

Dynamic Multi-Bus Dispatching Strategy with Boarding and Holding Control for Passenger Delay Alleviation and Schedule Reliability: A Combined Dispatching-Operation System

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Abstract—The continuing increase of the on-road private cars is contributing to a deterioration of the urban traffic system. Public transportation is widely used to tackle this issue due to its large ridership. In this paper, we propose a multi-bus dispatching strategy combined with the boarding and holding control (MBDBH) to improve bus utilization and further decrease the passenger excess delay. Dispatching adjustments and operation control are taken into account in the system. At the dispatching level, on the one hand, either a bus platoon or a single bus can be dispatched for each trip to provide adaptive bus capacity to match the highly-fluctuated stop demands, on the other hand, we adjust the bus dispatching time based on the existing timetable to minimize passenger excess waiting time to a large extent. Meanwhile, the operation level incorporates both holding strategy and boarding limit strategy to bring more flexible adjustments in improving bus service. Besides the efficiency, we also minimize the headway variation in order to maintain a high system reliability. The problem is formulated as a Mixed Integer Nonlinear Programming (MINP) problem, which is solved by the commercial solver Gurobi. With the computational complexity as a concern, we propose a distributed algorithm to implement dual decomposition based on the partial Lagrangian relaxation. Finally, numerical examples are investigated to illustrate the significant time reduction of distributed algorithm and the efficiency of our proposed strategy: The proposed MBDBH model can reduce roughly 50% and 30% of remaining passenger volumes when compared with the timetable-based fixed schedule and the optimized single-bus dispatching schedule, respectively.

Index Terms – Public transport systems, Multi-bus dispatching, Boarding limit, Holding control, Mixed-integer nonlinear programming, Lagrangian relaxation

I. INTRODUCTION

We are experiencing a remarkable urbanization process around the world, resulting in a dramatic increase of the car population, which makes traffic congestions a major obstacle for effective traffic management. Public transportation, as a

key component of city mobility, is an effective solution to tackle the above issue due to its large ridership. However, the heavy enroute traffic increases the interaction between buses and cars, which leads to high uncertainty and low service reliability for a bus system. To better improve the efficiency of public bus management, a variety of studies for different layers of the public planning system have been proposed in the literature.

One of the essential problem in tactical level of the public transportation system is the bus dispatching problem, which discusses the timetable design by selecting proper dispatching frequency or headway in order to not only minimize the passenger delay from demands' perspective but also keep the operational cost in a proper limit. Many traditional methods are proposed based on either historical data or some idealized assumptions: By deriving an analytic model, Newell proposes that the bus dispatching frequency is approximately proportional to the square root of the passenger arrival rates, which is under the assumption that the bus capacity can serve all waiting passengers [31]. Based on the historical passenger demand profile, Ceder [2][3] proposes the point check method and the ride check method to solve bus timetable problem, which are extended in [6][7] to achieve even-load level for all buses meanwhile reduce the deviation from the desired headway by determining bus dispatching times. Also, many mathematical models are proposed to tackle the bus dispatching problem. Han [23] and Furth [19] propose the bus allocation problem under the constraints of fleet size and loading feasibility by adopting the trip assignment model and formulating the budget relationship, respectively, which is later extended as a nonlinear programming problem in [33][34] incorporating the route patterns and ridership elasticity. Ceder et al. also create a timetable for a transit network by developing a mixed integer linear programming problem with the aim to maximize the bus synchronization [5][4]. Moreover, optimal dispatching time and corresponding bus capacity are determined by presenting a bi-objective optimization problem considering both passenger waiting time and bus occupancy rate [24].

With the increasing application and perfection of telematics (e.g., Automatic Vehicle Location (AVL), Automatic Passenger Counting (APC), etc.) in the bus management system [26][17][14], the collection of the real-time information becomes possible, which leads to studies on the periodic

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optimization of dispatching control, and this also indicates the trend of unclear boundary between the tactical studies and the operation studies. Both Eberlein et al. [16] and Hickman [25] propose a nonconvex quadratic programming problem by optimizing bus dispatching time and holding control at certain stops. However, their models fail to incorporate bus capacity constraint and assume all incoming passengers are allowed to board on the buses. Also, different from the rolling horizon mechanism in [16], the model in [25] is an event-based model and the bus dispatching or holding time is determined separately without taking the following buses into account to form a holistic optimization problem, which may lead to myopic results from the perspective of a long-horizon optimization. Moreover, Gkiotsalitis propose two quadratic programming problems to minimize the headway variation by adjusting bus dispatching time [21][20]: The dwell time in [21] is adopted from the traditional loading model without considering bus capacity limits, however, in [20], it is approximated based on the inter-arrival headway from [9]. Both models have no operation control, also, as some buses have already been dispatched and are operating on the road, the question that whether bus quantity in the terminal is sufficient to support all dispatching trips in the optimization horizon remains undiscussed in the paper.

Besides, the traditional tactical planning cannot guarantee a better service due to the highly stochastic demand patterns and complicated enroute environments. Therefore, real-time operational control strategies, e.g., holding, stop-skipping and boarding limit strategies, are proposed in order to address the issue. Eberlein et al. propose a mathematical model involving a combined holding and stop-skipping strategy, where the real data from Massachusetts Bay Transportation Authority (MBTA) Green Line in Boston is adopted to test the model, and results indicate that the cost reduction by combined control is approximately 37%, which is larger than any of the single strategies [15]. Compared with stop-skipping strategies (e.g., deadheading, short-turning and expressing), holding strategy is more common since it is easier to be implemented and relatively depresses passengers less than stop-skipping as passengers may fail to alight bus due to the bus stop skipping [16]. The holding problem can be divided into two major categories [38]: adaptive feedback control methods and rolling-horizon-based optimization methods. Daganzo [9] proposes a forward-headway-based holding strategy with the aim to stabilize the headway as well as maintain a high commercial speed for buses, while the stability of the control method is only guaranteed under small disturbances, which is extended in [10] by considering both forward-headway and backward-headway. Also, various adaptive control methods considering bus holding are compared [1], which divides different methods into three categories, naive methods, partial holding methods and prediction-based holding methods, and the paper finds that the prediction-based methods outperform the other two in compromise between headway regularity and holding time. On the other hand, iterative optimization methods are also discussed in the literature. By formulating a quadratic programming problem, a multiple control-point strategy is proposed by Koehler et al. [27] in order to minimize

the total passenger delay by implementing holding control. The onboard passenger variable is replaced by the estimated fixed value and updated by iterations to change the nonconvex cost into a convex quadratic cost, and guarantee a certain level of accuracy. Moreover, combining the holding control with the boarding limit method, Delgado et al. [13] propose a mathematical programming model via rolling horizon mechanism and apply control every time when a bus reaches a stop, which is later extended in [12] by connecting with the simulator to fulfill a complete approach, and the paper finds that the combination of the boarding limit and holding strategy can reduce the cost more than 22% when compared with the no control strategy. Moreover, passengers' perceived delay is discussed in [37] to incorporate the impacts of the passengers' psychological perception.

Although the operation methods mentioned above provide different perspectives for system reliability, most studies are proposed based on known bus dispatching time or schedule headways. Also, dispatching methods seldom consider operation control for buses already operating on the road. Meanwhile, more accurate passenger arrival patterns can be learnt with the help of the increasingly advanced machine-learning techniques [8][36]. Furthermore, the advanced communication systems [26][17][14] enable the implementation of the real-time control. In view of this, we fill the gap by formulating a holistic mixed integer nonlinear programming problem incorporating both dispatching adjustments and bus on-road operation control. The contributions of our proposed approach are listed as follows:

- Our strategy is a combined dispatching-operating optimization method, not only the bus dispatching time but also the holding and boarding control are applied to the system with the aim to further minimize the passenger delay.
- The traditional methods always dispatch one bus for each trip, also, instead of dispatching buses with different bus sizes, which requires multiple types of buses for each bus line and potentially increases the operating cost, we consider multi-bus dispatching to provide adaptive bus capacity to match with the highly-fluctuated stop demands.
- The re-dispatching mechanism is realistically captured in the proposed model by introducing the new logic constraint (constraint (11a) in Section II) even if the bus on-road driving time and stopping time at each stop are different for different trips.
- A distributed algorithm via Lagrangian decomposition is presented with the aim to reduce the computational time associated with a centralized approach and guarantee the high quality results.

Before proposing our multi-bus dispatching strategy in next section, the differences between the multi-bus dispatching and bus bunching are firstly emphasized herein. The bus bunching is a phenomenon resulting from the uncertainty of the road traffic and the fluctuation of the waiting passenger demand. The vacancy that the bunching buses bring does not match the passenger demands at the stop. The platoon dispatching

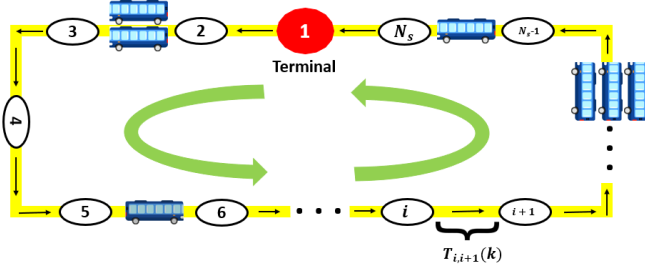


Fig. 1: A one-way loop bus line

is a strategy applied to better serve the passengers, and it is driven by the passenger demands, for example, the bus platoon can serve more passengers if there are large amount of people waiting at the stop, which is an impossible task for one single bus to accomplish in one time. The bus platoon in our multi-bus dispatching strategy is formed at the bus terminal, and the platoon is determined by the controller based on predicted travel demands at each bus stop.

Headway adjustment is just one way to schedule buses and we do incorporate this property in our model, besides that, the number of buses to be dispatched at each trip could also be considered as another decision variable, which leads to the result that the traditional single bus dispatching becomes a special case of our method. Imagine a loop bus line operating by a limited number of buses, optimizing bus frequency only provides one degree of freedom for bus management, however, by applying multi-bus dispatching, we provide another adjusting dimension to bring more flexibility and possibility under the same prediction horizon. Especially when the bus round trip distance is small, multi-bus dispatching could play an effective role since the short distance can guarantee enough bus quantity at the bus terminal to support the multi-bus dispatching strategy, and this largely improves the bus utilization, which is illustrated in Section IV-C in detail.

The paper is organized as follows: In Section II, our combined dispatching-operation optimization model is formulated to determine the optimal dispatching time and dispatching sequence associated with the boarding limit and holding time. In Section III, the transformation to MINP problem and dual decomposition method are introduced. In Section IV, simulation results on a specific bus line are analyzed to illustrate the efficiency of the proposed method. Finally, conclusion and future work are drawn in Section V.

II. PROBLEM FORMULATION OF A MULTI-BUS DISPATCHING STRATEGY UNDER BOARDING AND HOLDING CONTROL

In our model, we consider a system which involves a ring-shaped bus line with N_s bus stops and N_b buses dispatched from the terminal stop, as illustrated in Fig. 1. All buses dispatched from the terminal stop visit all stops on the bus line and wait to be re-dispatched once driving back to the terminal. The index of the bus stop starts from the terminal stop 1, and it increases monotonically till the last stop N_s .

A. Notations

In this section, we propose a Multi-Bus Dispatching strategy under Boarding and Holding control (MBDBH) with the aim to minimize the passenger delay time. In order to better describe the bus dispatching model, the parameters and decision variables used in our optimization model are summarized in Table I. To formally describe the model, a directed graph is

TABLE I: Notations and Definitions

Indices and Sets	
i, j	The index of the bus stop.
b	The index of the bus.
k	The index of the trip.
\mathcal{S}	The set of bus stops along the bus route, we have $i, j \in \mathcal{S}$.
\mathcal{B}	The set of buses, we have $b \in \mathcal{B}$.
Parameters	
N_b	The number of buses for the bus line.
N_s	The total number of bus stops along the bus line.
H_p	Prediction horizon, $H_p = N\Delta$, where N is the number of the prediction steps.
Cap_b	The capacity of the bus b .
DT_{max}	The maximum dwell time at each stop.
T_{min}	The minimum gap time between two adjacent trips.
α_1	The average boarding time for each passenger.
α_2	The average alighting time for each passenger.
t_{oc}	The bus door opening and closing time.
Variables	
$P_{i,j}(k)$	The number of passengers waiting at stop i with destination stop j when buses of trip k are coming.
$V_{b,i,j}(k)$	The number of passengers on bus b with destination stop j when bus b of trip k just reaches stop i .
$A_{b,i}(k)$	The number of alighting passengers of bus b of trip k at stop i .
$f_{i,j}(a, b)$	The incoming passenger flow at stop i with destination stop j between time a and time b .
$AR_i(k)$	The arrival time of buses of trip k at stop i , and $\forall i \in [1, N_b]$.
$DP_i(k)$	The departure time of buses of trip k at stop i , and $\forall i \in [1, N_b]$.
$LT_{b,i}(k)$	The loading time of bus b of trip k at stop i .
$DT_i(k)$	The dwell time of buses of trip k at stop i , in other words, the maximum of the loading time of buses from trip k at stop i .
$T_{i,i+1}(k)$	The traveling time between stops i and $i+1$ for buses dispatched at trip k .
$TA(k)$	The dispatching time of trip k specified in the pre-defined timetable.
$H_i(k)$	The headway between trips $k+1$ and k at stop i .
D^r	Round-Trip Distance.
Decision Variables	
$B_{b,i,j}(k)$	The number of boarding passengers of bus b of trip k at stop i with destination stop j .
$x_b(k)$	Dispatch indicator, which is a binary variable. Whether bus b is dispatched at the trip k . $= 1$ represents bus b is dispatched at k . $= 0$ indicates bus b is not dispatched at k .
$DS(k)$	The dispatching time of buses of trip k at the terminal.

defined as $\mathcal{G} = \{\mathcal{S}, \mathcal{L}\}$, where \mathcal{S} is the set of bus stops along the bus line, and \mathcal{L} is the link set, and each link denotes the bus path between two adjacent stops.

B. Assumptions

The following assumptions are considered:

- Bus overtaking and stop skipping are not allowed in the system.
- A pre-defined timetable, passenger arrival rates at each stop and estimated on-road traveling time are assumed to be known in advance.
- Simultaneous departure from each stop is required for buses in the same trip.

C. Problem Formulation

The MBDBH problem is formulated as a Mixed-Integer NonLinear Programming (MINLP) problem, where the objective considers the passenger total excess delay from efficiency perspective or the headway variation from reliability perspective, and the constraints incorporates the stop and bus volume dynamics, boarding and alighting flow constraints, bus dispatching and reschedule constraints, bus dwell time and travel time constraints, and bus arrival and departure constraints.

1) *Stop Volume Dynamics*: The following constraint defines the stop volume dynamic, the volume at stop i towards stop j is updated by the number of arrival passengers $f_{i,j}(AR_i(k+1), AR_i(k))$ subtracting the sum of the all boarding passengers $\sum_b B_{b,i,j}(k)$ towards stop j .

$$\begin{aligned} \forall k \in [1, H_p], (\forall i, j \in \mathcal{S} \wedge j > i) \wedge (j = 1 \wedge i > 1) \\ P_{i,j}(k+1) = P_{i,j}(k) + f_{i,j}(AR_i(k+1), AR_i(k)) - \sum_{b \in \mathcal{B}} B_{b,i,j}(k) \end{aligned} \quad (1)$$

The passenger incoming flow rate is a time-variant variable, and its value will become large during the high-peak hour when commuters all wait at the bus stop to catch a bus, therefore, the incoming flow rate depends on the time of the day. However, the study of the relationship between the passenger incoming flow rate and the time is beyond the scope of this paper, thus, we assume the average passenger incoming flow rate $f_{i,j}(k)$ is known during each discrete-time interval in the model. Accordingly, constraint (1) can be reformulated as follows:

$$\begin{aligned} P_{i,j}(k+1) = P_{i,j}(k) + (AR_i(k+1) - AR_i(k))f_{i,j}(k) - \sum_{b \in \mathcal{B}} B_{b,i,j}(k) \end{aligned} \quad (2)$$

where $f_{i,j}(k)$ is the passenger arrival rate of trip interval k at stop i with the destination stop j .

2) Bus Volume Dynamics:

$$\begin{aligned} \forall k \in [1, H_p], \\ \forall b \in \mathcal{B}, (\forall i, j \in \mathcal{S} \wedge j > i) \vee (j = 1 \wedge i > 1) \wedge (i \neq N_s) \\ \sum_j V_{b,i+1,j}(k) = \sum_j V_{b,i,j}(k) + \sum_j B_{b,i,j}(k) - A_{b,i}(k) \end{aligned} \quad (3)$$

Constraint (3) defines the bus volume dynamics, specifically, the passenger volume on bus b of trip k when it reaches stop $i+1$ is determined by its passenger volume when it reaches the adjacent upstream stop i , the boarding flow at stop i , $\sum_j B_{b,i,j}(k)$, and the alighting flow at stop i , $A_{b,i}(k)$.

3) *Boarding and Alighting Constraints*: Constraints (4a), (4b) and (4c) describe the boarding flow and the alighting flow.

$$\begin{aligned} \forall k \in [1, H_p], \forall b \in \mathcal{B} \\ (\forall i, j \in \mathcal{S} \wedge j > i) \vee (j = 1 \wedge i > 1) \\ \sum_j B_{b,i,j}(k) \leq Cap_b - \sum_j V_{b,i,j}(k) \end{aligned} \quad (4a)$$

$$\sum_b B_{b,i,j}(k) \leq P_{i,j}(k) \quad (4b)$$

$$\begin{aligned} (\forall i, q \in \mathcal{S} \wedge q < i) \\ A_{b,i}(k) = \sum_q V_{b,q,i}(k) \end{aligned} \quad (4c)$$

Constraint (4a) illustrates that the total number of passengers boarding bus b cannot exceed the remaining capacity of the bus b . Constraint (4b) indicates that the total number of boarding passengers at stop i with destination stop j cannot exceed the number of waiting passengers at stop i with destination stop j . The use of the inequalities in constraints (4a) and (4b) makes the boarding control in our system become possible. At the implementation stage, it is possible to control boarding flows via recent advanced communication technology (ICT) in the intelligent transportation system [14][29]. Imagine a message via mobile apps or screen board located at the bus stop could be sent or displayed to passengers when a bus is approaching the stop, and only passengers at the front queue will be selected to enter the designated boarding area at the bus stop. The identification of the front-queue passenger can be realized by the camera or Lidar installed at the bus stop. When the bus reaches the bus stop, only passengers who are waiting at the designated area are allowed to board the bus. This requires infrastructural enhancement at the bus stop, not only the area re-design but also the advanced sensors. By doing so, the control of the boarding passengers can be effectively realized. Fig. 2 gives an illustration for boarding control. In our model, all boarding passengers are allowed to get off the bus at their destination stop, thus, the passenger alighting flow of bus b of trip k at stop i is equal to those passengers who board from several upstream stops and prepare to alight at stop i , as illustrated in constraint (4c).

4) Dispatching Constraints:

$$\forall k \in [1, H_p], (\forall i, j \in \mathcal{S} \wedge j > i) \vee (j = 1 \wedge i > 1), \forall b \in \mathcal{B} \\ B_{b,i,j}(k) \leq x_b(k)Cap_b \quad (5a)$$

$$A_{b,i}(k) \leq x_b(k)Cap_b \quad (5b)$$

$$V_{b,i,j}(k) \leq x_b(k)Cap_b \quad (5c)$$

$$LT_{b,i}(k) \leq x_b(k)DT_{max} \quad (5d)$$

$$\sum_b x_b(k) \geq 1 \quad (5e)$$

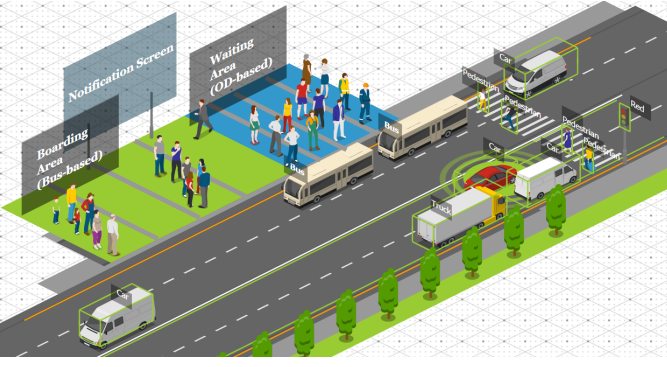


Fig. 2: Graphic illustration of boarding control at bus stop

Constraints (5a) - (5e) are dispatching constraints. Variables $B_{b,i,j}(k)$, $A_{b,i}(k)$, $V_{b,i,j}(k)$ and $LT_{b,i}(k)$ are all related to the bus dispatching indicator $x_b(k)$, in other words, only if the bus b is dispatched at trip k , the above mentioned variables belonging to trip k could be active in the model, otherwise, should be set as 0, as described in constraints (5a) - (5d). In this model, the bus dispatching times are not fixed values or known inputs of the specified timetable, but can be adaptive and adjusted according to the passenger boarding and alighting flows and the bus holding time. Also, every trip can dispatch either single bus or several buses, in other words, at least one bus is dispatched for each trip, as illustrated in constraint (5e).

5) Loading Time and Dwell Time Constraints:

$$\forall k \in [1, H_p], (\forall i, j \in \mathcal{S} \wedge j > i) \vee (j = 1 \wedge i > 1), \forall b \in \mathcal{B}$$

$$LT_{b,i}(k) \geq \alpha_1 \sum_j B_{b,i,j}(k) + t_{oc} x_b(k) \quad (6a)$$

$$LT_{b,i}(k) \geq \alpha_2 A_{b,i}(k) + t_{oc} x_b(k) \quad (6b)$$

$$LT_{b,i}(k) \leq DT_i(k) \quad (6c)$$

$$DT_i(k) \leq DT_{max} \quad (6d)$$

Loading time is defined as the time a bus spends at a stop to allow passengers to alight and board. In this paper, we adopt one of the simultaneous loading model recorded in Highway Capacity Manual [28]: $LT = \max(\alpha_1 B, \alpha_2 A) + t_{oc}$, where LT is the bus loading time at a stop (measured in seconds), B and A are the number of boarding passengers and alighting passengers at the stop, respectively. α_1 and α_2 are the average boarding time and alighting time for each passenger, respectively. t_{oc} is the door opening and closing time. Based on the above relationship, we can develop our bus loading time for bus b of trip k at stop i , $LT_{b,i}(k)$, as follows:

$$LT_{b,i}(k) = \max(\alpha_1 \sum_j B_{b,i,j}(k), \alpha_2 A_{b,i}(k)) + t_{oc} \quad (7)$$

To avoid conflicting with constraint (5d), the above equation is modified as follows:

$$LT_{b,i}(k) = \max(\alpha_1 \sum_j B_{b,i,j}(k), \alpha_2 A_{b,i}(k)) + t_{oc} x_b(k) \quad (8)$$

By converting the above equation, we can obtain the corresponding linear constraints (6a) and (6b). The loading time of each bus from the same trip at the same stop may be different.

In order to make them all act in concert, the bus which finishes loading earlier will wait until the latest bus in the platoon finish loading process. Then the whole stopping time at the stop, which is called the dwell time $DT_i(k)$ in the paper, is illustrated as follows:

$$DT_i(k) = \max_{b \in \mathcal{B}}(LT_{b,i}(k)) \quad (9)$$

Similarly, the above constraint can be converted to corresponding linear constraint (6c). Also, to guarantee the punctual arrival, each bus has a dwell time upper bound, as indicated in constraint (6d).

6) Arrival and Departure Constraints:

$$\forall k \in [1, H_p]$$

$$AR_{i+1}(k) = DP_i(k) + T_{i,i+1}(k), \quad \forall i \in \{\mathcal{S} \setminus N_s\} \quad (10a)$$

$$AR_1(k) = DP_i(k) + T_{i,1}(k), \quad \text{for } i = N_s \quad (10b)$$

$$DP_i(k) \geq AR_i(k) + DT_i(k), \quad \forall i \in \{\mathcal{S} \setminus 1\} \quad (10c)$$

$$DP_i(k) \geq DS(k) + DT_i(k), \quad \text{for } i = 1 \quad (10d)$$

$$AR_i(k) + T_{min} \leq AR_i(k+1), \quad \text{for } i \in \mathcal{S} \quad (10e)$$

$$DP_i(k) + T_{min} \leq DP_i(k+1), \quad \text{for } i \in \mathcal{S} \quad (10f)$$

$$DS(k) + T_{min} \leq DS(k+1) \quad (10g)$$

Constraints (10a) - (10f) describe the bus movement dynamics on the road. Constraints (10a) and (10b) illustrate that the bus arrival time of trip k at stop $i+1$ is equal to the bus departure time of trip k at stop i plus the travel time between stops i and $i+1$. Similarly, constraints (10c) and (10d) depict the bus departure time of trip k at stop i is no less than the summation of the bus arrival time of trip k at stop i and the dwell time of trip k at stop i . The reason why inequality is used in (10c) and (10d) is that the holding control can be incorporated as long as $DP_i(k)$ is not equal to $AR_i(k) + DT_i(k)$ or $DS(k) + DT_i(k)$, on the other hand, fewer variables lead to higher computational efficiency. T_{min} is the minimum gap time between two adjacent trips, which can avoid the overtaking phenomenon and thereby be consistent with the assumption. The gap of two adjacent trip arrival times, $AR_i(k+1) - AR_i(k)$, is required to maintain at least a minimum value T_{min} , as enforced by constraints (10e) and (10f), which is also regarded as the layover time and is added in constraint (10g). As the study of travel time is beyond the scope of this paper, we assume the traveling time $T_{i,i+1}(k)$ is known in advance during the period of interest.

7) Reschedule Constraints:

$$\forall k \in [2, H_p], \forall m \in [1, k-1], \forall b \in \mathcal{B}$$

$$x_b(k) = 1 \rightarrow DS(k) \geq \max_{m < k} \{AR_1(m) - M(1 - x_b(m))\} \quad (11a)$$

$$DS(k) \geq TA(k) \quad (11b)$$

Constraint (11a) discusses the re-schedule of the buses once they return back to the terminal. Constraint (11a) illustrates that a new trip of bus b can be generated only if the dispatching time of bus b at the terminal is no smaller than the latest arrival time of bus b at the terminal, and M is a large value as long as satisfying $M \geq \max_{m < k} AR_1(m)$ for $\forall k \in [2, H_p]$. Also, our dispatching is tuned based on the pre-defined timetable, and

the buses for a certain trip cannot be dispatched earlier than the specified timetable of that trip, as described in constraint (11b).

8) *Objective cost for the entire ring road*: In this model, our main objective is to minimize the total passenger excess delay for whole planning trips, on the other hand, the minimization on the headway gap is also considered in order to guarantee the system's reliability. Accordingly, the objective function is captured as follows:

$$J_{cost} = \beta_1 J_{td} + \beta_2 J_{hg} \quad (12)$$

where J_{td} is the passengers' total excess delay, and J_{hg} is the summation of the headway gap of any adjacent trips. β_1 and β_2 are binary variables, and they can activate different objectives to be achieved in the system.

a) *Passengers' total excess delay*: Compared with in-vehicle traveling time, stop waiting time is more suffering for passengers. Also, overlong waiting time could easily frustrates passengers. Therefore, we formulate the objective as the passengers' total excess delay, which is the multiplication of the remaining passenger volume and corresponding arrival time gap (or dispatching time gap at the terminal stop 1) for two adjacent trips through entire bus stops and planning horizon:

$$J_{td} = \sum_{k=1}^{H_p} \left\{ \left[\sum_{i \in \{S \setminus 1\}} \sum_{\substack{j \in S \wedge \\ (j > i \wedge j = 1)}} (P_{i,j}(k) - \sum_{b \in B} B_{b,i,j}(k)) (AR_i(k+1) - AR_i(k)) \right] + \left[\sum_{j \in S \wedge j > i} (P_{1,j}(k) - \sum_{b \in B} B_{b,1,j}(k)) (DS(k+1) - DS(k)) \right] \right\} \quad (13)$$

Many studies use $\sum_{k=1}^{H_p} \sum_{i \in \{S\}} (\sum_{j > i} \frac{f_{ij}}{2}) (AR_i(k+1) - AR_i(k))^2$ to capture the total passenger delay [16][27][13][12], which is under the assumption that there is no residue queue at each bus stop, in other words, all waiting passengers are allowed to board on the bus, which neglects the delay of those passengers who fail to board the bus, and this is actually the important delay significantly affecting the passengers' emotions [18][32][35], which is the reason why we use the multiplication of the remaining passenger volume $P_{i,j}(k) - \sum_{b \in B} B_{b,i,j}(k)$ and corresponding headway to represent the passenger waiting excess delay.

b) *Headway gap minimization*: In order to maintain the system reliability, the sum of headway gaps of any two adjacent trips is formulated as follows:

$$J_{hg} = \sum_{k=1}^{H_p-2} \sum_{i \in S} |H_i(k+1) - H_i(k)| \quad (14)$$

where

$$H_i(k) = \begin{cases} AR_i(k+1) - AR_i(k), & \forall i \in \{S \setminus 1\} \\ DS(k+1) - DS(k), & \text{otherwise} \end{cases}$$

III. SOLUTION ALGORITHM - MIXED INTEGER NONLINEAR PROGRAMMING

In this section, we propose two different algorithms to solve our optimization problem, that is, a centralized method and a distributed method, respectively. The proposed model

described in Section II incorporates some logic constraints, which are converted into the corresponding linear constraints and change the mixed-logic problem into a MINP problem in subsection III-A. Due to the large computation complexity involved in the centralized MINP problem, we partition our model into subsystems and solve the problem by adopting the Lagrangian relaxation in Section III-B.

A. Mixed Integer Nonlinear Programming

The MBDBH problem has been formulated as a mixed logical dynamic model, which involves a nonlinear cost function (12) with mixed logic constraints, e.g., the stop and bus volume dynamics (1)-(3), boarding and alighting flow constraints (4a)-(4c), bus dispatching constraints (5a)-(5e), bus dwell time and travel time constraints (6a)-(6d), bus arrival and departure constraints (10a)-(10g), and bus reschedule constraints (11a)-(11b). Among all these constraints and objective, the logic constraints include bus reschedule constraint (11a), also, the objective (14) involve the absolute variables in the expression.

Let M' be sufficiently large, and satisfies $M' \geq \max_{m < k} \{AR_1(m) - DS(k)\}$ for $\forall k \in [2, H_p]$, then the logic constraint (11a) can be converted into the following linear constraints:

$$\begin{aligned} \forall k \in [2, H_p], \forall m \in [1, k-1], \forall b \in B \\ DS(k) - AR_1(m) + M'(1 - x_b(m)) \geq -M'(1 - x_b(k)) \end{aligned} \quad (15)$$

Proposition 1: Replacing constraint (11a) with Inequality (15) in the model leads to the same solution.

The headway gap formulation (14) involves absolute value, and can be converted into the linear cost by introducing an auxiliary variable $\delta_i(k)$, then the cost can be equivalently transmitted into the following linear cost and linear constraints:

$$J'_{hg} = \sum_{k=1}^{H_p-2} \sum_{i \in S} \delta_i(k) \quad (16)$$

s.t.

$$\begin{aligned} \forall k \in [1, H_p - 2], \forall i \in \{S \setminus 1\} \\ 2AR_i(k+1) - AR_i(k) - AR_i(k+2) \leq \delta_i(k) \\ -2AR_i(k+1) + AR_i(k) + AR_i(k+2) \leq \delta_i(k) \end{aligned} \quad (17)$$

$$\begin{aligned} \forall k \in [1, H_p - 2] \\ 2DS(k+1) - DS(k) - DS(k+2) \leq \delta_1(k) \\ -2DS(k+1) + DS(k) + DS(k+2) \leq \delta_1(k) \end{aligned} \quad (18)$$

Proposition 2: Replacing objective (14) with objective (16) and inequalities (17)-(20) in the model leads to the same solution.

Finally, we can transform our MBDBH problem as a MINP problem below:

- Minimize $\beta_1 J_{td} + \beta_2 J'_{hg}$
- subject to
 - C1: stop volume dynamics: (1)
 - C2: bus volume dynamics: (3)
 - C3: boarding flow constraints: (4a)-(4b)

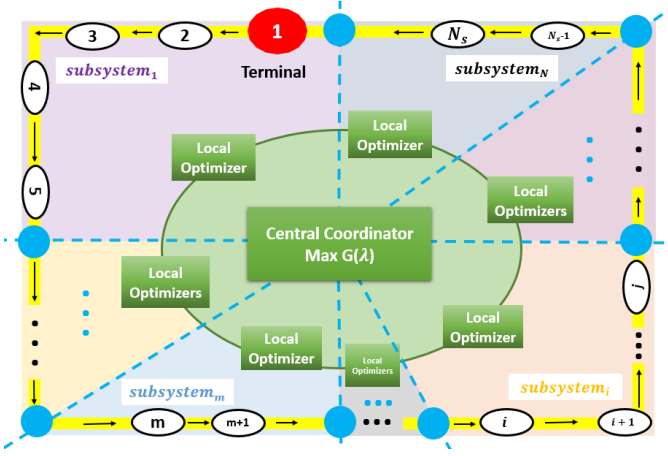


Fig. 3: Distributed optimization on a partitioned bus line

- C4: alighting flow constraint: (4c)
- C5: bus dispatching constraints: (5a)-(5e)
- C6: bus loading time constraints: (6a)-(6b)
- C7: bus dwell time constraint: (6c)-(6d)
- C8: bus arrival and departure constraints: (10a)-(10g)
- C9: bus reschedule constraints: (11b) and (15)
- C10: auxiliary constraints for objective (16): (17)-(20)

After converting into a MINP problem, the proposed MBDBH problem is solved by a commercial optimization solver, Gurobi [22], whose latest version supports solving a MINP problem.

B. Dual Decomposition

Obviously, our model includes many variables and constraints, and involves multiple decision judgements, such as bus dispatching indicator $x_b(k)$, boarding flow volume $B_{b,i,j}(k)$ and trip dispatching time $DS(k)$. To overcome the computational complexity, we partition our bus line into several line segments, as shown in Fig. 3, and conduct distributed optimization on each sub-system individually.

The original bus line in our paper is defined as a directed graph $\mathcal{G} = \{\mathcal{S}, \mathcal{L}\}$, where \mathcal{S} is the set of stops along the bus route, and \mathcal{L} is the link set, and each link denotes the bus route between two adjacent stops. The bus line in the distributed optimization is divided into several line segments and is defined as $\tilde{\mathcal{G}} = \{\tilde{\mathcal{S}}, \tilde{\mathcal{L}}\}$, where $\tilde{\mathcal{S}} = \mathcal{S} \cup \mathcal{R}$, and $r \in \mathcal{R}$ is the artificial stop located at the boundary of two adjacent line segments, which is also drawn as a blue circle in Fig. 3. Each original bus stop s in set \mathcal{S} belongs to only one line segment, while each additional bus stop in set \mathcal{R} is shared by two connected line segments. The link set is denoted as $\tilde{\mathcal{L}} \subseteq \tilde{\mathcal{S}} \times \tilde{\mathcal{S}} - \mathcal{R} \times \mathcal{R}$, e.g., each one-way link $(\tilde{s}, \tilde{s}') \in \tilde{\mathcal{L}}$ represents a bus route either from an internal bus stop \tilde{s} to another internal bus stop \tilde{s}' , or from the boundary stop $\tilde{s} = r$ to an internal bus stop \tilde{s}' , or from the internal bus stop \tilde{s} to the boundary stop $\tilde{s}' = r$. To formally develop the relationship between the boundary artificial stop and its connected original bus stops, we use $r_{s,s'}$ denote the boundary stop r connected with bus stops s and s' , where $s, s' \in \mathcal{S}$ and they both are from different line segments.

Assume the current bus line is partitioned into O line segments, we have $\{\tilde{P}_o \in \tilde{\mathcal{P}} | o \in \{1, 2, \dots, O\}\}$, and let $L(\tilde{P}_o)$ denote all edges in \tilde{P}_o , and $S(\tilde{P}_o)$ denote all stops existing in \tilde{P}_o . The constraints for the boundary stops should be defined to keep the consistency with the original centralized system. Accordingly, the boarding and alighting process should not be considered at boundary stops. Meanwhile, stop and bus volume dynamics are also omitted at boundary stops. Only the departure and arrival time at the boundary stop should be added, since no stopping is required at the boundary stop, we will have the constraints described as: $AR_{r_{ss'}}(k) = DP_s(k) + T_{s,r_{ss'}}(k)$, $DP_{r_{ss'}}(k) = AR_{r_{ss'}}(k)$ and $AR_{s'}(k) = DP_{r_{ss'}}(k) + T_{r_{ss'},s'}(k)$, where $(s, r_{ss'}) \in L(\tilde{P}_o)$ and $(r_{ss'}, s') \in L(\tilde{P}_{o'})$. We assume the terminal stop 1 is partitioned into the line segment \tilde{P}_1 and is the first original stop in \tilde{P}_1 . Based on the above description, the previous centralized model can be reformulated as follows:

$$\begin{aligned}
 & \min \sum_{\tilde{P}_o \in \tilde{\mathcal{P}}} J(\tilde{P}_o) \\
 & s.t. \quad \Phi(\tilde{P}_1) \\
 & \quad \forall \tilde{P}_o \in \{\tilde{\mathcal{P}} \setminus \tilde{P}_1\} : \quad \Phi(\tilde{P}_o) \setminus (11a) \text{ and } (11b) \\
 & \quad \forall r_{ss'} \in \mathcal{R}, \forall k \in [1, H_p] : \\
 & \quad \quad AR_{r_{ss'}}(k) = DP_s(k) + T_{s,r_{ss'}}(k) \quad (21) \\
 & \quad \quad DP_{r_{ss'}}(k) = AR_{r_{ss'}}(k) \quad (22) \\
 & \quad \quad AR_{s'}(k) = DP_{r_{ss'}}(k) + T_{r_{ss'},s'}(k) \quad (23) \\
 & \quad \forall k \in [1, H_p], \forall b \in \mathcal{B}, \forall o \in [2, O] \\
 & \quad \quad x_b^{\tilde{P}_o}(k) = x_b^{\tilde{P}_1}(k) \quad (24) \\
 & \quad \quad \forall k \in [1, H_p], \forall b \in \mathcal{B}, \forall o \in [2, O] \\
 & \quad \quad V_{b,1,j}^{\tilde{P}_o}(k) = V_{b,N_{sub_{o-1}},j}^{\tilde{P}_{o-1}}(k) \quad (25)
 \end{aligned}$$

where $\Phi(\tilde{P}_o)$ is a set of constraints associated with subsystem \tilde{P}_o . Since terminal stop 1 is partitioned into the subsystem 1, reschedule constraints (11a) and (11b) only exist in \tilde{P}_1 . Accordingly, the obtained bus dispatching sequence from \tilde{P}_1 is the input of the other subsystems, as shown in equation (24). Also, the bus volume at incoming boundary stop is the output from the upstream line segment, indicated as equation (25), where $N_{sub_{o-1}}$ is the last internal bus stop of subsystem $o-1$. To simply the notations, we define the new symbol $\Omega(\tilde{P}_o)$ indicating all constraints uniquely belonged to subsystem \tilde{P}_o . Accordingly, the only remaining constraints shared by both subsystems are (22), (24) and (25). To simplify the Lagrangian dual function explained in the later content, both equations (24) and (25) are incorporated into the subsystem o , since results $V_{b,N_{sub_{o-1}},j}^{\tilde{P}_{o-1}}(k)$ can be regarded as the inputs of subsystem o after optimizing subsystem $o-1$, similarly, results $x_b^{\tilde{P}_1}(k)$ can also be regarded as the inputs of subsystem o after optimizing the first subsystem. All passengers must be cleared after the buses drive back to the terminal, namely, the capacity of the newly dispatched buses from the terminal must equal to Cap_b , and because of this, the bus volume of the last subsystem is not a necessary knowledge

for optimizing subsystem 1 which includes the terminal stop, and this make the steps to sequentially solve each subsystem become possible. However, this cannot be applied for equation (22), as the returning times of buses from the last subsystem are the essential information for the first subsystem to make the dispatching decision, which is introduced into our partial Lagrangian dual function addressed in upcoming content. By doing so, we can obtain the simplified formulation as follows:

$$\begin{aligned} \min \quad & \sum_{\tilde{P}_o \in \tilde{\mathcal{P}}} J(\tilde{P}_o) \\ \text{s.t.} \quad & \forall \tilde{P}_o \in \tilde{\mathcal{P}} : \quad \Omega(\tilde{P}_o) \\ & \forall r_{ss'} \in \mathcal{R}, \forall k \in [1, H_p] : \\ & DP_{r_{ss'}}(k) = AR_{r_{ss'}}(k) \end{aligned} \quad (26)$$

Let $\lambda_{r_{ss'}}$ be a vector associated with different trips, after applying Lagrangian transformation, we can obtain the corresponding Lagrangian dual function and constraints:

$$\begin{aligned} \min \quad & \sum_{\tilde{P}_o \in \tilde{\mathcal{P}}} J(\tilde{P}_o) + \sum_{r_{ss'} \in \mathcal{R}} (\lambda_{r_{ss'}} AR_{r_{ss'}} - \lambda_{r_{ss'}} DP_{r_{ss'}}) \\ \text{s.t.} \quad & \forall \tilde{P}_o \in \tilde{\mathcal{P}} : \quad \Omega(\tilde{P}_o) \end{aligned}$$

Define $\lambda_{\tilde{P}_o} = [\lambda_{r_{ss'}}, \lambda_{r_{s''s'''}}]$, where $s \in S(\tilde{P}_{o-1})$, $s', s'' \in S(\tilde{P}_o)$ and $s''' \in S(\tilde{P}_{o+1})$. Then the separated Lagrangian dual function $G(\lambda)$ is illustrated as follows:

$$\min \quad \sum_{\substack{\tilde{P}_o \in \tilde{\mathcal{P}} \\ s', s'' \in S(\tilde{P}_o)}} J(\tilde{P}_o) + \lambda_{\tilde{P}_o} [AR_{r_{ss'}} - DP_{r_{s''s'''}}]^T$$

Finally, the partial Lagrangian dual problem is depicted in the following formulation:

$$\begin{aligned} \max_{\lambda \in \mathbb{R}} \quad & G(\lambda) \\ \text{s.t.} \quad & \forall \tilde{P}_o \in \tilde{\mathcal{P}} : \quad \Omega(\tilde{P}_o) \end{aligned}$$

To solve the above problem, the traditional subgradient method [11][30] is adopted. The corresponding algorithm is illustrated in Algorithm 1, where $\alpha_{r_{ss'}}^m$ is the stepsize at m^{th} iteration. The reason why solving problem sequentially instead of parallelly is that our partitioned subsystem requires the knowledge of the upstream subsystem as the inputs of current subsystem, namely, the dispatching sequence $x_b(k)$ and bus volume $V_{b,i,j}(k)$. Using $x_b(k)$ of subsystem 1 as the inputs of other subsystems is because the dispatching determination happens at the terminal in our model, in other words, we cannot completely separate each subsystem since they all share the same dispatching information. As for regarding bus volume $V_{b,i,j}(k)$ as the inputs of next subsystem, the purpose is to simplify the Lagrangian dual structure, which may become harder to find the results if too many constraints are relaxed. Moreover, due to the relaxation of boundary constraints illustrated in (22), satisfying (22) may not be achieved, which means the obtained cost for the Lagrangian dual problem only serves as a lower bound of the original primal problem illustrated in Section IV-A.

Algorithm 1 Subgradient method for Lagrangian dual problem

Select $\lambda_0 \geq 0$

Repeat until $\lambda_{r_{ss'}}$ converges or reaches the maximum iteration

- 1 solve each subsystem sequentially for iteration m
 - 2 update subgradient for each boundary stop, e.g.,
 $h(\lambda_{r_{ss'}}^m) = AR_{r_{ss'}}^m - DP_{r_{ss'}}^m$
 - 3 update Lagrangian multiplier: $\lambda_{r_{ss'}}^{m+1} = \lambda_{r_{ss'}}^m + \alpha_{r_{ss'}}^m h(\lambda_{r_{ss'}}^m)$
-

IV. SIMULATION RESULTS

In this section, we provide the simulation results to illustrate the efficiency of our proposed model: Firstly, we compare the computation time and results of proposed dual decomposition method and the centralized solver. Secondly, we test our model on a specific bus line and compare it with other methods from many perspectives to depict the advantages of our proposed model. Thirdly, sensitivity analysis of our model on different round-trip distances and time-variant demands is conducted to further illustrate the multi-bus dispatching strategy.

A. Computational complexity for centralized and distributed algorithms

In order to see the computational efficiency of our proposed method, we test our model with various bus sizes, stop numbers and trip horizons. After converting into a mixed integer problem, the model is solved by the optimization solver, Gurobi, coded in MATLAB under a PC with an Intel(R) Core(TM) CPU i7-10510U @2.30GHz and RAM 16GB. We only compare the linear cost, $J_{pv} = \sum_{k=1}^{H_p} \sum_{i,j \in \mathcal{S}} P_{i,j}(k)$, total passenger volume, in this section for simplicity. Table II illustrates the potential complexity involved in the model under different scales, which lists the total number of decision variables and the total number of constraints of 8 cases under different network sizes. Also, the corresponding computational time and the optimal results are included to further illustrate the computational complexity of the centralized algorithm.

TABLE II: Problem scale and processing time under centralized algorithm

	N_b	N_s	H_p	# of Constraints	# of Variables	Cost J_{pv}	Computation Time(s)
Case 1	5	10	5	3700	5553	3586	28.5920
Case 2	5	15	5	7600	10868	6383	66.1740
Case 3	5	20	5	12875	17933	10994	1046.6000
Case 4	7	10	7	7014	10345	4900	624.8500
Case 5	7	15	7	14434	20030	8454	3316.3000
Case 6	7	20	7	24479	32865	14034	4060.6000
Case 7	7	15	10	20620	28835	*	*
Case 8	10	10	10	13950	20428	*	*

* The computation time is more than 8 hours.

Clearly, the computational time of the centralized algorithm exponentially increases with the increase of the problem scale. We can find that Gurobi can solve a MBDBH problem under a bus line with 10 bus stops, 5 buses and 5 trip horizons in only 28.5920 second, which is equivalent to solving a mixed integer problem with 5553 integer and binary variables and 3700 constraints. This is definitely workable in practical

scheduling environment. However, when the problem scale becomes larger, the probability of successfully solving the dispatching-operation problem in short time becomes lower, e.g., a bus line with 7 buses, 20 bus stops and 7 trip horizons needs more than 3 hours to obtain a (near-)optimal signal profile, thus, the computational challenge requires more efficient algorithms. The star symbol in Table II means that MATLAB fails to run Gurobi due to memory insufficiency. By checking the Table II, we can also find that the computational time is not always monotonically increasing as the problem scale increases, e.g., Case 6 requires 14034 second to obtain the final results, however, Cases 7 and 8 need even more time to solve the problem even if their constraints and variables are smaller than that of Case 6. This is because of the predominant role of dispatching sequence $x_b(k)$ and dispatching time $DS(k)$ in determining computational time. Clearly, both dimensions of $x_b(k)$ and $DS(k)$ in Cases 7 and 8 are larger than that of Case 6. Just because of the dispatching pattern introduced into the operation control problem, our model leads to such higher computational complexity, which is also proofed in Section IV-B5. In view of this, we apply our distributed algorithm to test the corresponding computation time and performance results, as shown in Table III.

TABLE III: A comparison between the centralized algorithm and distributed algorithm

	Centralized Algorithm		Distributed Algorithm		Gap proportion
	Cost J_{pv}	Computation Time(s)	Computed lower bound G_λ	Computation Time(s)	
Case 3	10994	1046.6000	10115	694.7400	7.9953%
Case 4	4900	624.8500	4795	134.0600	2.1428%
Case 5	8454	3316.3000	7961	506.6100	5.8316%
Case 6	14034	4060.6000	10995	1107.2000	21.6546%

The computational time of centralized algorithm for Cases 1 and 2 are small, and it is redundant to partition the bus line when the problem scale is not large. Also, the results for Cases 7 and 8 cannot be obtained from the centralized way. Thus, we only compare the processing time and the result J_{pv} from Case 3 to Case 6, as shown in Table III. For all four cases, the bus line is partitioned into three sub-line segments. Cases clearly indicate that the proposed distributed approach significantly reduce the computational time of the larger-scale problems with longer prediction horizons. For example, the processing time for solving a 15 bus-stop system with 7 buses and 7 trip horizons is more than 50 minutes in centralized approach, which is reduced to roughly 8 minutes when adopting distributed approach. Moreover, the gap between the distribute algorithm and the centralized algorithm is still below 25% for all cases, which indicates that the proposed distributed algorithm provides a significant decrease of computation time meanwhile ensuring an acceptable degree of performance gap, especially when the problem scale is large.

B. Case study on a specific bus line

We test and compare the proposed strategy with three other methods. All four methods are applied on a loop bus line with 10 bus stops evenly spaced every 800 m, and the bus line

operates 7 buses with capacity of 80 passengers per bus. The pre-defined timetable TA starts from 360s towards 2520s with 6-min increments. Other initial parameters can be found from Table IV. The passenger arrival rate at each stop is shown in Fig. 4, which clearly illustrates the demand increases to the highest volume at the sixth stop and gradually decreases afterwards.

TABLE IV: Parameters used in case study

Parameters	Descriptions	Associated Values
N_b	Total serving buses	7
N_s	Bus stop number	10
H_p	Planning trips	7
TA	Pre-defined timetable	[360 720 1080 1440 1800 2160 2520]
Cap_b	Bus capacity	80
t_{oc}	Door open and close time	2s
α_1	Average boarding time for each passenger	2s
α_2	Average alighting time for each passenger	1s
T_{min}	Bus layover time	180s
DT_{max}	Maximum dwell time at each stop	180s

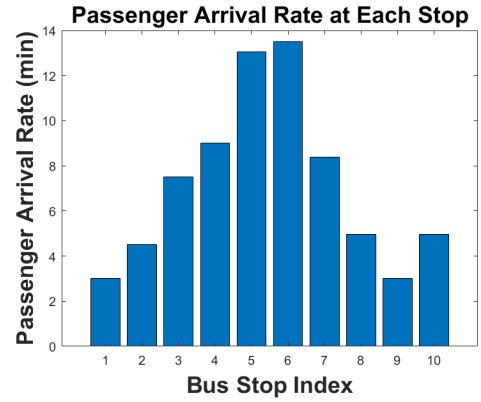


Fig. 4: The Passenger Arrival Rate at Each Stop

To clearly figure out the compared methods, we summarize four methods as follows:

- **MBDBH**: the objective function considers only the passengers' total delay, namely, $\beta_1 = 1$, $\beta_2 = 0$ for this method, and we consider dispatching time adjustment, boarding control and holding control in this method, also, the multi-bus dispatching is allowed to make full use of available buses.
- **MBDBH + Headway Control**: similar to MBDBH, the only difference is that the objective function considers both the passengers' total delay and the minimization of the headway gap, and β_1 , β_2 are set to 1 to fulfill this implementation.
- **Single Bus Dispatching with Boarding and Holding control (SBDBH)**: the objective is the same as the MBDBH, and this method also incorporates the dispatching time adjustment, boarding control and holding control. However, only one bus is allowed to be dispatched for each trip.
- **Timetable-based Fixed Dispatching (TFD)**: there is no control applied on this test case, the dispatching time strictly follows the pre-defined timetable, and each trip only dispatch one bus without applying boarding

and holding control, in other words, all dynamics are spontaneous evolution of the system.

1) *Passengers' Total Delay and Remaining Volume*: The total passenger excess delay of entire planning trips and total remaining passenger volume after entire planning trips for four different methods are summarized in Table V. The performance results respectively illustrate the total passenger delay J_{pd} and total remaining passenger volume J_{rpv} . By comparing with the TFD, the reduced proportion of the performance results when applying other methods is also listed. Clearly, MBDBH outperforms all other methods in both J_{pd} and J_{rpv} , and J_{pd} decreases roughly 77% compared to TFD. J_{rpv} also significantly drop 56% relatively to TFD. These all indicate that the efficient utilization of buses in MBDBH and the efficiency of the boarding and holding control. However, the advantages of MBDBH on minimizing J_{pd} and J_{rpv} decline a bit when headway control is also applied, which indicates that maintaining the system reliability enlarges a certain level of passenger excess delay, and this is really different with the results obtained from the traditional literature: the use of $\sum_{k=1}^{H_p} \sum_{i \in \{S\}} (\sum_{j>i} \frac{f_{ij}}{2})(AR_i(k+1) - AR_i(k))^2$ to capture the passenger total delay, instead of passenger excess delay in our paper, normally concludes that a positive correlation between the passenger delay and the headway variation. However, once incorporating on-board passenger delay, this positive correlation may not always be correct [38], which indicates that the relationship between the passenger delay and the headway variation depends on the type of the passenger delay. Also, the reduced proportions once considering headway variation are still significant compared to TFD, which are approximately 69% and 51%, respectively. With single-bus dispatching only, reductions of 38% and 31% are found when compared to TFD, which are roughly half improvement of the MBDBH, and this again indicates the proper utilization of the bus capacity when applying multi-bus dispatching.

TABLE V: Total passenger delay and remaining passenger volume under different methods

Different Methods	Performance Results		Reduced Proportion	
	Total Passenger Delay (s) J_{pd}	Total Remaining Passenger Volume J_{rpv}	$\frac{J_{pd}^{others} - J_{pd}^{TFD}}{J_{pd}^{TFD}}$	$\frac{J_{rpv}^{others} - J_{rpv}^{TFD}}{J_{rpv}^{TFD}}$
MBDBH	571200	955	-77.0137%	-56.3528%
MBDBH + Headway Control	755200	1072	-69.6092%	-51.0055%
SBDBH	1530500	1508	-38.4095%	-31.0786%
TFD	2484960	2188	-	-

2) *Bus Dispatching Time*: Fig. 5 depicts the gap between the actual dispatching time $DS(k)$ and the pre-defined dispatching time $TA(k)$ for each trip k . The first trip is assigned equal to the timetable in the simulation which leads to no gap value, thus, we only draw the gap from the second trip to the last trip. The blue bar, red bar and yellow bar respectively illustrate the dispatching time deviations under strategies MBDBH, MBDBH with headway control and SBDBH. Since strategy TFD strictly dispatches buses according to the timetable, only the other three methods are captured in Fig. 5. The time deviation of SBDBH is always the smallest one, and this is because the adjustment of the bus dispatching time has no need to consider the permutation and combination of the buses, and the phenomenon that at least one bus is waiting

at the terminal is very easy to be realized every time when corresponding dispatching time $TA(k)$ is reaching. However, this may not be applicable to MBDBH and MBDBH with headway control. Both multi-bus dispatching methods need to adjust the bus dispatching time by considering the permutation and combination of the buses so that the passenger total delay can be minimized, and the dispatching time could be increased in order to make sure that the terminal does have sufficient number of buses to be dispatched. By checking the figure, the time deviation reaches the maximum value at the third trip under MBDBH, to clearly find out the reason, the optimal results of the decision variables, dispatching indicator $x_b(k)$, for both MBDBH and MBDBH with headway control are listed in Table VI. The dispatched bus in each corresponding trip is highlighted in light blue in the table. By checking the Table VI(a), we can find that the first trip dispatches 4 buses and the second trip dispatches 3 buses, which indicates that all buses are released for the first two trips, and this leads to the large time deviation for the dispatching time of the third trip under MBDBH. Interestingly, the red bar in Fig. 5 increases monotonically with the increase of the trip index, and this phenomenon results from the minimization of the headway variation. The specific dispatching time of MBDBH with headway control can be found in Fig. 6, which clearly shows that the dispatching headway under headway control is fixed at 400s, however, the dispatching headway of timetable is fixed at 360s, and just because of this, the gap between pre-defined dispatching time and the actual dispatching time is becoming increasingly large.

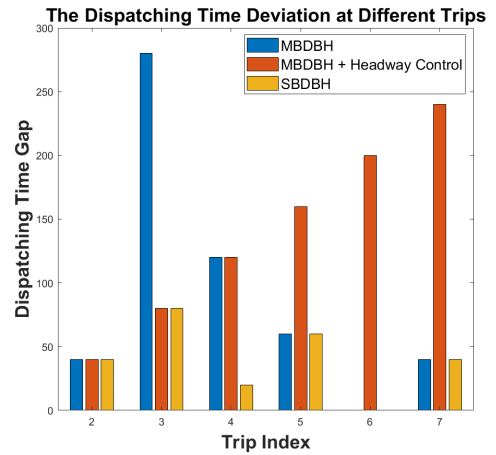


Fig. 5: The Gap between the Real Dispatching Time and the pre-defined Dispatching Time under Different Methods

3) *Bus Trajectories*: Fig. 7 illustrates the bus trajectories of different trips for four methods. Under the strategy of MBDBH, the distribution of bus trajectories is relatively messy, and a large gap can be found between the second trip and the third trip, and the reason has been explained in section IV-B2, as the previous two trips release out all buses from the terminal, which leads to a late allowed dispatching moment, since this moment must be larger than the arrival time of buses. Also, the uncertain dwell times at downstream several stops make the headway gap from trip three to trip seven deviate significantly,

TABLE VI: Bus Dispatching Sequence for Two Different Methods

$b \backslash k$	1	2	3	4	5	6	7
1	1	0	1	0	0	0	0
2	1	0	1	0	0	0	1
3	0	1	0	0	0	1	0
4	1	0	0	1	0	0	0
5	0	1	0	0	1	0	0
6	0	1	0	0	1	0	0
7	1	0	1	0	0	0	0

(a) Bus Dispatching Sequence for MBDBH

$b \backslash k$	1	2	3	4	5	6	7
1	1	0	0	1	0	0	0
2	1	0	0	1	0	0	0
3	1	0	0	1	0	0	0
4	0	1	0	0	1	0	0
5	0	0	1	0	0	1	0
6	1	0	0	1	0	0	1
7	0	1	0	0	1	0	0

(b) Bus Dispatching Sequence for MBDBH + Headway Control

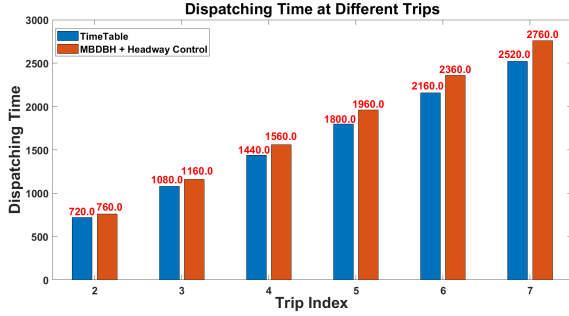


Fig. 6: The pre-defined Dispatching Time and the Actual Dispatching Time for MBDBH + Headway Control

which leads to low system reliability. We can also find that the large dwell time always appear after several stops, and this is consistent with the demand arrival rate in Fig. 4: A higher arrival volume starts from stop 5 and reaches the peak at stop 6, accordingly, which leads to relatively high boarding volume and dwell time at these stops. However, the dwell time can also be determined by the alighting process: the further downstream the stop located, the relatively larger alighting passengers the bus have, as the alighting flow is contributed by the sum of boarding flows in all upstream stops. Because of this, we can also find large dwell time after stop 6. To maintain a better

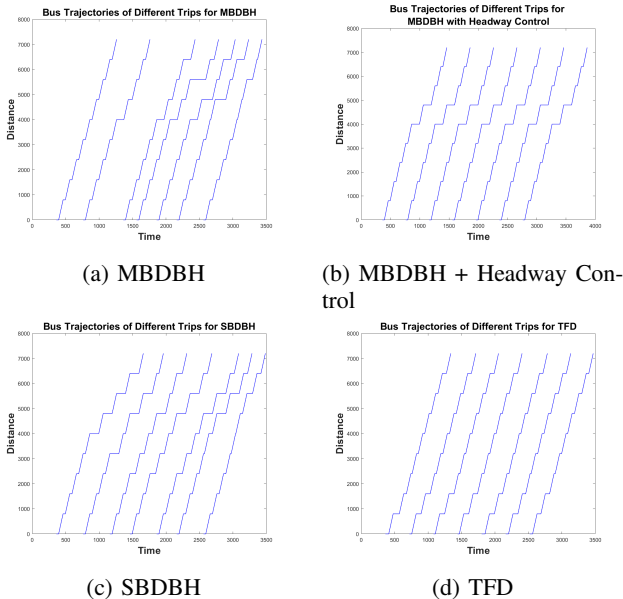


Fig. 7: Bus Trajectories under Different Methods

reliability, the headway control is applied in the objective function, and corresponding bus trajectories are shown in Fig. 7b. The headway distribution of bus trajectories is much more orderly compared to Fig. 7a. Also, the boarding process plays a dominant role in buses' dwell time relative to the alighting process, as the larger dwell time always can be found in stop 6 for each trip. For the strategy of SBDBH, the shape of its bus trajectories feels like an act of collaboration between Fig. 7a and Fig. 7b. Since the headway variation is not as obvious as trajectories under MBDBH, but it is also not so tidy as Fig. 7b, and this can be explained by its different dispatching way and the system objective function: On the one hand, the sufficient bus stock in the terminal determines a relatively low variation of headway at the initial dispatching stage under SBDBH, on the other hand, the bus movement dynamics will become efficiency-driven as the objective function only considers the total passenger delay without control of the headway, which leads to a relative messy distribution at several downstream stops. While for the last strategy, its bus trajectories also seem orderly. Since no control is applied in TFD, large dwell times are all found from first several stops, and after that, dwell times are small for remaining stops, and this is because the bus becomes full at very early stage and has no space to allow passengers at downstream stops to board. Although the distribution of bus trajectories looks quite tidy, the improper boarding arrangement largely wastes the resources and leads to much higher total passenger delay in the system.

4) *Boarding Volume*: Fig. 8 shows the boarding volume $\sum_j B_{b,i,j}(k)$ of each bus at different trips and stops under four different methods. For all subfigures in Fig. 8, the red bold line is the maximum value of all boarding volumes and the green bold line is the trip-based average boarding volume at different stops, namely, $\frac{\sum_k \sum_b \sum_j B_{b,i,j}(k)}{H_p}$. Due to multi-bus

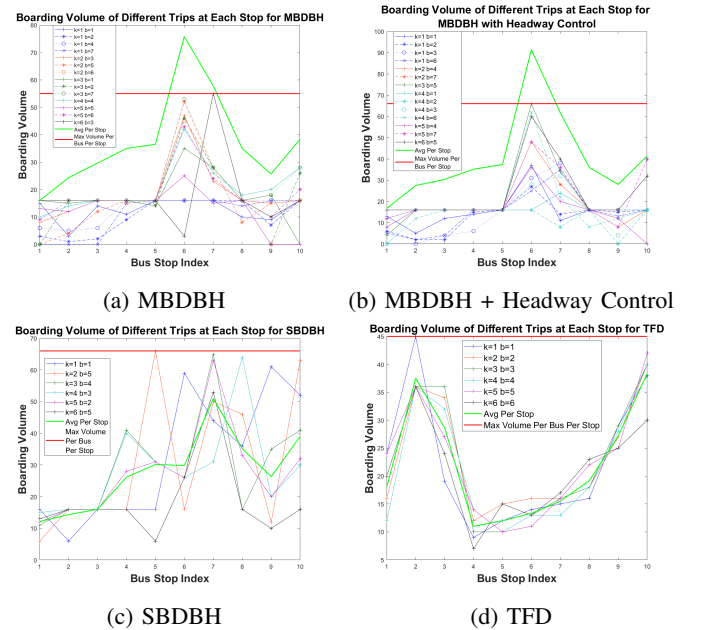


Fig. 8: Boarding Volume at Different Stops under Different Methods

dispatching is allowed in the first two methods, more lines are

drawn in Fig. 8a and Fig. 8b. For both strategies of MBDBH and MBDBH with headway control, most lines reaches the highest value at stop 6, which results in the average boarding volume, the bold green line, also reaches the largest value at stop 6, and this is consistent with the passenger arrival rate in Fig. 4. Also, it can be found that the largest average boarding volume on the bold green line in Fig. 8b has already exceeded the bus capacity, 80. Also, both average green lines in Fig. 8a and Fig. 8b are almost above all the other lines, and this is because the current average boarding volume is trip-based average and each trip can allow multi-bus dispatching in these two methods, while the other lines (those thin lines) drawn in the figure are the boarding volume targeted to each bus, which leads to this phenomenon in the figures. The average boarding volume in Fig. 8b is slightly larger than that in Fig. 8a, which means the strategy of MBDBH with headway control leads to larger boarding volume compared with MBDBH, but the total passenger delay of MBDBH in Table V is the smallest one. This may seem quite conflict but the reason behind is that although MBDBH with headway control brings larger boarding flow, it also leads to larger ending time for the same trip horizon, which can be clearly found from the x-axis of the bus trajectories figure. The finishing time of the last trip in Fig. 7b is larger than 3500s while the finishing time in Fig. 7a is less than 3500s, and under the same planning trips, the longer the total finishing time, the larger the introduced passenger volume into system, which leads to larger delay even if the boarding volume is relatively higher in MBDBH with headway control, and this also indicates that the addition of the headway control decreases the bus operating speed. For the strategy of SBDBH, the green bold line truly reflects the average of all other fine lines, as each trip only allows one bus to be dispatched. Although the convergent property of these fine lines is not obvious as the lines in Fig. 8a and Fig. 8b, they can still illustrate a certain level of consistency with the passenger arrival rate in Fig. 4. While for boarding volumes under TFD, we can find that the boarding volume reaches the largest value at the second stop and leads to low bus space for remaining downstream stops, especially missed the opportunity to board more passengers at stops with larger passenger volume, and it increases gradually at final several stops due to the spared space by moving out the alighting passengers at those middle stops. Remarkably, the largest boarding volume, 66, for each bus appears in both MBDBH with headway control and SBDBH, by checking all red bold lines of four subfigures, while the smallest value, 45, is found in TFD strategy, and this also reflects the efficient utilization of the buses in optimization methods. Also, the largest value appeared in boarding volume of each bus under SBDBH does not indicate that the SBDBH will have more total boarding volumes compared to MBDBH, since MBDBH allows multi-bus dispatching and it fails in the bus-based average but successes in the trip-based average compared to SBDBH.

5) *Computational Time for Different Optimization Strategies*: Table VII illustrates the computational time of three optimization methods: MBDBH, MBDBH + Headway Control and SBDBH. Both MBDBH and SBDBH have the same number of constraints and variables, the only difference is that

the constraint (5e) in MBDBH is an inequality but change to equality in SBDBH. The reduction on the dimension of dispatching sequence $x_b(k)$ significantly decrease the computational time for SBDBH, which is roughly 3 min listed in Table VII. On the other hand, the addition of the headway control introduces new variables and constraints, and make the solver spend much more time on trade-off between the efficiency and reliability. In our future work, we will try to enhance the current distributed algorithm or design new algorithms to solve the current problem in a more efficient way. On the other hand, the bus initial states (bus initial position and initial volume) shall be introduced into current model to make the rolling-horizon mechanism become possible. Under the structure of the model-predictive control, there is no need to consider a very long trip horizon, and this could largely reduce our constraints and decision variables involved in the model to enable the real-time control implementation.

TABLE VII: Computation time for different optimization methods

Different Methods	# of Constraints	# of Variables	Computation Time (s)
MBDBH	10345	7014	649.7100
MBDBH + Headway Control	10439	7064	1659.1880
SBDBH	10345	7014	193.709

C. Sensitivity Analysis

In this section, we provide the sensitivity analysis of multi-bus dispatching strategy when different round-trip distances and time-variant passenger demands are applied, with the aim to further understand the multi-bus dispatching strategy.

1) *Round-Trip Distance*: We test on a loop bus line with 10 bus stops evenly spaced, and the bus line is operating by 7 buses under the same capacity, passenger arrival rate and pre-defined timetable in Section IV-B, however, the round-trip distances vary from 8 km to 20 km. Fig. 9 illustrates the total passenger delay of MBDBH and SBDBH under different round-trip distances. Clearly, the passenger delay increases with the increase of the round-trip distance D^r for both methods, since longer loop distance leads to larger round-trip travel time and larger dispatching time interval even under the same number of dispatching trips. Interestingly, the delay gap between MBDBH and SBDBH becomes increasingly small as the round-trip distance increases, which indicates that the advantages of MBDBH is shrinking under a larger-loop bus line. This is also clearly reflected in the bus dispatching sequences $x_b(k)$ shown in Fig. VIII, which illustrates the value of $x_b(k)$ of MBDBH under four different round-trip distances. Trips with multi-bus services are highlighted in red. We can find that the number of multi-bus trips decreases from 4 to 2 as D^r increases from 8 km to 12 km. Although the number of multi-bus trips keep the same as D^r increases from 12 km to 20 km, the total number of dispatched buses in all multi-bus trips reduces from 7 when D^r is 12 km to 5 when D^r is 20 km. Due to the increase of the round-trip distance,

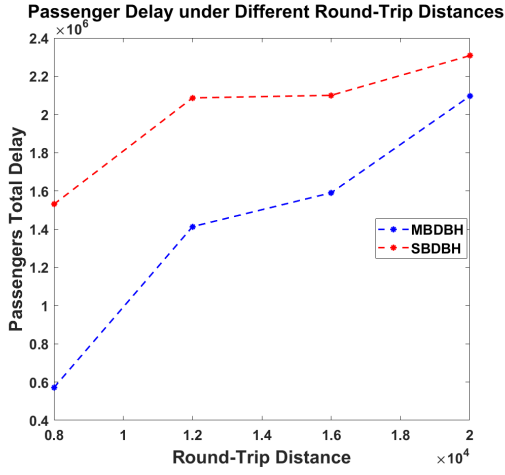


Fig. 9: Total Passenger Delay of MBDBH and SBDBH under Different Round-Trip Distances

the chance to dispatch multiple buses for one trip becomes increasingly small under the same number of trip horizons, and this indicates that the advantage of MBDBH is more noticeable when the bus line is a short-distance loop, which makes the efficient utilization of bus capacity feasible.

TABLE VIII: Bus Dispatching Sequence under Different Round-Trip Distances

$b \backslash k$	1	2	3	4	5	6	7
1	1	0	1	0	0	0	0
2	1	0	1	0	0	0	1
3	0	1	0	0	0	1	0
4	1	0	0	1	0	0	0
5	0	1	0	0	1	0	0
6	0	1	0	0	1	0	0
7	1	0	1	0	0	0	0

(a) D^r is 8km

$b \backslash k$	1	2	3	4	5	6	7
1	1	0	0	0	0	0	1
2	0	1	0	0	0	0	1
3	0	0	0	1	0	0	0
4	0	0	1	0	0	0	0
5	1	0	0	0	1	0	0
6	1	0	0	0	1	0	0
7	1	0	0	0	0	1	0

(b) D^r is 12km

$b \backslash k$	1	2	3	4	5	6	7
1	1	0	0	0	0	1	0
2	0	0	1	0	0	0	0
3	0	0	0	1	0	0	0
4	1	0	0	0	0	1	0
5	0	1	0	0	0	0	1
6	1	0	0	0	1	0	0
7	1	0	0	0	1	0	0

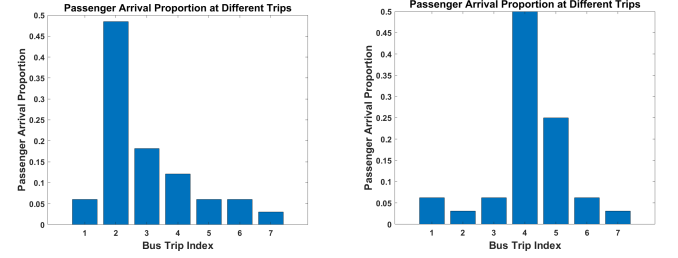
(c) D^r is 16km

$b \backslash k$	1	2	3	4	5	6	7
1	0	0	0	1	0	0	0
2	0	0	0	0	1	0	0
3	1	0	0	0	0	1	0
4	1	0	0	0	0	0	1
5	1	0	0	0	0	0	1
6	0	0	1	0	0	0	0
7	0	1	0	0	0	0	0

(d) D^r is 20km

2) *Time-variant Demand*: The advantage of MBDBH becomes prominent especially when the passenger arrival demands are timely fluctuated. The daily passenger arrival pattern can be learnt from the massive accumulated traffic historical data, accordingly, function $f_{ij}(AR_i(k+1), AR_i(k))$ in constraint (1) can be found to realistically describe the passenger arrival rate. Since the passenger arrival pattern, as a known input of our optimization model, is not the study emphasis of this paper, we use a constant arrival rate f_{ij} to test the proposed strategy in the above case study, which leads $f_{ij}(AR_i(k+1), AR_i(k))$ to $(AR_i(k+1) - AR_i(k))f_{ij}$. However, we do want to see how MBDBH dispatches buses when the demands are time-variant. Thus, we assume the passenger arrival rate changes according to the dispatching trip for simplicity, with the aim to see whether the number of dispatched buses at a certain trip will change with the changes

of passenger demands at that trip.



(a) First time-variant demand pattern (b) Second time-variant demand pattern

Fig. 10: Demand Patterns for Two Different Cases

TABLE IX: Bus Dispatching Sequence under Time-Variant Demands

$b \backslash k$	1	2	3	4	5	6	7
1	1	0	0	0	0	1	0
2	0	1	0	0	0	0	1
3	0	0	1	0	0	0	0
4	0	0	1	0	0	0	0
5	0	0	0	0	1	0	0
6	0	0	0	1	0	0	0
7	0	0	1	0	0	0	0

(a) $x_b(k)$ under first time-variant demand pattern

$b \backslash k$	1	2	3	4	5	6	7
1	1	0	0	0	0	1	0
2	0	0	0	0	1	0	0
3	0	1	0	0	0	0	1
4	0	0	0	1	0	0	0
5	0	0	1	0	0	0	0
6	0	0	0	0	1	0	0
7	1	0	0	0	0	1	0

(b) $x_b(k)$ under second time-variant demand pattern

The proposed method is still tested on a 8km-loop bus line with 10 bus stops evenly spaced, and the bus line operates 7 buses under the same capacity and pre-defined timetable in Section IV-B. The proportion of passenger demands across the whole stops is the same, but the demands change over the trip index, as shown in Fig. 10a and Fig. 10b. Table IX(a) and IX(b) illustrate the corresponding optimal dispatching sequence results of MBDBH under the passenger demands shown in Fig. 10a and Fig. 10b, respectively. The proposed method belongs to the discrete-time model, accordingly, the update of the passenger demands at each stop only occurs at the trip instant, as shown in constraint (1), thus, the changes of demands during trip interval k can only be reflected after trip instant $k+1$. Because of this, trips which dispatch multiple buses, shown in Table IX, are one-step lagging behind the demand patterns illustrated in Fig. 10. But from the overall view, the number of dispatched buses for each trip will change according to the changes of the passenger demands, which further illustrates the potential benefit of MBDBH, and this advantage stands out especially when a sudden demand increase happens, which could not be realized in a single bus dispatching strategy.

V. CONCLUSION AND FUTURE WORK

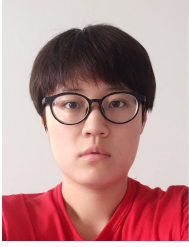
In this paper, a multi-bus dispatching strategy with operation control is proposed for a loop-shaped bus line system. Either a single bus or a bus platoon is allowed to be dispatched from the terminal in order to create adaptive bus capacities to meet the fluctuated passenger demands. Also, boarding control in each stop is applied with the aim to avoid the phenomenon that bus space is fully occupied at several initial

upstream stops which leaves no space for passengers waiting at the downstream stops. Additionally, holding control at each bus stop and headway control formulated in the objective function are implemented to fulfill the system reliability. The proposed model is converted into the MINP problem, which is firstly solved by the commercial solver, Gurobi. A distributed algorithm is also proposed via Lagrangian multipliers with the aim to further reduce the computational time. Moreover, we compare four different methods in a variety of different perspectives, e.g., from objective performance, bus trajectory, boarding volume to computational time. The results illustrate that the addition of the dispatching adjustment (dispatching sequence and time) into the operation system largely reduces the passenger delay, at the price of dramatically increased computational complexity.

To make this proposed approach practically implementable, we shall consider the following measures in our next step: besides meta-heuristic algorithms such as evolution algorithms to further reduce computational time, we will adopt a receding horizon strategy to minimize the number of decision variables and constraints within each optimization horizon. Also, the extended model based on the rolling-horizon mechanism could alleviate the impacts of the uncertainty involved in the passenger arrival patterns. All these results shall be presented in our future works.

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