RF}$ and $V_{DC}$ are the output of a TVC under test at RF and DC input voltages. $n$ is a sensitivity coefficient linearly relating a relative change in the input voltage $\Delta V/V$ and the corresponding relative change of the output voltage $\Delta E/E$.

C. Bilateral Comparison Results and Analysis

The test results of the traveling standard at each laboratory, together with their expanded uncertainties are shown in Fig. 2 and in Table I. Table I presents also the difference in results evaluated in term of the error normalized with respect to the stated uncertainties using the following formula [12]:

$$
\text{Error} = \frac{|\delta_{\text{NMC}} - \delta_{\text{NRC}}|}{\delta_{\text{NMC}}} \times 100\%
$$

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<tr>
<th>$f$ (MHz)</th>
<th>$\delta$(NMC)</th>
<th>$U$(NMC)</th>
<th>$\delta$(NRC)</th>
<th>$U$(NRC)</th>
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Table II. NMC Uncertainty Budget of the Calibration at 10 MHz, in $\mu$V/V.

<table>
<thead>
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<th>Uncertainty Source</th>
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<th>Standard Uncertainty</th>
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<td></td>
<td>55</td>
</tr>
</tbody>
</table>

Table III. NMC Uncertainty Budget of the Calibration at 10 MHz, in $\mu$V/V.
\[
E_n = \frac{\delta_A - \delta_B}{U_A^2 + U_B^2}
\]  
(3)

\(\delta_A\) and \(\delta_B\) are the test results of the relative RF–DC transfer difference, and \(U_A\) and \(U_B\) are the corresponding expanded uncertainties (at a 95% confidence level, \(k = 2\)) evaluated by each laboratory. According to [12], the discrepancies between the testing results are acceptable when \(|E_n| < 1\).

The normalized error \(E_n\) indicates good agreement of the results in the whole frequency range, from 1 kHz to 100 MHz. The observations demonstrate the measurement capability and degree of equivalence between RF–DC calibrations performed at both the laboratories, NRC Canada and NMC, A*STAR of Singapore. Only at 10 MHz \(E_n\) exceeded 0.5. As an example, uncertainty budgets at 10 MHz used for analyses in this study are provided in Tables II and III. The RF–DC transfer difference \(\delta\) of the traveling standard as evaluated by NMC, A*STAR and NRC, shows different behavior in the 0.5 MHz to 10 MHz frequency range; the NRC results indicate drooping frequency characteristic, while the NMC data show it rising, see Table I. In this frequency range, the characteristics of both NMIs’ references may require further examination.

III. EVALUATION OF RF–DC DIFFERENCE FOR THE CTVC UP TO 1 GHz

A. Mathematical Calibration Model

In the frequency band 100 MHz to 1 GHz the characterizations of the TVC RF–DC transfer difference are carried out either in terms of the power and impedance using thermistor mounts [13-14], or in terms of the effective efficiency of a thermistor mount and the RF and DC conductances at its input connector [15].

Moreover, it is well-known that when the operating frequency increases, the impedance mismatch at input connectors (e.g. type-N connector of the CTVC) almost always dominates in the RF and microwave power measurement uncertainty [16]. For this reason, besides using the well-established calibration method based on a thermistor mount [13-15], we decided to characterize the CTVC RF–DC difference in terms of the voltage reflection coefficient (a parameter which can indicate the degree of impedance mismatch) at the input of a thermistor mount. In the following, the proposed method is described in detail.

Theoretical Modeling: According to the transmission line theory [17], the time-average power \(P_{av}\) at a reference plane as shown in Fig.3 is

\[
P_{av} = \frac{|V|^2 (1 - |\Gamma|^2)}{Z_0 |1 + \Gamma|^2}
\]  
(4)

where \(V\) is the total voltage at the reference plane, and equal to \(V_{RF}\) for RF signal and \(V_{DC}\) for DC voltage respectively in this study. \(Z_0\) is the characteristic impedance of the coaxial line, while \(\Gamma\) is the voltage reflection coefficient and defined as

\[
\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}.
\]  
(5)

The effective efficiency \(\eta_{eff}\) of a thermistor mount is defined as [18]:

\[
\eta_{eff} = \frac{P_{av-DC}}{P_{av-RF}}
\]  
(6)

where \(P_{av-DC}\) and \(P_{av-RF}\) are the DC substituted power and the RF power absorbed at its input respectively. They can be calculated using (4) with the corresponding total voltage \(V\) (\(V_{DC}\) or \(V_{RF}\)) and the voltage reflection coefficient \(\Gamma\) (\(\Gamma_{DC}\) or \(\Gamma_{RF}\)). Combining (4) and (6) yields:

\[
\frac{V_{RF}}{V_{DC}} = \frac{1}{\eta_{eff} (1 - |\Gamma_{RF}|^2) \sqrt{\frac{1 + |\Gamma_{RF}|}{1 + |\Gamma_{DC}|}}}
\]  
(7)

It is noted that the calibration factor \(K\) of a thermistor mount can be derived from its effective efficiency \(\eta_{eff}\) as

\[
K = \eta_{eff} (1 - |\Gamma_{RF}|^2).
\]  
(8)

Therefore, equation (7) can be simplified to

\[
\frac{V_{RF}}{V_{DC}} = \frac{1}{\sqrt{K} \cdot (1 - |\Gamma_{DC}|^2) \cdot \frac{1 + |\Gamma_{RF}|}{1 + |\Gamma_{DC}|}}
\]  
(9)

Substituting (9) into (1), we obtain

\[
\delta = \frac{1}{\sqrt{K} \cdot (1 - |\Gamma_{DC}|^2) \cdot \frac{1 + |\Gamma_{RF}|}{1 + |\Gamma_{DC}|}} - 1.
\]  
(10)
If $|\Gamma_{DC}|^2 \ll 1$, (10) is simplified to:

$$
\delta = \sqrt{\frac{1}{K} \frac{|1 + \Gamma_{RF}|}{|1 + \Gamma_{DC}|}} - 1. \quad (11)
$$

\textbf{Shifting of the Reference Plane:} It is noted that the reference plane for deriving (11) is located at the input connector (type-N in this study). It is different from the usually defined reference plane while characterizing a CVTC, located at the middle of its built-in internal Tee. Therefore the RF–DC difference $\delta_N$ at the reference plane of type-N connector needs to be transferred to the usual reference plane for characterizing the CVTC.

In this study, transmission line model shown in Fig. 4 is used to transfer the RF–DC difference $\delta_N$ at the reference plane of the type-N connector to the desired RF–DC difference $\delta_T$ at the middle of the Tee. From the transmission line theory [17],

$$
\Gamma_T = \Gamma_N e^{-2\gamma l} \quad (12)
$$

where $\Gamma_N$ is the voltage reflection coefficient at the type-N connector, $\Gamma_T$ is the voltage reflection coefficient at the middle of the Tee, $\gamma$ is the complex propagation constant, and $l$ is the length of the coaxial line. Moreover,

$$
V_T = V_N e^{\gamma l} \frac{1 + \Gamma_T}{1 + \Gamma_N}. \quad (13)
$$

Therefore, the RF–DC difference $\delta_T$ at the middle of the Tee (reference plane T) is

$$
\delta_T = \frac{|V_{T,RF}|}{|V_{T,DC}|} - 1 = \frac{|V_{N,RF} e^{\gamma l}|}{|V_{N,DC} e^{\gamma l}|} \frac{1 + \Gamma_{T,RF}}{1 + \Gamma_{N,RF}} - 1. \quad (14)
$$

As $\delta_N = |\Gamma_{N,RF}| / |\Gamma_{N,DC}| - 1$, $\gamma_{DC} = 0$ and $\Gamma_T = \Gamma_N e^{-2\gamma l}$, we can simplify (14) to be

$$
\delta_T = (\delta_N + 1) \cdot \frac{e^{\gamma l} + \Gamma_{N,RF} e^{-\gamma l}}{1 + \Gamma_{N,RF}} - 1. \quad (15)
$$

That is, the RF–DC difference $\delta_N$ of a reference standard at its type-N connector can be transferred to the middle of the Tee using (15). The evaluated $\delta_T$ can be used in (2) as $\delta_{Std}$ to determine the RF–DC transfer difference $\delta_{DUT}$ of the TVC under test, the NRC CTVC in this study.

\textbf{B. Reference Standards for RF–DC Transfer Difference Up To 1 GHz at NMC, A*STAR}

In this study, the RF–DC transfer difference $\delta$ of the NRC CTVC (as a traveling standard) up to 1 GHz was characterized at NMC, A*STAR. Reference standards type NRV-Z51 (#32 and #33), manufactured by Rohde & Schwarz [19] were used for RF–DC transfer measurements up to 1 GHz. Their calibrations are traceable to a microwave power standard (a thermistor mount of NIST design fitted with a type-N connector and calibrated in terms of effective efficiency $\eta_{eff}$ directly by means of a micro-calorimeter [20]) using direct comparison transfer [21]. Typical values for the calibration factor $K$ and the reflection coefficients $\Gamma_{RF}$ of the NRV-Z51 reference power sensor at its type-N connector used in this study are summarized in Table IV.

During the calibrations, the RF–DC transfer difference $\delta_N$ of the reference standard NRV-Z51 at its type-N input connector is determined in terms of its calibration factor and the voltage reflection coefficients using (11). $\delta_N$ is then transferred to the middle of the built-in internal Tee using (15), and later works as $\delta_{Std}$ in (2).

\textbf{C. Performance Evaluation of the NRC CTVC for RF–DC Transfer Difference Up To 1 GHz}

The RF–DC transfer difference of the NRC CTVC (traveling standard) is evaluated at the middle of its built-in Tee using the method described in the above (Subsection A and B). Its performance is studied in the frequency band from 50 MHz to 1GHz. It is noted that the least frequency point of 50 MHz is due to the limitation of the thermistor mount. The results with the corresponding expanded uncertainties are shown in Fig. 5, and an example of uncertainty budgets at 1 GHz is reported in Table V and VI for reference. It is found that the dominant uncertainty contribution is from the reference standard as shown in Table V (or indirectly from the calibration factor $K$ and $\Gamma_{RF}$ as shown in Table VI). From our study, it is also noted that the uncertainty contribution from shifting the reference plane is very small and is negligible.

The evaluated RF–DC transfer difference of the NRC CTVC, traceable to the micro-calorimeter (reference standard for RF
power measurements at NMC, A*STAR), is also verified against the results evaluated using the EUR standards at 50 MHz and 100 MHz as shown in Fig. 5. It is found that the corresponding at both the frequencies is less than 0.2. This demonstrates that there is a good degree of equivalence between the two different methods for evaluating the RF–DC transfer difference of a thermal voltage converter. Moreover, comparing the results shown in Fig. 5 to the results evaluated by NMC, A*STAR for traveling standards in the key comparison CCEM.RF-K4.CL [10], it is found that the working frequency range of the CTVC designed by NRC could be extended up to 1 GHz. 

Furthermore if inversely using (15) and (11), the RF–DC difference $\delta$ at the middle of the Tee of a CTVC could be transferred to the calibration factor $K$ of a power sensor. The CTVC designed by NRC with a calculable frequency characteristic therefore, becomes a good candidate as a reference standard for calibrating a power sensor in terms of its RF–DC difference $\delta$. Its performance as the reference standard up to 1 GHz with calculable characteristic for power sensor calibrations is not covered in this study, and will be an interesting future work.

IV. CONCLUSION

This paper presents a mathematical modeling and performance evaluation of RF–DC voltage transfer difference for a NRC designed CTVC with the frequency up to 1 GHz. The first part of this study presents a bilateral comparison of the HF RF–DC difference at the frequency range of 1 kHz–100 MHz between NRC and NMC, A*STAR. A good agreement has been observed demonstrating the measurement capabilities and degree of equivalence of both NMIs, and further validating the CTVC as a traveling standard.

In the second part of this study, the performance of the NRC CTVC is evaluated at frequencies between 50 MHz and 1 GHz using a thermistor mount. Mathematical modeling of the RF–DC difference in terms of the calibration factor of a thermistor mount and the reflection coefficients at its input connector has been carried out. The performance of the proposed method has been verified using the EUR standards at 50 MHz and 100 MHz with at both the frequencies. Using this method, the NRC CTVC was then evaluated from 50 MHz further up to 1 GHz. In comparison to the traveling standards in the CCEM.RF-K4.CL key comparison [10], the evaluated results for the NRC CTVC up to 1 GHz look encouraging. However to be a reliable reference standard for RF–DC difference up to 1 GHz, further investigations for the CTVC are still required.

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REFERENCES


