Residual stress field of ballised holes †

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Abstract

Ballising, involving pushing a slightly over-sized ball made of hard material through a hole, is a kind of cold working process. Applying ballising process to fastener holes produces compressive residual stress on the edge of the holes, and therefore increases the fatigue life of the components or structures. Quantification of the residual stress field is critical to define and precede the ballising process. In this article, the ballised holes are modeled as cold-expanded holes. Elastic-perfectly plastic theory is employed to analyze the holes with cold expansion process. For theoretical simplification, an axially symmetrical thin plate with a cold expanded hole is assumed. The elastic-plastic boundaries and residual stress distribution surrounding the cold expanded hole are derived. With the analysis, the residual stress field can be obtained together with actual cold expansion process in which only the diameters of hole before and after cold expansion need to be measured. As it is a non-destructive method, it provides a convenient way to estimate the elastic-plastic boundaries and residual stresses of cold worked holes. The approach is later extended to the case involving two cold-worked holes. A ballised hole is looked upon as a cold expanded hole and therefore is investigated by the approach. Specimens ballised with different interference levels are investigated. The effects of interference levels and specimen size on residual stresses are studied. The overall residual stresses of plates with two ballised holes are obtained by superposing the residual stresses induced on a single ballised hole. The effects of distance between the centers of the two holes with different interference levels on the residual stress field are revealed.

Keywords: Cold-working; Cold expanded holes; Ballising; Ballised holes; Residual stress; Fatigue life

1. Introduction

In engineering structures, holes and bores are widely employed for many purposes such as joining or bolting two simple plates together with bolts and nuts or precision assembly--joining several intricate parts together to form a functional component. In such cases, holes and bores are subjected to some form of load in service. Where holes are designed for load bearing, failure represents a critical factor in the design of structures and components. Where static load is concerned, sufficient knowledge is available to the design engineer. Of interest is the dynamic load acting on a hole, which causes failure at loads far below the yield point of the material. The plastic enlargement of a hole by an expanding tool is known as a cold-working process. With cold-working process, the roughness and hardness of the surface and the roundness of the hole can be improved. In addition, cold-working induces compressive residual stresses on the surface of the hole as a result of the interference between the hole and the expanding tool. Previously, Lai and Siew [1] investigated how three cold-working processes would affect the fatigue life of holes and revealed that ballising with interference between 1-3.3% enhanced the fatigue lives by about 60-220%. Pinho et al. [2] showed that cold-working reduced stress intensity factors for a plastically deformed region for both a remotely loaded plate containing an open hole and a riveted joint. Matos et al. [3] analyzed the effect of cold-working of rivet holes on the fatigue life of an aluminum alloy. Pavier et al. [4] investigated the effect of residual stress around cold worked holes on fracture under superimposed mechanical load. All the studies revealed cold-working reduces stress intensity factors and significantly improves fatigue performance of the cold worked holes to that of non-cold worked holes.

To investigate the effects on fatigue performance due to cold-working of holes, although a thorough understanding of surface quality (i.e., surface roughness and integrity) is essential for the correct specifications of the surfaces that will provide optimum properties for fatigue performance, the most critical factor may be the introduction of compressive residual stresses surrounding the holes. Hence, an understanding of the residual stress field of cold worked holes is necessary. This will be helpful in design where it is necessary to choose the

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distance between holes for joining wide plates together. If reliable fatigue life estimates are to be made, it is necessary to characterize the residual stress field.

A comprehensive review of the prominent theories covering residual stress around a cold worked hole was presented by Poolsuk and Sharp [5]. The theories predicted a region of compressive residual stress surrounding the hole. However, all the investigations assumed the unloading process as elastic releasing, missing the information of reversed yielding. Alder and Dupree [6] performed an elastic-plastic finite element analysis of the two-dimensional stress field around a cold worked hole, confirming that within the characteristic residual stresses, a great amount of discrepancy exists between the exact magnitude of the stresses and the location of the elastic-plastic boundary as reviewed by Mann and Jost [7]. Experimental investigations into the exact location of the elastic-plastic boundary were carried out by Poolsuk and Sharp [5] and Stanford and Link [8]. Although some discrepancies on the exact location of the elastic-plastic boundary were encountered, their work confirmed the existence of the elastic-plastic boundary. Lai et al. [9] and Oh et al. [10] adopted a successive cracking method introduced by Kang et al. [11] to measure the residual stress fields surrounding a ballised hole. This method should be combined with fracture mechanics and finite element analysis to evaluate the residual stress fields, which is suitable for experimental investigation of residual stresses of cold worked holes. More recently, Liu et al. [12] investigated the residual stress field of cold expanded holes of an aluminum plate using finite element modeling and experimentally confirmed the fatigue performance enhancement via cold expanding process. These investigations require a large number of experiments and numerical analysis and are not convenient to arriving at a general estimation of residual stresses of cold worked holes.

In the present study, a detailed stress analysis of a cold-worked hole and estimation of residual stresses surrounding the hole are worked out using elastic-plastic theory. With the analysis and actual cold-working process, only two parameters, the distances of hole before and after cold working, need to be measured to compute the residual stress field and elastic-plastic boundaries. A ballising process is looked upon as a cold–working process. Based on experimental data of ballising, the residual stress field of a ballised hole is presented and analyzed. On the basis of the theoretical estimation of residual stress of a single ballised hole, the residual stress fields of plate with two ballised holes are investigated. The approach presented arrives at a convenient estimation of residual stresses and elastic-plastic boundaries of cold worked holes.

2. Problem formulation and solutions

A cold-working process (schematically shown in Fig. 1) is an interference process which involves pushing or pulling an oversized mandrel through an internally lubricated split sleeve. With the cold-working process, the wall of the cold-expanded

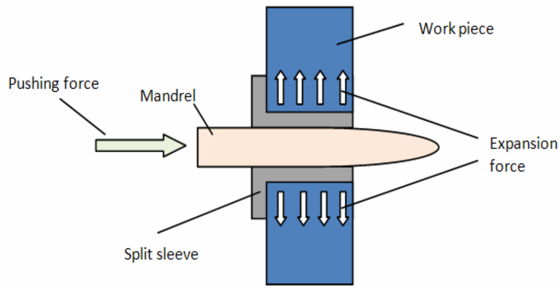

Fig. 1. Schematic of cold expanding process.

hole is brought with compressive residual stresses. The residual stress field can be estimated by elastic-plastic analysis of the circular plate with a central hole where the surface of the central hole is subjected to a uniform pressure and then the pressure is released. In the analysis, the material is assumed as ideally elastic-plastic where plastic deformation satisfies Tresca's yielding criterion.

2.1 Stresses surrounding the hole due to a uniform pressure

Referring to a circular plate with a central hole (diameter as $2a$) where a uniform radial pressure p is applied onto the surface of the hole, the pressure will probably result in elastic, elastic-plastic, or completely plastic deformation on the plate.

Consider a general case in the elastic region, the radial stress σ_r and the hoop stress σ_θ can be obtained by Lame's solution [13] as

$$\sigma_r = \frac{r_p^2 - p_e}{b^2 - r_p^2}\left(1 - \frac{b^2}{r^2}\right),\ \sigma_\theta = \frac{r_p^2 p_e}{b^2 - r_p^2}\left(1 + \frac{b^2}{r^2}\right)\quad r_p \le r \le b \tag{1}$$

where b is the outer radius of the circular plate, p_e the assumed radial pressure acting on the elastic-plastic boundary, and r_p the radius of the elastic-plastic boundary.

In the plastic region ($a \le r \le r_p$), where a is the initial radius of the central hole, the stresses can be obtained by the following differential equation and Tresca's yielding condition without considering strain hardening effects,

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \qquad \sigma_\theta - \sigma_r = \sigma_s$$

where σ_s is the yield stress of the plate material. The solutions are

$$\sigma_r = \sigma_s \ln\frac{r}{a} - p,\ \sigma_\theta = \sigma_s\left(1 + \ln\frac{r}{a}\right) - p \quad for \quad a \le r \le r_p. \tag{2}$$

At the elastic-plastic boundary, stress components obtained by Eqs. (1) and (2) are identical. Therefore, relationships among p_e, p, and r_p are obtained as

$$p_e = \frac{\sigma_s}{2}\left(1-\frac{r_p^{\ 2}}{b^2}\right), \quad p = \sigma_s \ln\frac{r_p}{a} + \frac{\sigma_s}{2}(1-\frac{r_p^{\ 2}}{b^2}). \tag{3}$$

It is obvious that when $r_p = a$, $p = \sigma_s/2(1-a^2/b^2)$ the whole plate will be within elastic state without plastic deformation. When $r_p = b$, then $p = \ln(b/a)$, the whole plate will enter into plastic state. It is convenient to denote $p = \sigma_s/2(1-a^2/b^2)$ as p_E, called elastic limit while $p = \ln(b/a)$ as p_P, the plastic limit.

2.2 Stress release due to unloading of uniform pressure p

Upon unloading the uniform pressure p, the released stresses σ_{ij}' can be calculated using elastic-perfect plastic theory to be dependent upon the value of p. However, the yielding criterion is different from that of loading. The new yielding condition is

$$\sigma_\theta' - \sigma_r' = 2\sigma_s \ . \tag{4}$$

(a) When the stress release follows completely elastic recovery, then

$$\sigma_r' = \frac{a^2 p}{b^2-a^2}\left(1-\frac{b^2}{r^2}\right), \quad \sigma_\theta' = \frac{a^2 p}{b^2-a^2}\left(1+\frac{b^2}{r^2}\right) \quad a \le r \le b. \tag{5}$$

(b) When the stress release follows elastic-plastic recovery, then, in the elastic region

$$\sigma_r' = \frac{r_p'^2 p_e'}{b^2-r_p'^2}\left(1-\frac{b^2}{r^2}\right), \quad \sigma_\theta' = \frac{r_p'^2 p_e'}{b^2-r_p'^2}\left(1+\frac{b^2}{r^2}\right) \quad r_p' \le r \le b, \tag{6}$$

where p_e' is an assumed radial pressure applied on the new elastic-plastic boundary, r'_p for unloading. In the plastic region, the stress components are given as

$$\sigma_r' = 2\sigma_s \ln\frac{r}{a} - p, \quad \sigma_\theta' = 2\sigma_s\left(1+\ln\frac{r}{a}\right) - p \quad a \le r \le r_p'. \tag{7}$$

Similarly, with the conditions that the stress components obtained by Eqs. (6) and (7) are identical at elastic-plastic boundary, the relationships among p'_e, p and r'_p can be written as

$$p_e' = \sigma_s\left(1-\frac{r_p'^2}{b^2}\right), \quad p = 2\sigma_s \ln\frac{r_p'}{a} + \sigma_s(1-\frac{r_p'^2}{b^2}). \tag{8}$$

When $r'_p = a$, then $p = \sigma_s(1-a^2/b^2)$. This means the unloading process completely follows elastic recovery while the uniform pressure is smaller than $\sigma_s(1-a^2/b^2)$. Denoting $p=\sigma_s(1-a^2/b^2)$ as p_E', p_E' is known as the unloading elastic-plastic limit.

During the loading process, the uniform pressure cannot exceed the plastic limit p_P. When $p \le p_E'$, there will be no yielding upon unloading. When $p_E' \le p \le p_P$, a new plastic region will be formed at the unloading of the uniform pressure p, depending on the critical ratio b/a. Let $p_P = p_E'$, i.e.,

$$\sigma_s \ln(b/a) = \sigma_s\left(1-a^2/b^2\right). \tag{9}$$

Solving Eq. (9) gives the critical ratio of approximately 2.22. When $b/a \ge 2.22$, $p_E' \le p_P$. If $p_E' \le p \le p_P$, secondary yielding occurs within a radius between the edge of the hole and the unloading elastic-plastic boundary r_p'. If $b/a \le 2.22$, then $p_E' \ge p_P$, there will be no secondary plastic deformation on the condition of $p \le p_P$.

2.3 Solutions for residual stress field

In general, if the stress components by loading process and unloading process are respectively expressed as σ_{ij} and σ_{ij}', then the residual stress components σ_{ij}^R can be written as

$$\sigma_{ij}^R = \sigma_{ij} - \sigma_{ij}'. \tag{10}$$

When the uniform pressure p is less than the elastic limit p_E, no plastic deformation will be produced. Upon unloading of the load p, the stresses will be totally released and hence no residual stresses will be induced.

When p is greater than the elastic limit, plastic deformation will be induced within the plate. The stress components in the elastic region follow Eq. (1), while the stress components in the plastic region satisfy Eq. (2).

On the release of p, for $b/a \le 2.22$, i.e., $p_P \le p_E'$, or $b/a \ge 2.22$, i.e., $p_P \ge p_E'$ and $p_E \le p \le p_E'$, the released stress components follow the elastic recovery as expressed in Eq. (5). The residual stresses are given as

$$\sigma_r^R = \begin{cases} \sigma_s \ln\dfrac{r}{a} - \dfrac{b^2 p}{b^2-a^2}\left(1-\dfrac{a^2}{r^2}\right) & a \le r \le r_p \\ \left(\dfrac{\sigma_s r_p^{\ 2}}{2b^2} - \dfrac{a^2 p}{b^2-a^2}\right)\cdot\left(1-\dfrac{b^2}{r^2}\right) & r_p \le r \le b \end{cases}, \tag{11}$$

$$\sigma_\theta^R = \begin{cases} \sigma_s\left(1+\ln\dfrac{r}{a}\right) - \dfrac{b^2 p}{b^2-a^2}\left(1+\dfrac{a^2}{r^2}\right) & a \le r \le r_p \\ \left(\dfrac{\sigma_s r_p^{\ 2}}{2b^2} - \dfrac{a^2 p}{b^2-a^2}\right)\cdot\left(1+\dfrac{b^2}{r^2}\right) & r_p \le r \le b \end{cases}. \tag{12}$$

When $b/a \ge 2.22$ and $p_E' < p < p_P$, there will be elastic deformation and a newly produced plastic deformation. In the elastic region, the released stress components are given in Eq. (6), while in the plastic region, Eq. (7) is followed. The residual stresses are written as

$$\sigma_r^R = \begin{cases} -\sigma_s \ln\dfrac{r}{a} & a \le r \le r_p' \\ \sigma_s \ln\dfrac{r}{a} - p - \dfrac{r_p'^2\sigma_s}{b^2}\left(1-\dfrac{b^2}{r^2}\right) & r_p' \le r \le r_p \\ \left(1-\dfrac{b^2}{r^2}\right)\dfrac{\sigma_s\left(r_p^{\,2}-2r_p'^2\right)}{2b^2} & r_p \le r \le b \end{cases}. \quad (13)$$

$$\sigma_\theta^R = \begin{cases} -\sigma_s\left(1+\ln\dfrac{r}{a}\right) & a \le r \le r_p' \\ \sigma_s\left(1+\ln\dfrac{r}{a}\right) - p - \dfrac{r_p'^2\sigma_s}{b^2}\left(1+\dfrac{b^2}{r^2}\right) & r_p' \le r \le r_p \\ \left(1+\dfrac{b^2}{r^2}\right)\dfrac{\sigma_s\left(r_p^{\,2}-2r_p'^2\right)}{2b^2} & r_p \le r \le b \end{cases}. \quad (14)$$

2.4 Solutions for displacements

To obtain the value of residual stresses σ_r^R and σ_θ^R given in Eqs. (11) to (14), the values of p, r_p and r'_p should be known. From elastic-plastic theory, even though the relations between p and r_p or p and r'_p have been depicted in Eqs. (3) and (8), one more relation is needed to evaluate the values of p, r_p and r'_p. This may be fulfilled with the aid of solutions from the radial displacements of the surface of the hole.

2.4.1 Displacement at elastic region

Under the conditions of plane stress, during the loading process, for partly elastic-plastic deformation, the displacement in the elastic region is given by [13],

$$u(r) = \frac{r_p^{\,2} p_e}{E\left(b^2 - r_p^{\,2}\right)}\left[\frac{(1+\nu)b^2}{r} + (1-\nu)r\right] \quad (15)$$

and

$$p_e = \frac{\sigma_s}{2}\left(1 - \frac{r_p^{\,2}}{b^2}\right)$$

therefore

$$u(r) = \frac{r_p^{\,2}\sigma_s}{2E\cdot b^2}\left[\frac{(1+\nu)b^2}{r} + (1-\nu)r\right]. \quad (16)$$

When $r_p = a$, then $p_e = p_E$. No plastic deformation occurs during the loading process when $p \le p_E$. Therefore, no residual stress is induced.

For $p_E \le p \le p_E'$, the loading process leads to elastic-plastic deformation while the unloading follows an elastic recovery. The recovered displacement due to the unloading process is given as

$$u(r) = \frac{a^2 p}{E\left(b^2 - a^2\right)}\left[\frac{(1+\nu)b^2}{r} + (1-\nu)r\right]. \quad (17)$$

2.4.2 Displacement at plastic region

The displacement in the plastic region can be obtained by considering that the volume strain $\varepsilon_r+\varepsilon_\theta+\varepsilon_z$ is elastic through the plate. The displacement is expressed as [13]

$$\frac{u}{r} = -\nu\varepsilon_z + (1-\nu)\frac{kc^2}{Gr^2} + (1-2\nu)\frac{\sigma_r}{2G} \quad (18)$$

where $c = r_p$ is the elastic-plastic boundary, $k = \sigma_s/2$, $G = E/2(1+\nu)$ is the shear modulus. The formula above can be further written as

$$u(r) = \frac{(1-2\nu)(1+\nu)}{E} r\sigma_r - \nu\varepsilon_z r + \frac{(1-\nu^2)\sigma_s}{Er} r_p^2. \quad (19)$$

This solution is valid throughout the plate. At the elastic-plastic boundary, it must coincide with that given by

$$u = r\varepsilon_\theta = \frac{r}{E}\left(\sigma_\theta - \nu\sigma_r\right) = \frac{1+\nu}{E}\left[(1-\nu)\sigma_\theta - \nu\sigma_r\right]r - \nu\varepsilon_z r. \quad (20)$$

For the loading process, substituting σ_r, σ_θ from Eq. (1) at $r = r_p$ into Eqs. (19) and (20), and using Eq. (3), leads to

$$u(r) = \frac{1-\nu}{E} r\sigma_r + \frac{\sigma_s r_p^2}{E\cdot r}. \quad (21)$$

Similarly, for the unloading process, substituting σ_r, σ_θ from Eq. (6) at $r = r'_p$ into Eqs. (19) and (20), by Eq. (8), leads to

$$u(r) = \frac{1-\nu}{E} r\sigma_r + \frac{2\sigma_s r_p'^2}{Er}. \quad (22)$$

2.5 Computational procedure

To determine the residual stresses of a cold worked hole, it is necessary to measure the diameter of the hole before and after the cold-working process, and to determine the appropriate equations to employ. The procedure involves some judgments on the uniform pressure p produced by the cold-working process to judge whether it has produced plastic deformation and whether the unloading process will induce secondary plastic deformation.

Let a and a_2 respectively denote the radius of the hole before and after the cold-working process, and a_1 denote the radius of the expanding tool. During the cold-working process, the radial displacement due to the loading process is assumed as a_1-a and that due to the unloading process is a_1-a_2. Making use of the expressions for radial displacement and the relationship between p and r_p or r'_p, the values of r_p and r'_p

can be obtained, and p can then be calculated. With these values, the residual stresses are estimated according to Eqs. (11) to (14).

2.5.1 Determination of p according to the process of elastic recovery

For $p_E \leq p \leq p_E'$, the loading process leads to elastic-plastic deformation and the unloading follows a completely elastic recovery. The recovered displacement due to unloading process follows Eq. (17). When $r = a$, then $u = a_1 - a_2$. With Eq. (17), it can be obtained that

$$p = \frac{E(a_1 - a_2)(b^2 - a^2)}{a\left[(1+\nu)b^2 + (1-\nu)a^2\right]}. \tag{23}$$

2.5.2 Determination of r_p according to loading process

On the loading process no plastic deformation will be induced when $p \leq p_E$. The material will totally recover to its original state on the exit of the load. When $p \geq p_E$, yielding will be produced around the hole. Within the plastic zone, the radial displacement is given as Eq. (21), i.e.

$$u(r) = \frac{1-\nu}{E} r\sigma_r + \frac{\sigma_s r_p^2}{Er} \qquad a \leq r \leq r_p \tag{24}$$

where

$$\sigma_r = \sigma_s \ln\frac{r}{a} - p, \tag{25}$$

$$p = \sigma_s \ln\frac{r_p}{a} + \frac{\sigma_s}{2}\left(1 - \frac{r_p^2}{b^2}\right). \tag{26}$$

When $r = a$,

$$u(a) = a_1 - a \quad \sigma_r = -\sigma_s \ln\frac{r_p}{a} - \frac{\sigma_s}{2}\left(1 - \frac{r_p^2}{b^2}\right). \tag{27}$$

Substituting them into Eq. (24), r_p may be given as

$$A + B\ln\frac{r_p}{a} = Cr_p^2 \tag{28}$$

where

$$A = Ea(a_1 - a) + \frac{\sigma_s}{2}(1-\nu)a^2, \tag{29}$$

$$B = (1-\nu)a^2\sigma_s \tag{30}$$

$$C = \frac{\sigma_s a^2}{2b^2}(1-\nu) + \sigma. \tag{31}$$

From Eq. (28), r_p can be calculated. With r_p known, p and r_p' can be determined as well.

2.5.3 Determination of r_p' by the unloading process

When $p_E' \leq p \leq p_P$, the unloading process does not follow the elastic recovery, but a different plastic deformation is induced. This plastic boundary has been defined as r_p'. Within the plastic zone, the radial displacement is written as

$$u(r) = \frac{1-\nu}{E} r\sigma_r' + \frac{2\sigma_s r_p'^2}{Er} \tag{32}$$

where

$$\sigma_r' = 2\sigma_s \ln\frac{r}{a} - p, \tag{33}$$

$$p = 2\sigma_s \ln\frac{r_p'}{a} + \sigma_s\left(1 - \frac{r_p'^2}{b^2}\right). \tag{34}$$

When $r = a$,

$$u(a) = a_1 - a_2, \quad \sigma_r' = -2\sigma_s \ln\frac{r_p'}{a} - \sigma_s\left(1 - \frac{r_p'^2}{b^2}\right). \tag{35}$$

Substituting these into Eq. (32), r_p' may be written as

$$A' + B'\ln\frac{r_p'}{a} = C'r_p'^2 \tag{36}$$

where

$$A' = E \cdot a \cdot (a_1 - a_2) + (1-\nu)\sigma_s a^2, \tag{37}$$

$$B' = 2(1-\nu)\sigma_s a^2, \tag{38}$$

$$C' = 2\sigma_s + (1-\nu)\sigma_s \frac{a^2}{b^2}. \tag{39}$$

2.6 Residual stresses of two cold worked holes

In engineering structure and components, not only are bores and holes used for joining purposes such as bolting two simple plates together with bolts and nuts, but also expanded to arrays of holes joining a wide plate to another that constitutes the body of a structure. This leads to the familiar rivet holes. In these cases, it is necessary to understand the overall residual stress fields of plate with multiple cold worked holes. One of the objectives in the present work is to understand the overall residual stress fields of plates with multiple cold worked holes.

An array of holes may be simplified by a model of plate with two cold worked holes with the same size and same interference. In the present investigation, the residual stress field of a plate with two cold worked holes is obtained by a numerical superposition of the residual stress field induced by each single cold worked hole. We note that for an arbitrary point in the plate, it is not likely to superpose the residual stress field of each single ballised hole, as the stress components (σ_r^R, σ_θ^R) are in different directions for the two ballised

Table 1. Material properties of ballised specimen.

Condition	Hardness	Young's modulus	Poisson ratio	Yield stress
Unannealed	210	1.83×10^5 (MPa)	0.193	340-350 (MPa)

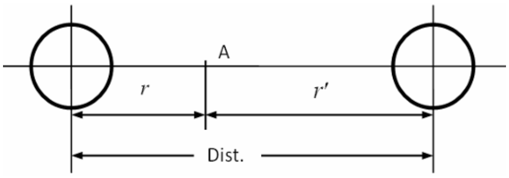

Fig. 2. Two cold worked holes.

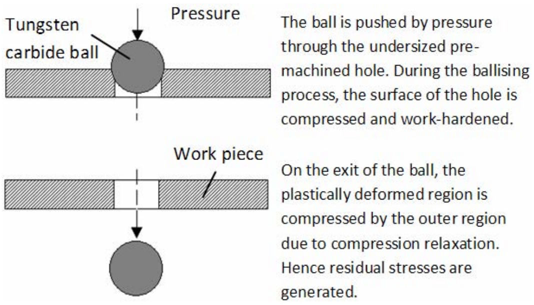

Fig. 3. A schematic of ballising process.

holes. However, in any points in the straight line between the two holes, the directions of stress components for the two ballised holes are identical, and therefore the superposition method applies. It is reasonable as long as the sum of residual stress does not exceed the material yield stress. Consequently, the overall residual stress field will be affected significantly by two factors: the interference and the distance between the centers of the two holes.

Let point A be on the straight line between the centers of the two neighboring holes as shown in Fig. 2. *Dist*. stands for the distance between the centers of the two holes. r and r' are the radial distances of point A to the centers of the left hole and right hole respectively. Assuming that residual stresses induced by each cold worked hole are not affected by cold-working of the other hole, Eqs. (11)-(14) can be used to calculate the residual stress field induced by the cold worked hole on the left (S_L) and those induced by cold worked hole on the right (S_R) at this point. The total residual stress due to the two cold worked holes at point A would be

$$S = S_L + S_R . \tag{40}$$

3. Residual stresses of ballised holes

3.1 Ballising process

Ballising is a finishing process that involves pushing a hardened spherical ball through a slightly under-sized hole without rotation, as shown in Fig. 3. The ballising process is looked upon as a cold working process, and thus the ball is looked upon as the expanding tool. Several specimens of material ASS760B steel rectangle plate, with the dimension 200×76×6 mm and the initial central holes close to 19.00 mm, are shown in Fig. 4. The material properties of the ballised specimen are listed in Table 1. It is noted that the theoretical analysis has been derived on the assumption of circular plate with central hole, whereas the specimens in the actual experimental work are rectangular plates with central holes. This may produce some discrepancy between the theoretically evaluated and the actual stress states. The dimensions of the specimen are chosen in accordance with previous work done by Oh et al. [10] so that comparison could be made. Theoretical calculations of the elastic-plastic boundary are carried out to ascertain the width of the test plate. In the computation, a ball diameter of 19.05 mm (3/4 inch) is chosen as a_I while half width of the plate is set as b, the outer radius of the assumed circular plate. The interference between the ball and the hole is defined as

$$I = \frac{ball\,diameter - initial\,hole\,diameter}{initial\,hole\,diameter} \times 100\% . \tag{41}$$

Based on experimental data of the ballising process, Table 2 shows the diameters before and after ballising for three specimens and the calculated interferences. The effects of interference level and specimen size on residual stresses are studied. The overall residual stresses of plate with two ballised holes are obtained by numerically superposing the residual stresses induced on single ballised hole. The distance between the centers of the two holes is varied in the computation for different interference levels.

Table 2. Diameters of holes before and after the ballising process.

No. of specimen	Diameter before ballising (mm)	Diameter after ballising (mm)	Interference (%)
1	18.77	19.00	1.492
2	18.72	18.99	1.763
3	18.44	18.91	3.308

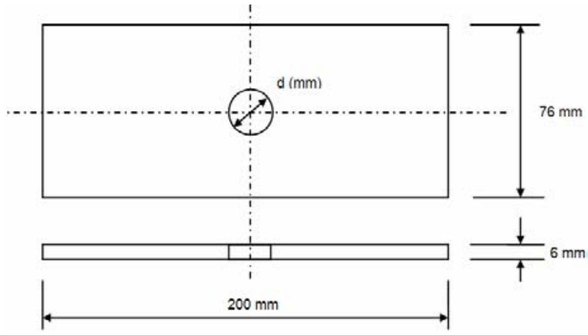

Fig. 4. The designed specimen used for ballising process.

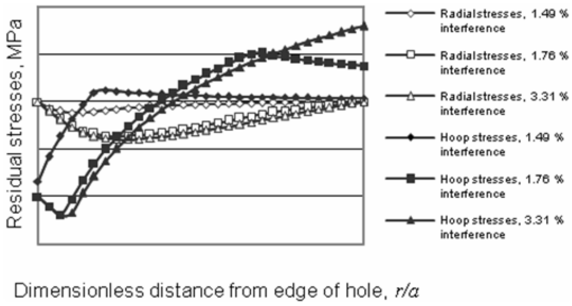

Fig. 5. Residual stresses of ballised holes.

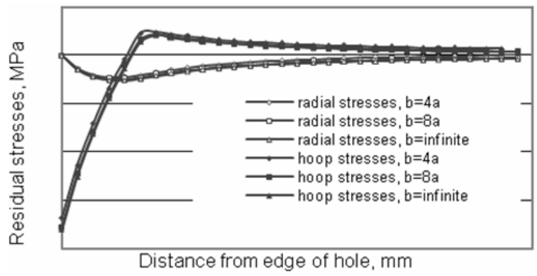

Fig. 6. Residual stresses for different plate size with interference of 1.49%.

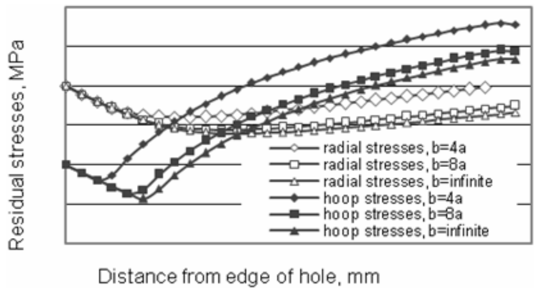

Fig. 7. Residual stresses for different plate size with interference of 3.31%.

3.2 Results and discussion

3.2.1 Residual stresses of single ballised hole

The residual stresses of a ballised hole, with interference levels of 1.49, 1.76 and 3.31%, are shown in Fig. 5, where *b* equals 4*a* (the outer radius of the plate is four-times the radius of the hole). The residual radial stresses are always compressive, zero at the edge of the hole, reach a minimum value (maximum compression) between the edge of the hole and the elastic-plastic boundary and then return to zero. The maximum compressive stresses can be observed to increase with increasing interference level.

For the residual hoop stresses, it can be seen that the extent of the plastic deformation during expansion characterizes the residual stress distribution. The maximum tensile hoop stress occurs at the elastic-plastic boundary. At low level of interference, the maximum residual compressive hoop stress occurs at the edge of the hole. With increasing level of interference, the maximum compressive hoop stress tends to move progressively away from the edge of the hole. More accurately, the maximum compressive stress occurs at the unloading elastic-plastic boundary r'_p.

From the present computed results, r'_p begins to form when the interference level is between 1.49% and 1.76%. The residual hoop stresses at the edge of the hole always remain the same irrespective of the different interference levels and their values equal the compressive yielding stress $(-\sigma_s)$ of the material. The maximum compressive hoop stress also remains constant at the unloading elastic-plastic boundary but is displaced slightly further away from edge of the hole as described by Eqs. (13) and (14).

Figs. 6 and 7 show the computed residual stress of ballised holes with different outer radius of the circular plate for the interference levels of 1.49% and 3.31%, respectively. The effects of the outer radius on residual stress field can be considered as that of plate size, such as the half width of the plate, on the residual stresses. Increasing the size of the plate does not significantly affect the elastic-plastic boundary and slightly increases the unloading elastic-plastic boundary at interference level of 3.31%. However, the magnitudes of the residual stress field change with the increase in outer radius. Generally, increasing the outer radius of the plate decreases the residual tensile stresses but increases the residual compressive stresses. This is more obvious at higher interference level and beneficial for the purpose of fatigue life enhancement. The phenomenon is probably caused by the increase in constraint of the plate, making it more resistant to external loads. At interference level of 1.49%, changing the specimen size results in only slight changes in the residual stresses. On the other hand, at interference level of 3.31%, the change in residual stresses is dramatic.

Note that the solutions are obtained by axisymmetric assumption, while the actual specimen is a rectangular plate with a central hole. The half width of the plate is used as the outer radius of the assumed circular plate. The size of the finite plate may affect the computation accuracy. For smaller interference level at 1.49%, the effects of specimen size are insignificant. For higher interference level at 3.31%, the specimen size has significant effect on the residual stress field. However, it can be seen from Fig. 7 that with further increasing the specimen size to $b=8a$ and above, the results obtained for the finite circular plate are close to that of an infinite plate.

3.2.2 Residual stresses of two ballised holes

In a point in the straight line between the two holes, the overall residual stresses of a plate with two ballised holes are obtained by numerical superposition of the residual stresses induced by each single ballised hole. The residual stress field of two ballised holes is significantly affected by the level of interference and the distance between the centers of the two holes. In the computation, the hole size and interference of each of the two ballised holes were assumed to be identical. As shown in Figs. 8-10, the residual radial stresses for plates with two ballised holes are always compressive for different

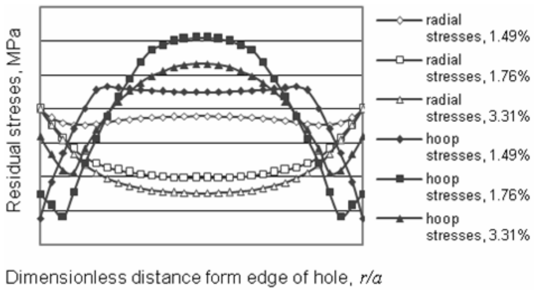

Fig. 8. Residual stresses of two ballised holes (*Dist.* = 5*a*), with interference of 1.49, 1.76, 3.31%.

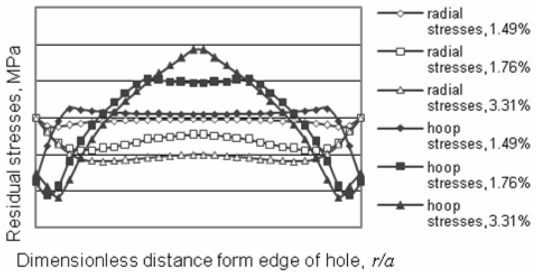

Fig. 9. Residual stresses of two ballised holes (*Dist.* =8*a*), with interference of 1.49, 1.76, 3.31%.

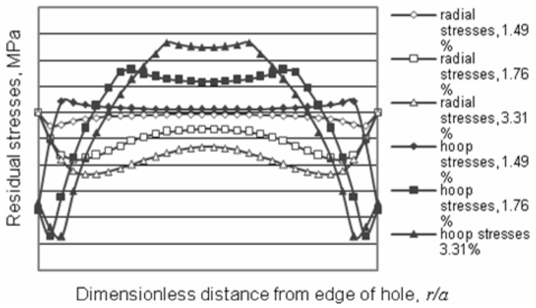

Fig. 10. Residual stresses of two ballised holes (*Dist.* =10*a*), with interference of 1.49, 1.76, 3.31%.

interference levels. With increasing level of interference, the absolute magnitude of the compressive stresses increases. This trend is the same as that of plate with single ballised hole. With increase in the distances between the two holes, the positions of maximum stress tend to move away from the hole surface, while the magnitude of maximum compressive stress changes slightly.

Residual hoop stresses for plates with two ballised holes are more complicated than those in the radial direction. For small interference such as 1.49%, no plastic deformation occurs on the unloading process. The maximum compressive stress is at the edge of hole. It then increases and reaches maximum tensile at the elastic-plastic boundary and then tapers off. The tensile stress is gentler in comparison to the compressive stress induced at the edge of hole. This trend is similar to that of a plate with single ballised hole. When the interference level is increased to the extent when unloading plastic deformation takes place, such as 1.76 and 3.31%, the higher interference level does not always correspond to high maximum compressive and maximum tensile stresses. This is not the same as that for the plate with a single ballised hole. From Fig. 8, for *Dist.* = 5*a*, i.e., the distance between two holes is five-times that of the hole radius, the maximum interference level appears to give the lowest absolute compressive residual hoop stress at the edge of the hole, whereas the lowest interference level corresponds to the highest absolute compressive stress value due to the effects of superposition. By considering the superposing effect, the maximum tensile state is probably not at the elastic-plastic boundary. The tensile stress curve will probably contain one or two peak values, depending on the distance between the two holes and the interference level. Figs. 9 and 10, for the case with *Dist.*= 8*a* and 10*a,* respectively, show the change in the overall stress pattern from the case where *Dist.* = 5*a*. From Fig. 9, the interference level of 1.76% yielded two peak positions of tensile state, at the elastic-plastic boundary of each ballised hole, while for the interference of 3.31% the residual hoop stress curve peaks at one position, and manifests among the three interference levels the highest maximum tensile stress. The reason for such occurrence is that the distance between the centers of the two holes is nearly two-times that of the elastic-plastic boundary induced by each ballised hole. For a ballised hole, the elastic-plastic boundary corresponds to maximum tensile stress, and therefore, at a distance corresponding to two times the elastic-plastic boundary, the tensile stress of each ballised hole is superimposed to give two maximum tensile stresses. With the distance between the centers of the two holes increasing further (*Dist.* = 10*a* for example), as shown in Fig. 10, even for an interference of 3.31%, the two peak positions for the tensile stresses are still being manifested. The non-coincidence of the peak stresses leads to a decrease in maximum superposed tensile stress. If the spacing between the two holes was large enough, the overall stress field of the plate would return to a state where the residual stresses of the two ballised holes would not have any interaction with each other.

4. Discussions

4.1 Validity of the model

The purpose of this study is to investigate the residual stress fields around ballised holes. However, the theoretical formulation is derived on the basis of cold expansion of hole where a uniform pressure acts on the edge of the hole and then is released. With the information involved in the ballising process, the residual stress fields of ballised holes can be conveniently obtained. The theory is basically valid for ballised holes. In the ballising process, a hardened ball is forced through an undersized hole. As the ball enters the hole, the region of the wall immediately ahead of the ball is subjected to a force acting at some angle not parallel to the wall of the hole as shown in Fig. 11. The force can then be resolved into two mutually perpendicular components, one acting on a plane normal to

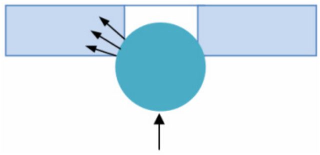

Fig. 11. Mechanics of the ballising process.

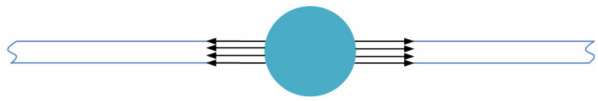

Fig. 12. Approximation of ballising a thin plate to a state of pure shear.

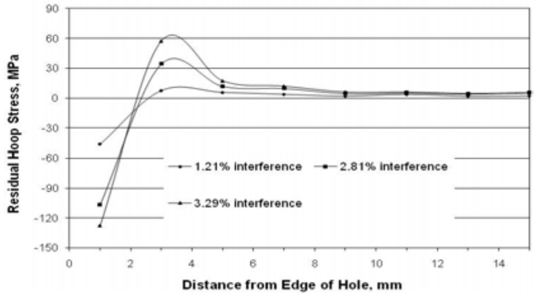

Fig. 13. Residual hoop stress of ballised holes obtained by successive cracking method (Oh et al., 1993).

the direction of the hole (radial force) and the other parallel to the hole direction (longitudinal force). As the ball penetrates further, the direction of the force acting on the hole becomes increasingly more normal to the wall of the hole, evidently due to the curvature of the surface of the ball-assuming negligible or no deformation of the ball. The maximum radial force, and therefore deformation, is achieved when the full diameter of the ball enters the hole. The force acting on the normal plane at this point is purely radial and therefore in a state of pure shear, as shown in Fig. 12. As the ball travels along the hole, every element in the normal plane ahead of the ball will be subjected to identical stress state, and therefore the ballsing process is essentially a ring of travelling pressure where the stress state of the element immediately ahead of the ball changes from three dimensional to pure shear. Similarly, elastic recovery on the exit of the ball can be approximated as pure shear. The above assumption approximates the ballising process well if ploughing of material ahead of the ball can be neglected.

4.2 Comparison with previous work

Before the present study, the only previous investigation on residual stress distribution of ballised holes was carried out by using fracture mechanics approach by Oh et al. [10]. In the approach, the relationship between the residual stress, stress intensity factor and displacement associated with release of residual stress by the introduction of a crack at the edge of the hole was developed. The displacement as the crack was extended incrementally was used to calculate the residual stress field around the hole. Fig. 13 shows the results of the residual hoop stress obtained by this method. The results are in agreement in the residual stresses trend with those obtained in the current research, although some discrepancies exist. The differences lie in the locations and the magnitude of the maximum tensile and compressive stresses. The discrepancies may probably be due to experimental inaccuracy, while the current study refers to an ideal cold-expansion of the holes. In addition, the experimental method was based on linear elastic fracture mechanics which may not be suitable for ductile material such as ASSAB 760, a medium carbon steel. As X-ray diffraction can only measure the residual stresses on the surface (1-2 μm in depth) of a plate, the successive crack method appears to be the only experimentally feasible way to measure the mean value of the residual stress along the depth of a hole with high gradients on the surface. The theoretical model set up in this study, however, is a convenient way to estimate the residual stresses of ballised and other cold worked holes. It can easily be applied to the holes with different dimensions. The future study on residual stress field of ballised hole would consider the strain hardening effects of different material, and explore effective experimental approach for residual stress measurement as well.

Finite element modeling and surface stress measurement using X-ray on cold expanded holes by Liu et al. [12] revealed similar results as presented in this paper. That is, the elastic-plastic boundary is increased with the increasing of the interference level, and an unloading elastic-plastic boundary is observed. The differences in the computed residual stress values and the locations of the elastic-plastic boundaries are due to the fact that the materials used for the analysis in the current paper and finite element modeling are different. In the current research, the material used is ASS760B steel, while in the finite element modeling it is aluminum.

5. Conclusions

This study investigates the residual stress distribution around cold worked holes. Theories of elasticity and plasticity are employed to derive the residual stress fields and elastic-plastic boundaries. The ballising process is considered a cold working process, and hence the residual stresses of ballised holes are obtained. The method presented in this study is a non-destructive method, and therefore offers a convenient way to obtain an estimation of the residual stresses and elastic-plastic boundaries. This is later extended to the case involving two ballised holes. Through ballising several specimens and theoretical computation, the following conclusions can be made.

The study confirms that cold-working, e.g., ballising, introduces large compressive residual stresses onto the surface of a cold worked hole, and thus has the potential to give significant enhancement in the fatigue performance of the hole.

The residual radial stress of a ballised hole is always compressive. The value is zero at the edge of the hole, attains a minimum value (maximum for compressive) between the edge of the hole and the elastic-plastic boundary, and then goes towards zero. The maximum compressive stress increases with the increase in interference. Residual hoop stress is compressive at the edge of the hole. The stress decreases in magnitude away from the edge of the hole or at the location of maximum compressive stress, changes further to a tensile residual stress. It reaches a maximum tensile state at the elastic-plastic boundary and then tapers off.

A critical interference value exists between 1.49% and 1.76%. When the interference exceeds the critical value, reversed yielding occurs at the unloading of the cold working process.

The size of the plate does not significantly affect the elastic-plastic boundary and the unloading elastic-plastic boundary. Increasing the plate size merely produces some changes in the computed values in that the residual tensile stress decreases while the residual compressive stress increases in magnitude. This trend is more beneficial for the purpose of fatigue life enhancement.

The overall residual stresses of two cold worked holes are governed by the interference and the distance between the two holes. For identical hole sizes and interference, when the spacing is two-times the computed elastic-plastic boundary of each cold worked hole, a superposing of the maximum residual hoop tensile stresses occurs.

Nomenclature

a : Initial radius of hole
a_1 : Radius of expanding tool
a_2 : Radius of hole after cold-*working*
b : Outer radius of circular plate
r : Radius from the center of the hole
σ_r : Radial stress due to the expanding process
σ_θ : Hoop stress due to the expanding process
σ'_r : Radial stress due to the unloading process
σ'_θ : Hoop stress due to the unloading process
σ_r^R : Residual radial stress
σ_θ^R : Residual hoop stress
p : Radial pressure acting on the hole produced by the expanding
r_p : Radius of elastic-plastic boundary due to the expanding
r'_p : Radius of elastic-plastic boundary due to the unloading
p_e : Assumed radial pressure acting on the elastic-plasticboundary
σ_s : Yielding stress
E : Young's modulus
G : Shear modulus
ν : Poisson's ratio
p_E : Pressure of elastic limit
p_E' : Pressure of unloading elastic limit
p_P : Plastic limit, maximum radial pressure that the plate can withstand
p_e' : Assumed radial pressure acting on the unloading elastic-plastic boundary
I : Interference between expanding tool and hole
σ_{ij} : Stress components due to expanding process
σ'_{ij} : Released stress components due to unloading process
σ_{ij}^R : Residual stress components
$u(r)$: Radial displacement expression
ε_r : Radial strain
ε_θ : Hoop strain
ε_z : Axial strain
Dist. : Distance between centers of two ballised holes

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