Distributed Discrete Level Energy Scheduling for Residential Load Control in Smart Grids

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Abstract— **An optimization framework is presented for distributed discrete level energy scheduling from the energy provider's perspective, which maximizes general total utility functions for multiple residential users, for a general total energy cost constraint and energy generation constraint. However, the load selection criteria involves integer constraints, and results in a mixed integer programming problem which is difficult to solve. We develop a novel approach to derive new closed form optimal solutions for load selection and energy scheduling. A new distributed scheme that maximizes the total utility is proposed by exploiting two-way communications between the energy provider and each residence. The solutions for distributed load selection and discrete level energy scheduling optimize individual utility from each user's perspective, as well as their total utility. Moreover, the proposed scheme also maximizes the total revenue or sale at the energy provider through pricing control, a desirable objective which has been neglected in previous works.**

I. INTRODUCTION

In smart grid infrastructures, smart meters are used to enable two-way communications and electricity flow between the energy providers and the users. Smart meters are usually installed at the users' premises or residences to monitor and manage energy consumption, and to report energy consumption profiles throughout the day to the energy providers. Energy providers can also send pricing information from their databases to the smart meters. By equipping each residence with a energy management system (EMS) with sufficient computational power, the energy providers can provide suitable energy scheduling and pricing control, while adapting to the energy consumption patterns of various electrical appliances or devices, to maximize users utility or satisfaction and minimize the electricity bill.

In general, recent research works on the energy scheduling for residential users and pricing control for smart grids can be categorized as either from the users' perspective or from the energy provider's perspective. From users' perspective, the commonly adopted objective is either to maximize the user's utility, minimize the total electricity bill, or minimize the waiting time based on predicted electricity pricing. In [1] [2] [3], the authors study energy scheduling or power dispatch to multiple users using game-theoretic approach. However, the users' satisfaction or welfare is neglected in these works, and no closed form solution has been presented for load control or energy scheduling.

From the energy provider's perspective, several prior studies on energy scheduling and real-time pricing control for multiple residential users have adopted the objective that maximizes the total social welfare across the users [4] and the energy provider, such as [5] [6] [7] [8] and references therein. More specifically in [5] [6] [8], the objective function for optimization is defined as a linear combination of total utility and total cost of energy or electricity. Distributed scheduling algorithms and convergence results are presented in [8] for the scenario of incomplete advance metering infrastructure messages. However, in all these works, the residential users have to fix both starting and ending times of their loads prior to the execution of the proposed schemes.

In this paper, we derive new distributed load selection and discrete level energy scheduling scheme for multiple residential users by exploiting two-way communications between the energy provider and multiple users. We formulate the problem setup from the energy provider's perspective, which maximizes the total utility at the users, given a total energy cost constraint and a contracted energy generation constraint at the energy provider.

The contributions in this work are summarized as follows. Firstly, we consider discrete level energy scheduling with variable starting time and termination time, and take into account variable starting and ending times of elastic loads which depend on the users' need. However, our consideration results in a mixed integer programming problem, which is difficult to solve, especially in a distributed way.Secondly, we develop a novel approach to reformulate and solve this problem to derive closed form solutions that facilitate distributed implementation. These solutions maximize the total utility from the users' perspective, as well as maximize the total revenue from the energy provider's perspective, a desirable objective which has been neglected in previous works. Thirdly, we propose a distributed load selection and discrete level energy scheduling scheme that achieves distribution of computation complexity to users, while each user can still converge to its local sub-optimal solutions. The proposed scheme requires only aggregate load and pricing information exchanges between the energy provider and each user.

II. System model

Consider a set of residences or users $\mathcal{K}$ that are supplied with electricity from the same energy provider or utility company. We shall use the terms residence and user interchangeably throughout this paper. The system model is illustrated in Figure 1. Each residence is equipped with a EMS and a smart meter that communicates with its various electrical devices, and also with the EMS central controller at the energy provider. Two-way communications between the energy provider and the smart meters are supported by appropriate communication network protocols, for example, through a local area network. We assume that the energy provider knows in advance the parameters of the energy cost function $C_t(l_t)$ for the time period $t \in \mathcal{T}$. Each scheduling time slot t can represent, for example, one hour or a few ten minutes depending on the time resolution of the pricing and energy consumption data.

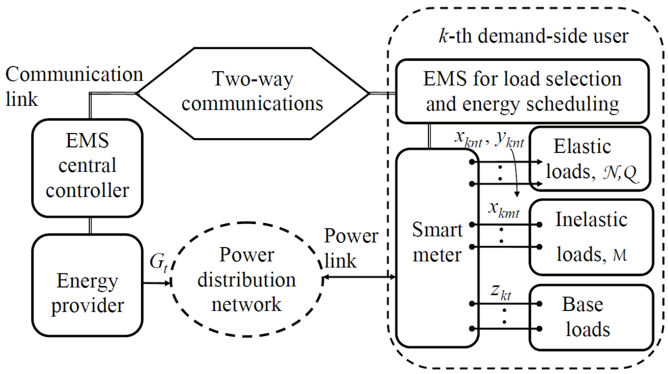

Fig. 1. Two-way communications and power links between each residential user and the energy provider in a smart grid.

A. Load selection and discrete level energy scheduling

In this work we consider discrete level energy scheduling with variable starting time and termination time. To model that, we first define the load selection index, y_{knt}, that is associated with the k-th user's n-th device or appliance at the t-th hour,

$$y_{knt} = \begin{cases} 1; & \text{if the } k\text{−th user switches on the } n\text{−th} \\ & \text{device at the } t\text{−th hour,} \\ 0; & \text{otherwise.} \end{cases} \tag{1}$$

Let x_{knt}, $n \in \mathcal{M} \cup \mathcal{N} \cup \mathcal{Q}$, $k \in \mathcal{K}$, denote the energy consumption of the k-th user's n-th device at the t-th hour, $t \in \mathcal{T}$. We consider that each residential user has a base load which is nondeferrable, and the following three typical classes of deferrable devices:

1) Class $\mathcal{M}$ devices which are categorized as inelastic loads with inelastic time schedule in energy consumption. Each device has an accumulated energy requirement E_{km} that has to be met between the prescribed start time S_{km} and termination time T_{km}, that is,

$$\sum_{t=S_{km}}^{T_{km}} x_{kmt} = E_{km}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M}. \tag{2}$$

One typical example of such device is a cooking appliance. We assume that the prescribed time schedules and accumulated energy requirements of the inelastic loads must be met with a higher priority than the following two classes of elastic loads.

2) Class $\mathcal{N}$ devices which are categorized as elastic loads with adjustable power and elastic time schedule in energy consumption. Each device has an operating energy of x_{knt}, a variable starting time $\tilde{S}_{kn}$ and a variable termination time $\tilde{T}_{kn}$ depending on the user need, where $n \in \mathcal{N}$. Moreover, each x_{knt} is subject to the minimum limit x_{kn}^{min} and maximum limit x_{kn}^{max}. For this class of devices, the user may choose to operate away from a nominal point, if by doing so can further optimize the utility or minimize the electricity bill. Hence, it is practical to assume no accumulated energy requirement for such devices. One example from this class of load is an air-conditioning unit.

3) Class $\mathcal{Q}$ devices which are also categorized as elastic loads with adjustable power and elastic time schedule in energy consumption. In addition, each device has an accumulated energy requirement E_{kq} that has to be met between the variable start time $\tilde{S}_{kq}$ and variable termination time $\tilde{T}_{kq}$, that is,

$$\sum_{t=\tilde{S}_{kq}}^{\tilde{T}_{kq}} y_{kqt} x_{kqt} = E_{kq}, \forall k \in \mathcal{K}, \forall q \in \mathcal{Q}. \tag{3}$$

One example of such loads is an battery energy storage. Moreover, for both Class $\mathcal{Q}$ and Class $\mathcal{N}$ devices, each x_{knt} is subject to the minimum limit x_{kn}^{min} and maximum limit x_{kn}^{max}. This constraint is expressed as,

$$x_{kn}^{min} \leq x_{knt} \leq x_{kn}^{max}, \forall k \in \mathcal{K}, \forall n \in \mathcal{N} \cup \mathcal{Q}, \forall t \in \mathcal{T}. \tag{4}$$

III. Problem formulation

We model the response of each user's appliance or device using the utility function [5] [6], that provides a reasonable indication of a user's satisfaction with respect to its electricity consumption. A general utility function, denoted as $U_{kn}(x_{knt})$, is assumed to be a concave and non-decreasing function of x_{knt}. The first derivative of $U_{kn}(x_{knt})$ with respect to x_{knt} is a non-increasing function of x_{knt}, which models the saturation of $U_{kn}(x_{knt})$ with respect to the increasing x_{knt}. The total utility from all users' devices over the time span $\mathcal{T}$ is adopted as our objective function to provide for user cooperation in electricity or energy comsumption,

$$U(\boldsymbol{X}_k, \boldsymbol{Y}_k, \forall k \in \mathcal{K}) = \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{M} \cup \mathcal{N} \cup \mathcal{Q}} y_{knt} U_{kn}(x_{knt}). \tag{5}$$

For the k-th user, $\boldsymbol{X}_k$ is a matrix consists of the elements x_{knt}, $n \in \mathcal{M} \cup \mathcal{N}$, whereas $\boldsymbol{Y}_k$ is a matrix consists of the elements y_{knt}.

The optimization problem is formulated as,

$$\max_{\boldsymbol{X}_k, \boldsymbol{Y}_k} U(\boldsymbol{X}_k, \boldsymbol{Y}_k, \forall k \in \mathcal{K})$$

subject to

$$\sum_{k\in\mathcal{K}} z_{kt} + \sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}\cup\mathcal{Q}} y_{knt}x_{knt} + \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} x_{kmt} \leq G_t,$$

$\forall t \in \mathcal{T}$,

(2), (3), (4),

$y_{knt} \in \{0, 1\}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T}, \forall n \in \mathcal{N} \cup \mathcal{Q}$,

$$\sum_{t\in\mathcal{T}} C_t(\sum_{k\in\mathcal{K}} z_{kt} + \sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}\cup\mathcal{Q}} y_{knt}x_{knt} + \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} x_{kmt}) \leq B. \tag{6}$$

In the first set of constraints, the sum of the base loads z_{kt}, elastic loads and inelastic loads is not to exceed the contracted energy generation G_t to be supplied by the energy provider or utility company, who is likely to own multiple generators. The constraints (2) and (3) are the accumulated energy requirements of each k-th user's Class $\mathcal{M}$ inelastic loads, and Class $\mathcal{Q}$ elastic loads, respectively. The constraints (4) impose minimum and maximum limits on x_{knt} of each user's Class $\mathcal{N}$ and Class $\mathcal{Q}$ elastic loads. In the last constraint, the cost function $C_t(\cdot)$ is used to represent the energy cost that the energy provider incurs in order to provide electricity to the user over time slot t. The B is the maximum total energy cost to support all the users within the given time period $\mathcal{T}$. Note that our analysis throughout this paper is valid for any valid utility function and cost function.

IV. Analysis and optimization of load selection and energy scheduling

We can view the energy scheduling for inelastic loads as a deterministic case because their time schedules and accumulated energy requirements are prescribed, which must be met with a higher priority. From optimization perspective, the original problem (6) can be tackled by first solving the deterministic energy scheduling for inelastic loads, and then modifying the constraints for subsequent optimization of inelastic loads. After energy scheduling for the inelastic loads, the first constraint in (6) can be written as,

$$\sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}\cup\mathcal{Q}} y_{knt}x_{knt} \leq L_t, \forall t \in \mathcal{T}, \tag{7}$$

where $L_t = G_t - \sum_{k\in\mathcal{K}} z_{kt} - \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} x^*_{kmt}$ can be interpreted as the equivalent contracted energy generation capacity that caters for the elastic loads.

the total cost constraint can be written as,

$$\sum_{t\in\mathcal{T}} \tilde{C}_t(\sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}\cup\mathcal{Q}} y_{knt}x_{knt}) \leq B, \tag{8}$$

where $\tilde{C}_t(l_t) = \tilde{a}_t l_t^2 + \tilde{b}_t l_t + \tilde{c}_t$, $\tilde{a}_t = a_t$, $\tilde{b}_t = b_t + 2a_t u_t$, $\tilde{c}_t = c_t + a_t u_t^2 + 2b_t u_t$, and $u_t = \sum_{k\in\mathcal{K}} z_{kt} + \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} x^*_{kmt}$.

By incorporating the results of (7) and (8), we only need to optimize the load selection and energy scheduling for elastic loads in subsequent steps.

A. Optimization of load selection and discrete level energy scheduling for elastic loads

For each elastic load $n \in \mathcal{N} \cup \mathcal{Q}$, both of its y_{knt} and x_{knt} are to be optimized to maximize the total utility. The y_{knt} is to be optimized to either 1 or 0 during the period $\tilde{S}_{kn} \leq t \leq \tilde{T}_{kn}$, with an arbitrary start time $\tilde{S}_{kn}$ and a termination time $\tilde{T}_{kn}$. Moreover, for each elastic load $q \in \mathcal{Q}$, its x_{kqt} is to be scheduled to meet the constraint (3).

By incorporating (7) and (8), the original optimization problem (6) can be written as,

$$\begin{aligned} &\max_{\boldsymbol{X}_k, \boldsymbol{Y}_k} U(\boldsymbol{X}_k, \boldsymbol{Y}_k, \forall k \in \mathcal{K}) \\ &\text{subject to } (3), (4), (7), (8), \\ &\qquad y_{knt} \in \{0, 1\}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T}, \forall n \in \mathcal{N} \cup \mathcal{Q}. \end{aligned} \tag{9}$$

The problems in (6) and (9) are mixed-integer programming problems, each involves $|\mathcal{K}| \times |\mathcal{N}| \times |\mathcal{T}|$ binary variables y_{knt} and as many continuous variables x_{knt}. Consider the typical values of $|\mathcal{T}| = 24$ and $|\mathcal{K}| \times |\mathcal{N}|$ being in the hundreds, each of (6) and (9) is a mixed-integer programming problem with thousands of integer variables plus thousands of continuous variables, which constitute a prohibitively large dimensionality for any standard computational method (e.g. branch and bound). Both problems (6) and (9) also require centralized computation, which is hard to implement in practice because it requires the load scheduling information or parameters of the users' utility functions, which are supposed to be private. Moreover, the users would not want to provide and share details information about their load selection and energy consumption schedules, at all times, with either the energy provider or other users, let alone relinquishing control over their controllable devices to any third party.

The main contributions in this work are the derivations of closed form solutions that solve the mixed integer programming problem (9), and facilitate distributed computation. To achieve that, we first define a set of auxiliary variables $\hat{l}_{kt}$, each of which represents the local load threshold of k-th user at time t. The constraints in (7) can be written as,

$$\sum_{n\in\mathcal{N}\cup\mathcal{Q}} y_{knt}x_{knt} \leq \hat{l}_{kt}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T}, \tag{10}$$

$$\sum_{k\in\mathcal{K}} \hat{l}_{kt} \leq L_t, \forall t \in \mathcal{T}. \tag{11}$$

Then, we define another set of auxiliary constants e_{kn} and set their values such that $e_{kn} \gg \max_{\forall t\in\mathcal{T}} x_{knt}$. We can now remove the integer constraints of $y_{knt} \in \{0, 1\}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T}, \forall n \in \mathcal{N} \cup \mathcal{Q}$, from (9) by replacing the constraints (10) with the following,

$$\begin{aligned} &\sum_{n\in\mathcal{N}\cup\mathcal{Q}} y_{knt}x_{knt} \leq \hat{l}_{kt} + \sum_{n\in\mathcal{N}\cup\mathcal{Q}} (1 - y_{knt})e_{kn}, \\ &\forall k \in \mathcal{K}, \forall t \in \mathcal{T}. \end{aligned} \tag{12}$$

It can be verified that both constraints (10) and (12) are equivalent. Furthermore, we note that $\tilde{C}_t(\sum_{k\in\mathcal{K}}\hat{l}_{kt})$ is an upper bound to the left hand side of (8) because $\tilde{C}_t(\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}\cup\mathcal{Q}}y_{knt}x_{knt}) \le \tilde{C}_t(\sum_{k\in\mathcal{K}}\hat{l}_{kt})$. Therefore in subsequent derivations, we substitute the constraint (8) with the following,

$$\sum_{t\in\mathcal{T}}\tilde{C}_t(\sum_{k\in\mathcal{K}}\hat{l}_{kt}) \le B. \tag{13}$$

By consolidating the above results, we can write the optimization in (9) as follows,

$$\begin{aligned}&\max_{\boldsymbol{X}_k,\boldsymbol{Y}_k} U(\boldsymbol{X}_k,\boldsymbol{Y}_k,\forall k\in\mathcal{K})\\ &\text{subject to } (3),(4),(11),(12),(13).\end{aligned} \tag{14}$$

We apply partial dual decomposition to (14) by multiplying the Lagrange multipliers ρ_{kn} and λ_t to the constraints (3) and (12), respectively, and added them to the objective function. This yields,

$$\begin{aligned}&\max_{x_{knt},y_{knt}}\sum_{k\in\mathcal{K}}\mathfrak{L}_k(x_{knt},y_{knt})+\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\lambda_t\hat{l}_{kt}\\ &\text{subject to } (4),(11),(13).\end{aligned} \tag{15}$$

where $\lambda_t > 0$, $\rho_{kn} > 0$, and

$$\begin{aligned}\mathfrak{L}_k(x_{knt},y_{knt}) &= \sum_{t\in\mathcal{T}}[\sum_{n\in\mathcal{N}\cup\mathcal{Q}}y_{knt}U_{knt}(x_{knt})\\ &-\lambda_t(\sum_{n\in\mathcal{N}\cup\mathcal{Q}}y_{knt}x_{knt}-\sum_{n\in\mathcal{N}\cup\mathcal{Q}}(1-y_{knt})e_{kn})]\\ &-\sum_{n\in\mathcal{Q}}\rho_{kn}(\sum_{t=S_{kq}}^{T_{kq}}y_{kqt}x_{kqt}-E_{kq}).\end{aligned} \tag{16}$$

From (15), we observe that the optimization problem can be decoupled into two sets of optimization problems: (i) one maximization of individual utility function for each user, (ii) one maximization to update the aggregated load threshold and pricing at the energy provider. We derive new closed form solutions for load selection and energy scheduling from these two sets of optimization problems, which are presented subsequently.

B. Maximization of utility function for each user

Based on (15) and (16), the optimization for each user can be decoupled as individual subproblem as follows. For any given nonnegative λ_t, $\forall t\in\mathcal{T}$, each k-th user, $\forall k\in\mathcal{K}$, determines the optimal load x_{knt} and y_{knt} from the following,

$$\begin{aligned}&\max_{x_{knt},y_{knt},\forall t,\forall n}\mathfrak{L}_k(x_{knt},y_{knt})\\ &\text{subject to } (4).\end{aligned} \tag{17}$$

1) Optimal solution for energy scheduling: For any given combination of selected elastic loads, the optimization in (17) is convex in x_{knt} because the objective function (16) is a linear combination of concave utility and linear functions. By setting $\partial\mathfrak{L}_k(x_{knt},y_{knt})/\partial x_{knt}=0$, for any selected elastic load (with $y_{knt}=1$), it can be shown that for a general utility function $U_{knt}(x_{knt})$, the optimal energy x^*_{knt} should satisfy,

$$\frac{\partial U_{knt}(x^*_{knt})}{\partial x_{knt}} = \begin{cases}\lambda_t; & \text{for Class } \mathcal{N} \text{ device}, \forall n\in\mathcal{N},\\ \lambda_t+\rho_{kn}; & \text{for Class } \mathcal{Q} \text{ device}, \forall n\in\mathcal{Q}.\end{cases} \tag{18}$$

This is a general result that is valid for any valid utility function. In other words, given any utility function, the optimal x^*_{knt} can be solved directly from (18). To verify the above result, we substitute the utility function $U_{knt}(x_{knt})=\log(1+\eta_t x_{knt})$ into (18) and arrive at the water-filling solution [4] for the optimal x^*_{knt}, with the so-called water-level $\mu_{knt}=1/\lambda_t,\forall n\in\mathcal{N}$, and $\mu_{knt}=1/(\lambda_t+\rho_{kn}),\forall n\in\mathcal{Q}$.

2) Optimal solution for load selection: The optimal solutions for y_{knt} should satisfy $\partial\mathfrak{L}_k(x_{knt},y_{knt})/\partial y_{knt}=0$. For any given nonnegative λ_t, the solution for y_{knt} is derived as,

$$y^*_{knt} = \begin{cases}1; & \text{if } U_{knt}(x_{knt})/(\lambda_t e_{kn})\\ & =\max_{i,j}\{U_{ijt}(x_{ijt})/(\lambda_t e_{ij})\}>1,\\ 0; & \text{otherwise.}\end{cases} \tag{19}$$

where $i\in\mathcal{K}$ is the user index, and $j\in\mathcal{N}\cup\mathcal{Q}$ is the index for elastic loads.

On the right hand side of (19), the $U_{knt}(x_{knt})/\lambda_t$ can be interpreted as utility per unit price. It provides an interpretation which is intuitively pleasing: At any time t, among all the users' elastic loads that satisfy $U_{ijt}(x_{ijt})/(\lambda_h e_{ij})>1$, $i\in\mathcal{K}$, $j\in\mathcal{N}\cup\mathcal{Q}$, the optimal scheme should select the first few elastic loads which are associated with the largest utility per unit price. However, such optimal scheme requires a central controller which knows $U_{ijt}(x_{ijt})/(\lambda_t e_{ij})$, $\forall i\in\mathcal{K}$, and $\forall j\in\mathcal{N}\cup\mathcal{Q}$.

To achieve distributed implementation, we can further modify the solution (19) into a suboptimal scheme as follows: Each user is to select the first few elastic loads of its own which are associated with the largest utility per unit price. For any given user, this can be achieved by simply replacing the operator $\max_{i,j}\{\cdot\}$ in (19) with an operator $\max_j\{\cdot\}$. The main motivation or advantage of doing so is to achieve distribution of computation complexity to all users, while each user can still converge to its local sub-optimal solution.

Finally, the total elastic load of k-th user at time t is computed as,

$$l^*_{kt}=\sum_{n\in\mathcal{N}\cup\mathcal{Q}}y^*_{knt}x^*_{knt}. \tag{20}$$

Note that each x^*_{knt}, y^*_{knt} and l^*_{kt} is computed locally at the k-th user. That is how the computational load is distributed among the users.

C. Maximization at the energy provider

As derived in (15), the energy provider is to determine the optimal local load thresholds $\hat{l}_{kt}$ for each k-th user at time t from the following optimization problem,

$$\begin{aligned}&\max_{\hat{l}_{kt},\forall k\in\mathcal{K},\forall t\in\mathcal{T}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\lambda_t\hat{l}_{kt}\\ &\text{subject to } (11),(13).\end{aligned} \tag{21}$$

The objective function in (21) is equivalent to the total sale of energy or total revenue, given that the targets $\hat{l}_{kt}$'s are met. Recall that our original optimization problem has been decomposed into subproblems in (17) and (21). Therefore, while maximizing the total utility from the users' perspective, we expect that the solutions obtained also maximize the total revenue from the energy provider's perspective, for given capacity constraint (11) and total cost constraint (13).

1) Optimal solution for load thresholds: By formulating the Lagrange equation $\mathfrak{L}(\hat{l}_{kt}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T})$ for (21) and evaluating $\partial\mathfrak{L}(\hat{l}_{kt}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T})/\partial\hat{l}_{kt} = 0$, it can be shown that for a general cost function $\tilde{C}_t(\sum_{k\in\mathcal{K}} \hat{l}_{kt})$, the optimal $\hat{l}_{kt}$ should satisfy,

$$\frac{\partial\tilde{C}_t(\sum_{k\in\mathcal{K}} \hat{l}_{kt})}{\partial\hat{l}_{kt}} = \frac{\lambda_t - \gamma_t}{\nu}, \forall t, \forall k, \tag{22}$$

where γ_t and ν are Lagrange multipliers associated with the total load constraint and total energy cost constraint, respectively.

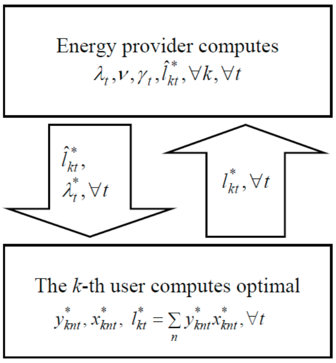

Fig. 2. Two-way communications between the energy provider and each k-th user for the proposed load selection and energy scheduling scheme.

The solutions in (18) to (22) are combined to formulate the distributed load selection and discrete level energy scheduling scheme, as conceptualized in Figure 2, by exploiting two-way communications between the energy provider and each k-th user, For aggregate load control, the λ_t, γ_t and ν can be updated by the standard iterative subgradient algorithm, which details are omitted here due to page constraint. Note that equations (18) to (22) are general results that are valid for any valid utility function and cost function, respectively. In other words, given any utility function or cost function, their x^*_{knt}, y^*_{knt} and $\hat{l}^*_{kt}$ can be solved respectively.

V. Simulation results and discussions

We present simulation results to give further insights into the proposed distributed load selection and energy scheduling scheme. The scheduling interval follows 1-hour time slots. The time horizon in the simulation is 24 hours, corresponding to 8 am, 9 am, 10 am, etc., until 7 am of the next day. To provide a more realistic assessment of the proposed scheme, the base load data of 50 residences are generated using the domestic energy demand model developed in [9], which took into account the time periods and the number of persons at home, as well as their various activities within a day, which associated with various home devices or appliances. In addition to the base loads, there are inelastic and elastic

TABLE I

Parameters and initial values generated for the simulations.

Parameters	Initial values
E_{km}(kWh)	Uniform on {10.0, 11.0, 12.0}
x^{min}_{km}(kWh)	Uniform on {0.001, 0.002, ..., 0.01}
x^{max}_{km}(kWh)	Uniform on {1.1, 1.2, 1.3, 1.4, 1.5}
S_{km}	Uniform on {5pm, 6pm, 7pm, 8pm, 9pm}
T_{km}	Uniform on {5am, 6am, 7am, 8am}
x^{min}_{kn}(kWh)	Uniform on {0.001, 0.002, ..., 0.005}
x^{max}_{kn}(kWh)	Uniform on {2.1, 2.2, 2.3, 2.4, 2.5}
$\tilde{S}_{kn}$	{9am, 10am, 11am, 12pm}
$\tilde{T}_{kn}$	{10pm, 11pm, 12am, 1am, 2am}
$\{\gamma_t, \lambda_t, \nu\}$	{0.1, 0.2, 15.0}

loads that are to be scheduled by the proposed scheme. The parameters of the inelastic and elastic loads, and initial values of the Lagrange multipliers are listed in Table I. Each residence has one inelastic load and 4 elastic loads.

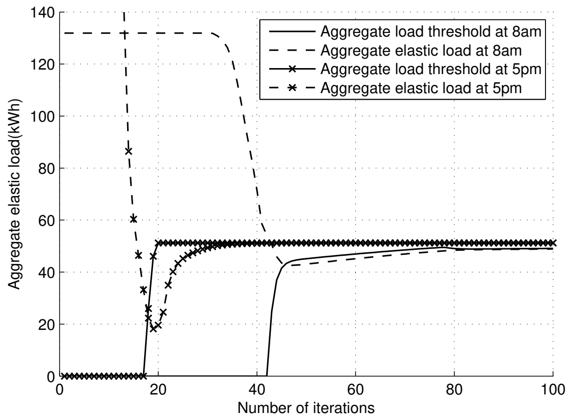

Fig. 3. Convergence of aggregate elastic load $\sum_{k\in\mathcal{K}} l^*_{kt}$ which is optimized by the users versus the iteration index, is plotted as the dash line. Convergence of aggregate load threshold $\sum_{k\in\mathcal{K}} \hat{l}^*_{kt}$ which is optimized by the energy provider versus the iteration index, is plotted as the solid line. Note how these curves converge to each other iteratively.

To provide a further insight into our proposed scheme, in Figure 3, we highlight the convergence of the aggregate elastic load $\sum_{k\in\mathcal{K}} l^*_{kt}$ (computed and updated at the users), and the aggregate load threshold $\sum_{k\in\mathcal{K}} \hat{l}^*_{kt}$ (computed and updated at the energy provider), versus the iteration index. This constitutes the basis of our proposed distributed scheme: the aggregate load threshold $\hat{l}^*_{kt}$ serves as a local target for each user to track and update its elastic loads and selection indices at each time t in a distributed way, thus forgoing the need for any centralized optimization or computation. While the results for two specific time slots (8am and 5pm) are shown here, the same trend is observed for each hour of the day.

The optimal load selection and energy scheduling results over the 24 hours are presented in Figure 4 for the case of

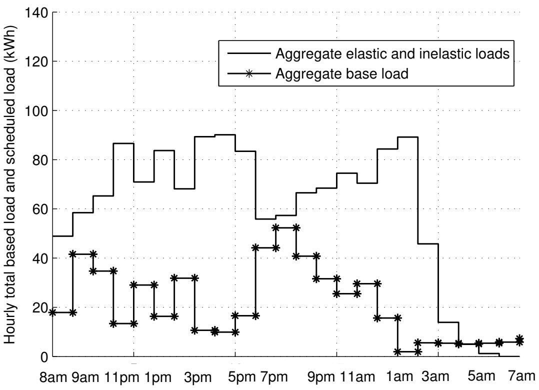

Fig. 4. Hourly energy schedule versus time after optimization by the proposed distributed load selection and energy scheduling scheme, for the simulation setup with 50 residential users, each with a linear marginal benefit function.

linear marginal benefit function. As expected, more elastic loads are scheduled at the time periods when the base load is low, due to the relatively low electricity demand and low electricity price λ_t at those periods (not shown here).

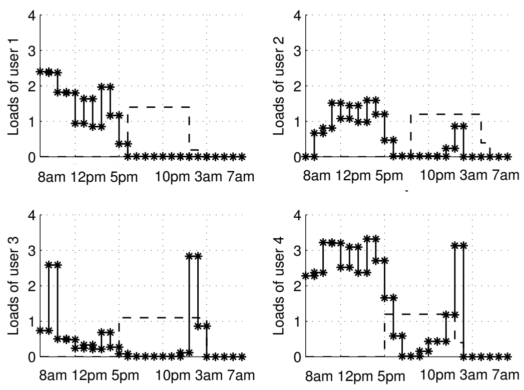

Fig. 5. Aggregate elastic load and inelastic load (in kWh) versus time for randomly selected 4 users (out of 50), each with a linear marginal benefit function. The dash lines correspond to the inelastic load whereas the solid lines correspond to the sum of elastic loads.

In Figures 5 and 6, we plot the aggregate elastic load and inelastic load of randomly selected 4 users for the cases of linear marginal benefit function [5] [10] and water-filling based solution [4], respectively. In comparison, the water-filling based solution guarantees fairness by ensuring similar energy scheduling profiles for all users in Figure 6, while the case of linear marginal benefit function (Figure 5) does not. We also observed that the computation time increases linearly with respect to the total number of iterations I, and approximate linearly with respect to the total number of elastic and deferable loads $|\mathcal{K}| \times |\mathcal{N}|$ for a given $|\mathcal{T}|$.

VI. Conclusions

An optimization framework is developed to solve the mixed integer programming problem of distributed discrete level

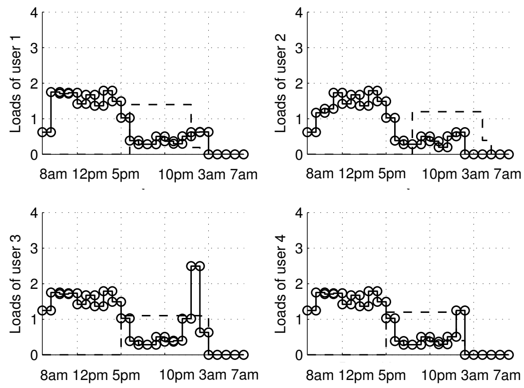

Fig. 6. Aggregate elastic load and inelastic load (in kWh) versus time for randomly selected 4 users (out of 50), each with a water-filling based solution. The dash lines correspond to the inelastic load whereas the solid lines correspond to the sum of elastic loads.

energy scheduling for residential load control, based on general utility function and energy cost function. The general solutions are also applicable to other utility and energy cost functions. Based on these solutions, a distributed load selection and energy scheduling scheme is formulated. The proposed scheme is suitable for distributed implementation because only aggregate load and pricing information exchanges between the energy provider and each user are required.

References

[1] A.-H. Mohsenian-Rad, V. W. S. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, "Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid," *IEEE Trans. Smart Grid*, vol. 1, no. 3, pp. 320–331, Dec. 2010.

[2] I. Atzeni, L. G. Ordóñez, G. Scutari, D. P. Palomar, and J. R. Fonollosa, "Noncooperative and cooperative optimization of distributed energy generation and storage in the demand-side of the smart grid," *IEEE Trans. Signal Processing*, vol. 61, no. 10, p. 245412472, May 2013.

[3] L. Du, S. Grijalva, and R. G. Harley, "Game-theoretic formulation of power dispatch with guaranteed convergence and prioritized best response," *IEEE Transactions on Sustainable Energy*, vol. 6, no. 1, pp. 51–59, 2015.

[4] M. Shinwari, A. Youssef, and W. Hamouda, "A water-filling based scheduling algorithm for the smart grid," *IEEE Transactions on Smart Grid*, vol. 3, no. 2, pp. 710–719, June 2012.

[5] P. Samadi, A.-H. Mohsenian-Rad, R. Schober, V. W. S. Wong, and J. Jatskevich, "Optimal real-time pricing algorithm based on utility maximization for smart grid," in *Proc. IEEE First International Conference on Smart Grid Communications (SmartGridComm)*, Oct. 2010, pp. 415–420.

[6] N. Li, L. Chen, and S. H. Low, "Optimal demand response based on utility maximization in power networks," in *Proceedings of IEEE Power and Energy Society General Meeting*, 2011.

[7] S. Caron and G. Kesidis, "Incentive-based energy consumption scheduling algorithms for the smart grid," in *Proc. IEEE First Int. Conf. on Smart Grid Communications (SmartGridComm)*, Oct. 2010, pp. 391–396.

[8] N. Gatsis and G. B. Giannakis, "Residential load control: Distributed scheduling algorithms and convergence with lost AMI," *IEEE Trans. Smart Grid*, vol. 3, no. 2, pp. 770–786, Jun. 2012.

[9] I. Richardson, M. Thomson, D. Infield, and C. Clifford, "Domestic electricity use: A high resolution energy demand model," *Energy and Buildings*, vol. 42, no. 10, pp. 1878–1887, 2010.

[10] M. Fahrioğlu and F. L. Alvarado, "Designing cost effective demand management contracts using game theory," in *Proc. Power Engineering Society 1999 Winter Meeting*, Jan.-Feb. 1999, pp. 427–432.