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An Integrated Plant/Control Design Method and Application in Hard Disk Drives

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One approach in servo control to achieve a high track density in hard disk drives is to minimize the H_2 norm from disturbances to position error signal. The H_2 performance optimization is then deemed as a matter of great significance. This paper presents an integrated design method involving plant modification and controller design sequentially to achieve the H_2 performance requirement. A linear matrix inequality based approach is developed for the plant damping ratio modification using the plant output. The proposed model modification method is then applied to the voice coil motor plant in hard disk drives, followed by the optimal H_2 controller design using the Riccati equation method with the modified plant. It turns out that the modified plant leads to better H_2 performance, stability margins than the original plant.

Keywords: Damping ratio; H_2 performance; Integrated plant/control design; H_2 optimal control; Hard disk drives

1. Introduction

Conventionally, the sequential design method is employed in the control design problem of mechanical systems, involving the mechanical plant design satisfying the requirements of stiffness, strength, weight, etc., followed by the controller design. For the controller design, the advanced control techniques in improving precision and accuracy has been well recognized in the field of positioning control for hard disk drives (HDDs) (Devasia, Eleftheriou, and Moheimani 2007, Du, Xie, Lewis, and Wang 2009, Chen and Tomizuka 2012, Nie, and Horowitz 2009). However, the optimal performance of the whole system is limited because this sequential design method fails to consider the freedom of plant modification. Therefore, the integrated plant/control design method, which aims to numerically optimize the parameters of both plant and controller, has attained much attention (Grigoriadis and Wu 1997, Lu and Skelton 2000, Hiramoto and Grigoriadis 2006, Pang et al. 2012, Tan et al. 2013). And it turns out that the system modification on structural parameters can improve the ultimate performance. Unfortunately, this integrated plant/control design method results in a bilinear matrix inequality (BMI) problem. It requires iterative algorithm and intractable computation leading to a local optimal solution,

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which depends on the initial conditions and conservativeness of the adopted iterative algorithm. Besides, this method misses investigating the relationship between plant parameters and system performances.

Therefore, another integrated plant/control design method, including plant modification followed by optimal controller design, is proposed. It turns out that some plant properties such as finite frequency positive real(Iwasaki, Hara, and Yamaguchi 2003), condition Π (Kanno, Hara, and Onishi 2007) and finite frequency high-gain (Hara, Iwasaki, and Shimizu 2002) are crucial to achievable control performances. Then, the plant which is modified to satisfy such properties is called good or easily controlled plant. Although there is not much theoretical justification for these three concepts, they have been proven to be efficient in tackling the placement problem of actuator and sensor to enhance the controlled performance (Hara, Iwasaki, and Shimizu 2002, Iwasaki, Hara, and Yamaguchi 2003, Kanno, Hara, and Onishi 2007). However, when the plant modification comes to design the damping parameters, it results in a BMI problem again.

Recently, the damping parameter design in structural systems such as the vector second order systems with collocated sensors and actuators has drawn much attention (Bai, Grigoriadis, and Demetriou 2004, Hiramoto and Grigoriadis 2008, Bai and Grigoriadis 2009, Mohammadpour, Meisami-Azad, Grigoriadis 2009). Different from the simultaneous design method and the FFPR condition, the damping parameter design of such structural systems can be formulated as a convex optimization problem. This method has advantage over the other integrated design methods in the computational efficiency, especially for large scale structures. In particular, for a single-input-single-output (SISO) system, collocated sensor and actuator condition leads to an in-phase property of the mechanical systems, which has been claimed to be helpful to expect a good control performance (Ono and Teramoto 1992). However, the collocated sensors and actuators condition is not practically satisfied for HDDs (Iwasaki, Hara, and Yamaguchi 2003). Furthermore, the above mentioned integrated plant/control design method does not consider the disturbance model of the system. Therefore, it is meaningful to investigate the integrated plant/control design method for the non-collocated system considering the disturbance models.

The lower H_2 performance is a crucial requirement for the positioning control in HDDs, because it relates to less track misregistration (TMR) during the track following. Therefore, it is meaningful to figure out an integrated plant/control design method including the plant modification on the damping parameters for less TMR followed by the optimal H_2 controller design. In this paper, the integrated plant/control design problem based on the vector second order system modeling is presented. And by using the plant output, only damping parameter part of the plant model is modified in order to achieve better control performances. Then, the optimal H_2 controller is designed using the Riccati equation method. The resultant control performances of the original plant and the modified plant are further compared after the completion of the integrated design.

2. Problem formulation

In this section, the formulation of a plant with disturbance models is discussed. It can be represented as the vector second order system, which describes the damping parameter of the plant in explicit structure. Such an explicit expression is significant to the integrated plant/control design.

Consider a dynamic plant $P(s)$ with the input disturbance d_1, the output disturbance d_2 and measurement noise d_n, as shown in Fig. 1. In HDDs, d_1 represents all torque disturbances and windage, and d_2 includes the disturbances due to disk motion, motor vibration, and airflow induced vibrations. $D_1(s)$ and $D_2(s)$ are the input and the output disturbance models, respectively, and N is a constant representing the noise level. ω_1, ω_2 and ω_3 are independent white noises with zero mean and unit variance. Reference $r = 0$. Generally, the plant $P(s)$ can

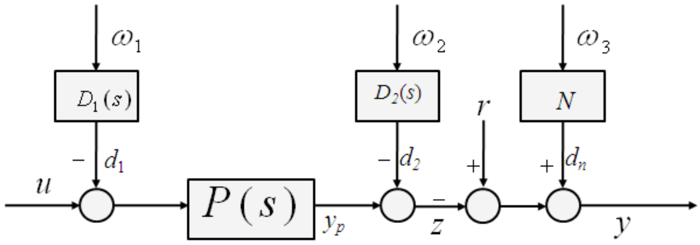

Figure 1. A plant with disturbances.

be modeled as follows.

$$P(s) = \frac{k_{p_0}}{s^2} + \sum_{i=1}^{n} \frac{k_{p_i}}{s^2 + 2\xi_{p_i}\omega_{p_i} s + \omega_{p_i}^2}. \tag{1}$$

where $\frac{k_{p_0}}{s^2}$ is the rigid part, the rest part involves n resonance modes with resonance frequencies at $\omega_{p_i} = 2\pi f_{p_i}$, damping ratios of $\xi_{p_i} > 0$ and residues of k_{p_i} $(i = 1, 2, \cdots, n)$. In general, the rigid part can also be written as the resonance mode $\frac{k_{p_0}}{s^2+2\xi_{p_0}\omega_{p_0}s+\omega_{p_0}^2}$ with resonance frequency at $\omega_{p_0} = 2\pi f_{p_0}$, damping ratio of $\xi_{p_0} > 0$ and residue of k_{p_0}. In this situation, the transfer function of plant $P(s)$ can be realized by the following vector second order system with non-collocated actuators and sensors.

$$M_q\ddot{q}(t) + D_q\dot{q}(t) + K_q q(t) = -B_q d_1(t) + B_q u(t), \tag{2a}$$

$$z(t) = C_q q(t) - d_2(t), \tag{2b}$$

$$y(t) = -z(t) + d_n(t), \tag{2c}$$

where $q(t)$, $u(t)$, $y_p(t)$ and $z(t)$ are the generalized coordinate vector, control input, plant output and measured output vector, respectively. Matrices M_q, D_q and K_q are symmetric positive definite matrices that represent the structural system mass, damping and stiffness matrices of the mechanical system, respectively. Matrices B_q and C_q are the distribution matrices on the control input and the control output, respectively. It is easy to know that Equ. (1) can be described in the form of (2a)-(2c), where

$$M_q = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, D_q = \begin{bmatrix} 2\xi_{p_0}\omega_{p_0} & 0 & \cdots & 0 \\ 0 & 2\xi_{p_1}\omega_{p_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2\xi_{p_n}\omega_{p_n} \end{bmatrix},$$

$$K_q = \begin{bmatrix} \omega_{p_0}^2 & 0 & \cdots & 0 \\ 0 & \omega_{p_1}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_{p_n}^2 \end{bmatrix}, B_q = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, C_q = \begin{bmatrix} k_{p_0} & k_{p_1} & \cdots & k_{p_n} \end{bmatrix}.$$

Assume that the input disturbance model D_1 is a constant gain and the output disturbance model takes the form of

$$D_2(s) = \sum_{i=1}^{m} \frac{k_{d_i}}{s^2 + 2\xi_{d_i}\omega_{d_i}s + \omega_{d_i}^2} \tag{3}$$

with m narrow-band disturbances at frequencies $\omega_{d_i} = 2\pi f_{d_i}$, bandwidth determined by $\xi_{d_i} > 0$ and amplitude determined by k_{d_i} $(i = 1, 2, \cdots, m)$. The disturbance model D_2 can also be written as the following vector second order system

$$M_d\ddot{q}_d(t) + D_d\dot{q}_d(t) + K_dq_d(t) = B_d\omega_2(t), \tag{4a}$$

$$d_2(t) = C_dq_d(t), \tag{4b}$$

where $q_d(t)$ is the coordinate vector of the output disturbance model, matrices M_d, D_d and K_d are symmetric positive definite matrices, and

$$M_d = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, D_d = \begin{bmatrix} 2\xi_{d_0}\omega_{d_1} & 0 & \cdots & 0 \\ 0 & 2\xi_{d_1}\omega_{d_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2\xi_{d_r}\omega_{d_r} \end{bmatrix},$$

$$K_d = \begin{bmatrix} \omega_{d_1}^2 & 0 & \cdots & 0 \\ 0 & \omega_{d_2}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_{d_r}^2 \end{bmatrix}, B_d = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, C_d = \begin{bmatrix} k_{d_1} & k_{d_2} & \cdots & k_{d_r} \end{bmatrix}.$$

Denote $\bar{x}(t) = [q^T(t)\ \ q_d^T(t)\ \ \dot{q}^T(t)\ \ \dot{q}_d^T(t)]^T$ and $\omega(t) = [\omega_1^T(t)\ \ \omega_2^T(t)\ \ \omega_3^T(t)]^T$. The state-space representation of the plant $P(s)$ with the disturbances d_1 and d_2 and measurement noise d_n is given by

$$\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}_1\omega(t) + \bar{B}_2u(t), \tag{5a}$$

$$z(t) = \bar{C}\bar{x}(t), \tag{5b}$$

$$y(t) = -\bar{C}\bar{x}(t) + D\omega, \tag{5c}$$

where

$$\bar{A} = \begin{bmatrix} 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ -M_q^{-1}K_q & 0 & -M_q^{-1}D_q & 0 \\ 0 & -M_d^{-1}K_d & 0 & -M_d^{-1}D_d \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -M_q^{-1}B_qD_1 & 0 & 0 \\ 0 & M_d^{-1}B_d & 0 \end{bmatrix}, \tag{6a}$$

$$\bar{B}_2 = \begin{bmatrix} 0 \\ 0 \\ M_q^{-1}B_q \\ 0 \end{bmatrix}, \ \bar{C} = [C_q \ \ -C_d \ 0 \ 0], \ and \ D = [0 \ 0 \ N]. \tag{6b}$$

3. Integrated damping parameters and controller design for optimal H_2 performance

This section provides a linear matrix inequality (LMI) approach for the integrated damping parameters and controller design. The plant is modified firstly by using the output feedback to change the damping parameter part of the plant. Then, the optimal controllers are designed for the original plant and the modified plant, respectively.

3.1. *Plant model modification*

Consider the vector second order system (5a)-(5c) with an output feedback controller

$$u_G(t) = Gv(t) = G\left[-C_q\dot{q}(t) + C_d\dot{q}_d(t) + \dot{d}_n\right]. \tag{7}$$

Then, the system, as shown in Fig. 2, can be formulated as follows.

$$\dot{\bar{x}}(t) = A_{cl}\bar{x}(t) + B_{cl}\bar{\omega}(t) + \bar{B}_2u(t), \tag{8a}$$

$$z(t) = \bar{C}\bar{x}(t), \tag{8b}$$

$$y(t) = -\bar{C}\bar{x}(t) + \bar{D}\bar{\omega}, \tag{8c}$$

where $\bar{\omega}(t) = [\omega_1^T(t) \ \ \omega_2^T(t) \ \ \omega_3^T(t) \ \ \dot{\omega}_3^T(t)]^T$,

$$A_{cl} = \begin{bmatrix} 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ -M_q^{-1}K_q & 0 & -M_q^{-1}\tilde{D} & -M_q^{-1}B_qGC_d \\ 0 & -M_d^{-1}K_d & 0 & -M_d^{-1}D_d \end{bmatrix}, \tilde{D} = D_q - B_qGC_q,$$

$$B_{cl} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -M_q^{-1}B_qD_1 & 0 & 0 & -M_q^{-1}B_qGN \\ 0 & M_d^{-1}B_d & 0 & 0 \end{bmatrix}, \bar{D} = [0 \ \ 0 \ \ N \ \ 0],$$

$\bar{B}_2$ and $\bar{C}$ are seen in (6b).

The objective is to use the plant output to design the feedback gain in Equ. (7) to modify the plant model (5a)-(5c) so that the modified plant (8a)-(8c) is stable and the H_2 norm $\|T_{z\bar{\omega}}\|_2$ of the transfer function from $\bar{\omega}$ to z is minimized. Before we proceed to find the solution, we first introduce the following lemmas.

Lemma 3.1: *(Scherer, Gahinet, and Chilali 1997) For the stable system (5a)-(5b) with no control input, i.e.* $u = 0$*, the minimal* H_2 *norm* γ *of the transfer function* $T_{z\omega}$ *can be calculated*

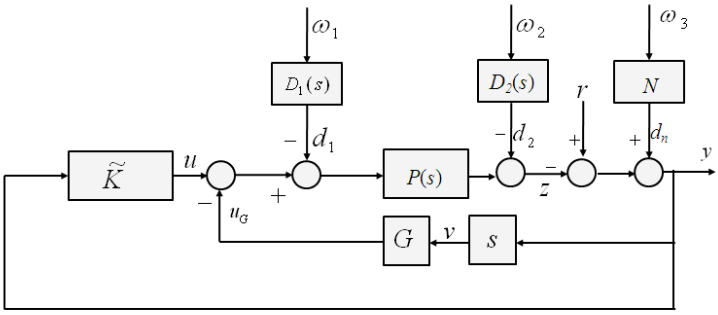

Figure 2. Structure of vector second order system with output feedback loop.

by the following minimization:

$$\min trace(Z) < \gamma^2 \tag{9}$$

subject to

$$\begin{bmatrix} \bar{A}^T Q + Q\bar{A} & Q\bar{B}_1 \\ * & -I \end{bmatrix} < 0, \tag{10a}$$

$$\begin{bmatrix} Q & \bar{C}^T \\ * & Z \end{bmatrix} > 0, \tag{10b}$$

where Q and Z are symmetric positive-definite matrices.

Lemma 3.2: *(Boyd, Ghaoui, Feron, and Balakrishnan 1994)The block matrix*

$$\begin{bmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{bmatrix} > 0, \tag{11}$$

where S_1 and S_3 are symmetric matrices, is equivalent to

$$S_1 > 0, S_3 - S_2^T S_1^{-1} S_2 > 0,$$

or

$$S_3 > 0, S_1 - S_2 S_3^{-1} S_2^T > 0.$$

Using the above formulation and lemmas, the plant modification problem, where the plant output is used as the feedback to change the damping parameter matrix, is solvable in terms of LMIs.

Theorem 3.3 : *Consider the vector second order system (5a)-(5c), there exists the controller (7) such that $\|T_{z\bar{\omega}}\|_2 < \gamma$ if the following optimization admits a solution for matrix Z, G, the scalar α and the damping parameters $\xi_{p_i} (i = 0, 1, \cdots, n)$ in D_q:*

$$\min trace(Z) < \gamma^2, \tag{12}$$

subject to

$$\begin{bmatrix} -2D_q + B_qGC_q + (B_qGC_q)^T + \alpha D_1^2 B_q B_q^T & -B_qGC_d & B_qGN \\ * & -2D_d + \alpha B_d B_d^T & 0 \\ * & * & -\alpha I \end{bmatrix} < 0, \tag{13a}$$

$$\begin{bmatrix} \alpha K_q & 0 & C_q^T \\ * & \alpha K_d & C_d^T \\ * & * & Z \end{bmatrix} > 0. \tag{13b}$$

Proof: From Lemma 3.1, the H_2 norm of closed loop system (8a)-(8b) with no input can be computed by $\min trace(Z)$ subject to (10a)-(10b).

Choose the candidate Lyapunov matrix Q as $Q = \alpha \begin{bmatrix} K_q & 0 & 0 & 0 \\ * & K_d & 0 & 0 \\ * & * & M_q & 0 \\ * & * & * & M_d \end{bmatrix}$, where α is a positive scalar to be used as an additional degree of freedom in order to reduce the conservativeness of the H_2 norm bound. Then, the LMI (10a) can be simplified as follows.

$$\begin{bmatrix} -\alpha(\tilde{D} + \tilde{D}^T) & -\alpha B_qGC_d & -\alpha D_1 B_q & 0 & -\alpha B_qGN \\ * & -2\alpha D_d & 0 & \alpha B_d & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0.$$

Considering $\alpha > 0$ and Lemma 3.2, we can obtain

$$\begin{bmatrix} -(\tilde{D} + \tilde{D}^T) + \alpha D_1^2 B_q B_q^T & -B_qGC_d & BqGN \\ * & -2D_d + \alpha B_d B_d^T & 0 \\ * & * & -\alpha I \end{bmatrix} < 0. \tag{14}$$

Replacing $\tilde{D}$ in (14) with $\tilde{D} = D_q - B_qGC_q$ results in the LMI (13a) with respect to α, G and $\xi_{p_i} (i = 0, 1, \cdots, n)$ in D_q.

Furthermore,

$$\begin{bmatrix} Q & \bar{C}^T \\ * & Z \end{bmatrix} > 0$$

is equivalent to LMI (13b) by using Lemma 3.2 again. □

Remark 1: The design problem of the damping parameters $\xi_{p_i} (i = 0, 1, \cdots, n)$ in D_q and the output feedback gain G can be solved as the H_2 optimization problem as follows:

$$\min_{\alpha, \xi_{\mathrm{p_i}}, \mathrm{G}} \ \gamma^2$$
$$\text{subject to } \ (13a) - (13b)$$

Remark 2: In fact, the closed-loop system from u to y in Fig. 2 after the damping parameters and output feedback gain design in this section can be seen as a modified plant with damping matrix $\tilde{D} = D_q - B_qGC_q$ (see Fig. 2). It is well known that the plant having good open-loop

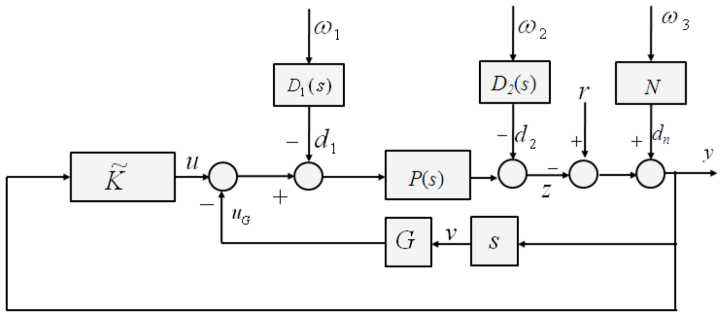

Figure 3. H_2 control scheme for the modified plant with disturbance models.

specifications is beneficial to ease the control and improve the overall performance. Advanced control techniques can be then applied to realize better control performance.

3.2. H_2 *controller design*

The feedback control loop for the modified plant is shown in Fig. 3. Let $P : (A_P,\ B_P,\ C_P,\ 0)$ with the state $x_p = \begin{bmatrix} q^T\ \dot{q}^T \end{bmatrix}^T$, $D_2 : (A_D, B_D, C_D, 0)$ with the state $x_d = \begin{bmatrix} q_d^T & \dot{q_d}^T \end{bmatrix}^T$, and $v = -\tilde{C}_q x_p + \tilde{C}_d x_d + \dot{d}_n$ with $\tilde{C}_q = \begin{bmatrix} 0 & C_q \end{bmatrix}$, $\tilde{C}_d = \begin{bmatrix} 0 & C_d \end{bmatrix}$. Define state x as $x = \begin{bmatrix} x_p^T & x_d^T \end{bmatrix}^T$.

The augmented system for the feedback controller design is obtained as follows.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1\bar{\omega}(t) + B_2u(t), \\ z(t) &= C_1x(t) + D_{12}u(t), \\ y(t) &= -C_1x(t) + D_{21}\bar{\omega}(t), \end{aligned}$$

where $A = \begin{bmatrix} A_P + B_PG\tilde{C}_q & -B_PG\tilde{C}_d \\ 0 & A_D \end{bmatrix}$, $B_1 = \begin{bmatrix} -B_PD_1 & 0 & 0 & -B_pGN \\ 0 & B_D & 0 & 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} B_P \\ 0 \end{bmatrix}$, $C_1 = \begin{bmatrix} C_P & -C_D \end{bmatrix}$, $D_{12} = 0$, $D_{21} = \begin{bmatrix} 0\ 0\ N\ 0 \end{bmatrix}$, with $A_P = \begin{bmatrix} 0 & I \\ -M_q^{-1}K_q & -M_q^{-1}D_q \end{bmatrix}$, $B_P = \begin{bmatrix} 0 \\ M_q^{-1}B_q \end{bmatrix}$, $C_P = \begin{bmatrix} C_q\ 0 \end{bmatrix}$, $A_D = \begin{bmatrix} 0 & I \\ -M_d^{-1}K_d & -M_d^{-1}D_d \end{bmatrix}$, $B_D = \begin{bmatrix} 0 \\ M_d^{-1}B_d \end{bmatrix}$, $C_D = \begin{bmatrix} C_d\ 0 \end{bmatrix}$.

The optimal H_2 controller $\tilde{K}(s)$ is subsequently designed for the above system using algebraic Riccati equation method. Because $D_{12} = 0$, it is a singular H_2 problem. Thus, a perturbation is needed by adding a dummy output to the system(Li, Guo, Chen, and Lee 2001). We then have

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1\bar{\omega}(t) + B_2u(t), \\ \tilde{z}(t) &= \tilde{C}_1x(t) + \tilde{D}_{12}u(t), \\ y(t) &= -C_1x(t) + D_{21}\bar{\omega}(t), \end{aligned}$$

where $\tilde{z} = \begin{bmatrix} z \\ \epsilon x \\ \epsilon u \end{bmatrix}$, $\tilde{C}_1 = \begin{bmatrix} C_1 \\ \epsilon I \\ 0 \end{bmatrix}$, $\tilde{D}_{12} = \begin{bmatrix} D_{12} \\ 0 \\ \epsilon I \end{bmatrix}$. The H_2 controller $\tilde{K}$ can be solved using the

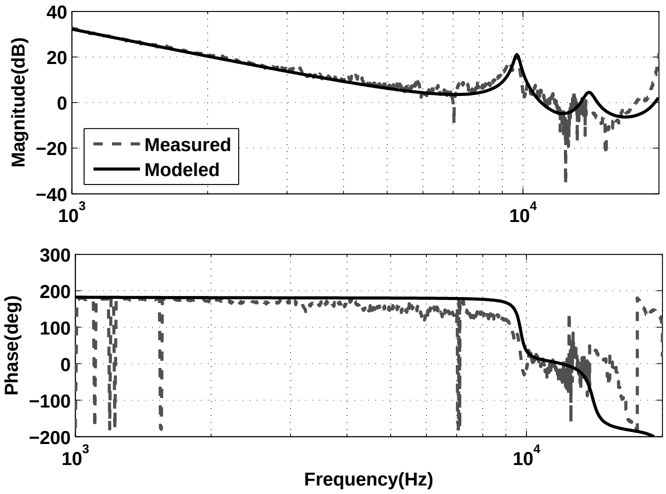

Figure 4. Frequency responses of the VCM plant.

Table 1. Parameters of the original plant

mode i	$k_{p_i}(\times 1.8701 \times 10^9)$	ξ_{p_i}	ω_{p_i} (rad/s)
0	1	0.2	2π 100
1	-0.8	0.02	2π 9700
2	-0.5	0.03	2π 14000
3	-1	0.03	2π 20500

following Riccati equations:

$$A^T X + XA - (XB_2 + \tilde{C}_1^T \tilde{D}_{12})(\tilde{D}_{12}^T \tilde{D}_{12})^{-1}(\tilde{D}_{12}^T \tilde{C}_1 + B_2^T X) + \tilde{C}_1^T \tilde{C}_1 = 0,$$
$$YA^T + AY - (-YC_1^T + B_1 D_{21}^T)(D_{21} D_{21}^T)^{-1}(D_{21} B_1^T - C_1 Y) + B_1 B_1^T = 0.$$

The solution of the controller $\tilde{K} : (A_K, B_K, C_K, 0)$ is given by

$$A_K = A + B_2 F - KC_1, B_K = -K, C_K = F,$$

where $F = -(\tilde{D}_{12}^T \tilde{D}_{12})^{-1}(\tilde{D}_{12}^T \tilde{C}_1 + B_2^T X)$, $K = -(-YC_1^T + B_1 D_{21}^T)(D_{21} D_{21}^T)^{-1}$.

4. Application in HDDs

In this section, we will apply the results obtained in the above sections to design the damping parameters of the voice coil motor (VCM) plant in HDDs. It has been shown in the mechanical engineering community that the damping configuration for head actuator influences its tracking dynamics (Jiang and Miles 1999).

We consider the VCM plant in a 2.5 inch HDD with three resonance modes, i.e., $n = 3$, to show the efficiency of the integrated design method. The frequency response of the original plant is shown in Fig. 4, and the model parameters are seen in Table 1.

Based on the power spectrum of the non-repeatable runout (NRRO) of a position error signal (PES) without servo control as shown in Fig. 5, we can model the input disturbance as the constant gain and the output disturbance as a narrow-band disturbance with the form in Equ.

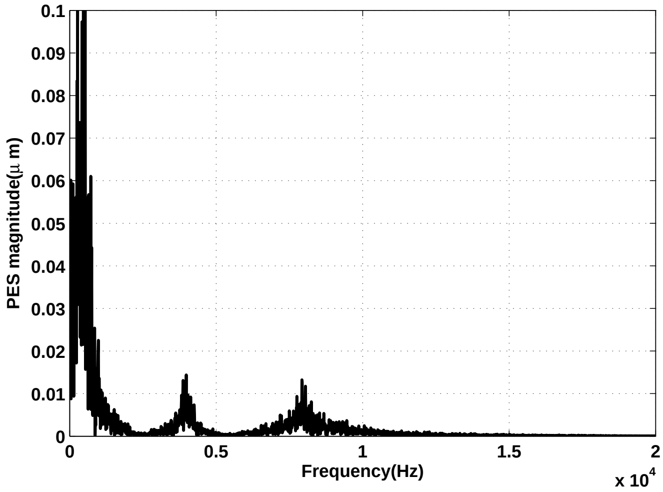

Figure 5. Power spectrum of a PES NRRO without servo control.

Table 2. Parameters of the output disturbance

mode i	k_{d_i}	ξ_{d_i}	ω_{d_i} (rad/s)
1	0.04	0.3	2π 500
2	0.005	0.03	2π 4000
3	0.01	0.05	2π 8000

(3) with $m = 3$. The parameters of the output disturbance model D_2 are shown in Table 2, and three main peaks at 500, 4k and 8k Hz are considered.

Theorem 3.3 is used to determine the damping ratio ξ_{p_i} and feedback gain G such that the H_2 performance $\|T_{z\bar{\omega}}\|_2$ of the modified plant (8a)-(8c) is minimized. After obtaining the damping parameters and the feedback gain, we can calculate the modified plant model $\tilde{P}(s)$. Here, we take two cases for example. As shown in Fig. 6, the modified plant$_1$ is obtained with the condition $\sum_{i=1}^{4} \xi_{p_i} < 2$, and the modified plant$_2$ is with the condition $\sum_{i=1}^{4} \xi_{p_i} < 4$. These conditions lead to the result that the modified plant$_2$ has larger damping parameters than the modified plant$_1$, and subsequently the modified plant$_2$ has a better H_2 performance with H_2 norm of 3.9435 than the modified plant$_1$ with H_2 norm of 4.4112. As for the original plant, its H_2 norm is 4.9703 nm and larger than the modified plants'.

As seen in Fig. 6, compared with the original plant, there exists slight phase lag between 7 kHz and 10 kHz, but phase lead between 10 kHz and 20 kHz for the damped modified plants. This little phase lag, which is much far away from the expected bandwidth seen in Table 3, will not affect the stability of the closed loop system. After all, the modified plants which are closer to the rigid mode will potentially lead to a better control performance than the original plant.

H_2 controller $\tilde{K}$ design is conducted for the above two modified plants as well as the original plant. The comparison of the control performances is summarized in Table 3. The modified plants $\tilde{P}(s)$ have higher phase margin, higher bandwidth and the same gain margin, compared with the original plant $P(s)$. The H_2 performance of the closed loop systems of the modified plants $\tilde{P}(s)$ is lower than that of the original plant $P(s)$. The sensitivity functions are also compared in Fig. 7. From this figure, we can see that the modified plants $\tilde{P}(s)$ have a higher bandwidth, but same sensitivity function hump, compared with the original plant $P(s)$. The power spectrum of the PES NRRO (y in Fig. 3) of the original plant and the modified plant$_2$ are compared in Fig. 8, from which we can observe that the modified plant$_2$ yields a considerable improvement

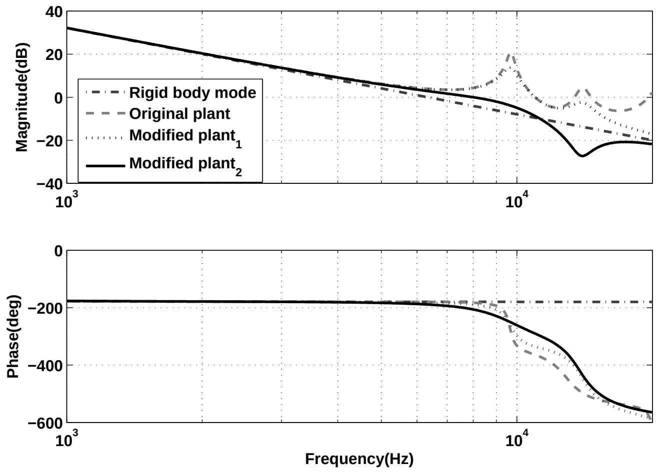

Figure 6. Comparison of the frequency responses of the rigid body mode, original plant and the modified plants.

in the low frequency up to 2 kHz. The H_2 norm of the modified plants is lower than the original plant, as shown in Table 3. Moreover, Fig. 6 shows that the modified plant$_2$ is closer to the rigid body model than the modified plant$_1$, which agrees well with their corresponding performances in both Table 3 and Fig. 7.

Table 3. Control performances comparison

Performances	Rigid body mode	Original plant	Modified plant$_1$	Modified plant$_2$
Open-loop 0dB crossover frequency (kHz)	1.82	1.44	1.51	1.70
Gain margin (dB)	7.96	7.96	7.96	7.96
Phase margin (deg)	30.6	28.2	28.3	29.4
H_2 performance (σ_z) (nm)	2.4927	2.8434	2.6921	2.5365

5. Conclusions

An LMI based approach to modify the damping ratios of the plant through the output feedback control has been developed in this paper. This method is to take the freedom of the plant property, which can guarantee that the modified plants have a better H_2 optimal performance than the original plant. Furthermore, we apply the result into the VCM plant in HDD to show the validity and effectiveness of the proposed method. Two modified plants have been proposed and compared with the original VCM plant. After the model modification, H_2 optimal controllers have been designed for each case. It has been shown that the proposed modified VCM plants have the lower H_2 performance, higher phase margin and bandwidth.

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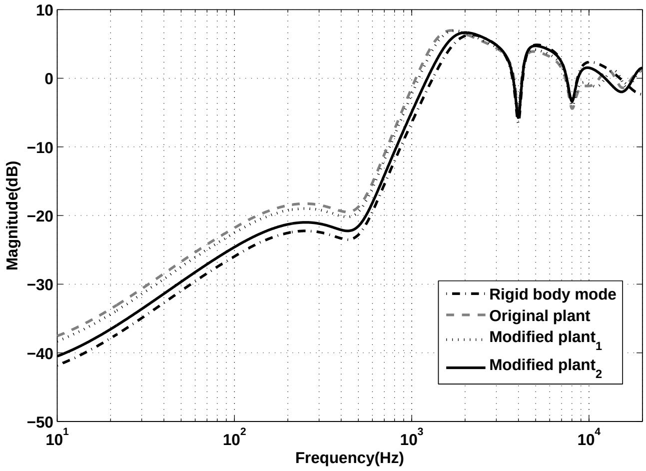

Figure 7. Sensitivity function comparison of the rigid body mode, the original plant and the modified plants.

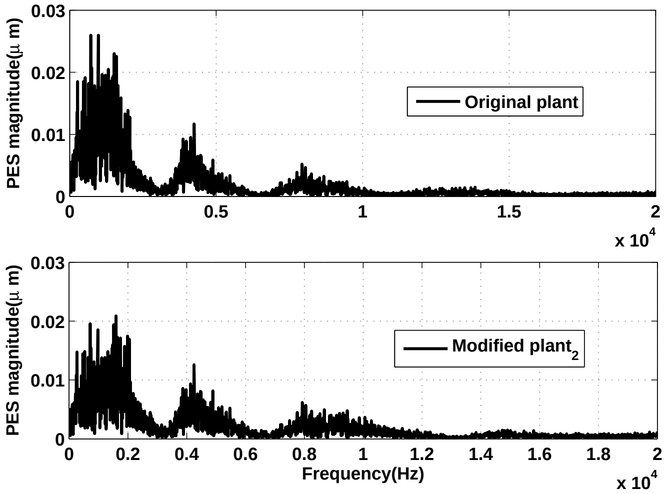

Figure 8. Power spectrum comparison of the PES NRRO with the original plant and the modified plant$_2$.

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