Error Analysis and Uncertainty Estimation for a Millimeter-wave Phase-shift Measurement System at 325 GHz

T. Y. Wu[a,*], Y. L. Lu[b]

[a] *National Metrology Centre, A*STAR, 1 Science Park Drive, Singapore 118221*
[b] *School of Electrical and Electronic Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798*

Abstract

Error analysis and uncertainty estimation for a millimeter-wave phase-shift measurement system at 325 GHz are presented. The measurement errors due to inductive voltage divider, receiver noise, drift, mismatch, gauge-block attenuator, leakage and repeatability have been identified and evaluated. The standard uncertainty of these error components have been estimated and combined to obtain the expanded uncertainty in phase-shift measurement. The differential phase-shift measurement uncertainty for a 0–60 dB WR-03 rotary vane attenuator is estimated to be 0.57°–1.3° at 325 GHz.

Keywords: Phase-shift measurement, error analysis, uncertainty estimation, millimeter-wave metrology standard, heterodyne receiver

---

1. Introduction

Millimetre-wave is increasingly being used in broadband communications, radar, biomedical imaging, security imaging and non-destructive testing [1–6]. Accurate measurement of millimetre-wave transmission phase-shift is important in many applications. A 70 GHz phase-shift measuring system has been developed for plasma diagnosis [7]. This system used a single-channel homodyne receiver to derive the millimeter-wave phase-shift through a phase-comparator at 2 MHz. Millimeter-wave phase-shift measurement standard has been studied using a vector network analyser (VNA) and an air line with 1.85 mm connector for frequencies up to 65 GHz [8]. The increasing demand for fast wireless data transmission applications requires com-

*Corresponding author. Tel: +65 62791928
Email address: thomas_wu@nmc.a-star.edu.sg (T. Y. Wu)

munication systems around 300 GHz [4, 9, 10]. A millimeter-wave network analyzer [11] or a dual-channel VNA [12, 13] can be used to measure phase-shift at 50–1000 GHz. The expanded uncertainty of attenuation measurement for a 30 dB attenuator using a VNA is estimated to be 0.78 dB at 220–325 GHz [13]. Based on the attenuation measurement uncertainty, the expanded uncertainty of transmission phase-shift measurement for a 30 dB attenuator using VNA can be estimated to be 5.4° at 220–325 GHz [12]. We have developed a millimeter-wave phase-shift measurement system at 220–325 GHz to provide phase-shift measurement with high accuracy[14]. A variable phase changer or a waveguide shim measured by this system can serve as a phase-shift reference standard to provide reference for VNA or other millimeter-wave instruments. The detailed measurement error analysis had not been given in [14]. In this paper, we will provide a detailed analysis of the measurement errors in the phase-shift measurement system at 325 GHz and estimate the overall measurement uncertainty.

2. Comments on the Measurement System

The differential phase-shift of a variable attenuator or phase changer and the insertion phase-shift of a waveguide shim or a fixed attenuator at 220–325 GHz can be measured using a dual-channel heterodyne receiver. A block diagram of the measurement system is given in Fig. 1. A millimeter-wave signal at 220–325 GHz is generated by a microwave synthesizer via a multiplier chain ($\times 18$). Two subharmonic mixers are used to down-convert the signal to a main and a reference intermediate frequency (IF) signal at 2.5 GHz, which are further down-converted to 5.02 kHz. The differential or insertion phase-shift of a device under test (DUT) is measured by detecting the phase changes of the audio frequency (AF) signal using the LIA.

The attenuation of the DUT will decrease the AF signal level at the output of main radio frequency (RF) mixer. An inductive voltage divider (IVD) is used to keep the magnitude of the AF signal at a constant level when it is measured by the LIA so that the nonlinearity of LIA does not affect the phase measurement accuracy. The measured phase-shift is the difference between two measurements of the phase angle detected by the LIA, thus the phase angle detection need to be stable to ensure good measurement precision and accuracy. The receiver has been designed to reduce the effect of thermal noise and phase noise so that the random fluctuation and short-term drift in phase angle reading can be minimized.

3. Error analysis and uncertainty estimation

There are multiple sources of error in the measurement of phase-shift using our system. These include errors due to IVD, receiver noise, receiver phase drift, mismatch error at the test port, leakage, measurement repeatability and phase-shift measurement error of gauge-block attenuator (for DUT with attenuation more than 20 dB). Detailed analysis of these measurement errors and estimation of their standard uncertainties are given in the following subsections. The standard uncertainty of each error component will be combined to obtain the combined and expanded uncertainty of the measurement system [15].

3.1. Phase error due to IVD

The millimeter-wave signal is down-converted to a signal at 5.02 kHz, which goes through an IVD before its phase is measured by the LIA. A seven-decade IVD is used in our system. The IVD input voltage, V_{in}, and output voltage, V_{out}, is related by

$$V_{out} = V_{in}\ d\ (1 + \alpha + j\beta) \tag{1}$$

where d is the ratio setting of the IVD, α and β are the in-phase and quadrature deviation of IVD, respectively[16, 17]. Values of α and β for the first 3 dials of the IVD are obtained from calibration of IVD. α ranges from 3.1×10^{-6} to 1.6×10^{-4}. β ranges from -5.0×10^{-5} to -2.0×10^{-5} for $0.5 \leq d \leq 0.8$ and from 4.8×10^{-5} to 6.9×10^{-4} for $0.008 \leq d < 0.5$. The above equation can be rewritten as

$$V_{out} = V_{in}\ d\ \sqrt{(1+\alpha)^2 + \beta^2}\ e^{j\theta_e} \tag{2}$$

where

$$\tan\theta_e = \frac{\beta}{1+\alpha} \approx \beta \tag{3}$$

since α is much smaller than 1. θ_e indicates the phase angle error introduced by IVD.

The IVD ratio changes involved in our measurement is from 0.008 to 0.8. We assume the phase-shift measurement error due to IVD is uniformly distributed in

$[-e_i, e_i]$. Based on the β values provided by IVD calibration report, the maximum absolute error in phase-shift measurement due to IVD is estimated to be $e_i = 0.015°$ for DUT with attenuation variation below 20 dB. According to the Guide to the expression of Uncertainty in Measurement (GUM)[15], the standard uncertainty in phase-shift measurement caused by IVD is estimated to be

$$u_i = \frac{e_i}{\sqrt{3}} = 0.009°. \tag{4}$$

Similarly, the maximum absolute error in phase-shift measurement due to IVD is estimated to be $e_i = 0.042°$ when DUT attenuation variation is 0–60 dB. Hence the standard uncertainty in phase-shift measurement caused by IVD for this case is estimated to be

$$u_i = \frac{e_i}{\sqrt{3}} = 0.024°. \tag{5}$$

Table 1 shows the standard uncertainties for phase error due to IVD for DUT attenuation variation from 0 to 60 dB.

3.2. Receiver noise

The detected signal by the LIA has random fluctuations in its phase reading which is caused by receiver thermal noise. The phase-shift measurement error due to receiver noise can be assumed to be uniformly distributed in $[-e_n, e_n]$, where the maximum absolute value, e_n, can be measured by our system.

When the DUT attenuation is below 20 dB, e_n is measured to be 0.1° and the standard uncertainty due to the receiver noise can be estimated as

$$u_n = \frac{e_n}{\sqrt{3}} = 0.058°. \tag{6}$$

The fluctuation increases when the noise effect is more significant at higher attenuation levels. When the DUT attenuation variation is 0–60 dB, e_n is measured to be 0.5° and the standard uncertainty due to the receiver noise can be estimated as

$$u_n = \frac{e_n}{\sqrt{3}} = 0.29°. \tag{7}$$

The standard uncertainties due to the receiver noise for DUT attenuation variation at 0–30 dB, 0–40 dB and 0–50 dB can also be estimated using the corresponding maximum fluctuations obtained via measurement by LIA.

3.3. Mismatch error

3.3.1. Mismatch error for differential phase-shift measurement

When a 2-port network (with scattering parameters s_{11b}, s_{12b}, s_{22b} and s_{21b}) inserted between the test ports changes to another 2-port network (with scattering parameters s_{11e}, s_{12e}, s_{22e} and s_{21e}), the differential phase-shift between the two networks can be derived as

$$\theta_D = \arg\left(\frac{s_{21e}\cdot[(1-\Gamma_G s_{11b})(1-\Gamma_L s_{22b})-\Gamma_G\Gamma_L s_{12b}s_{21b}]}{s_{21b}\cdot[(1-\Gamma_G s_{11e})(1-\Gamma_L s_{22e})-\Gamma_G\Gamma_L s_{12e}s_{21e}]}\right) \tag{8}$$

where Γ_G and Γ_L are the reflection coefficient of port 1 and port 2.

Mismatch error in differential phase-shift measurements has been investigated in [18]. The mismatch error can be derived as

$$\begin{aligned} E_{mis} &= \arg\left\{\frac{(1-\Gamma_G s_{11e})(1-\Gamma_L s_{22e})-\Gamma_G\Gamma_L s_{12e}s_{21e}}{(1-\Gamma_G s_{11b})(1-\Gamma_L s_{22b})-\Gamma_G\Gamma_L s_{12b}s_{21b}}\right\} \\ &\approx \arg\{1-\Gamma_G s_{11e}-\Gamma_L s_{22e}+\Gamma_G\Gamma_L(s_{11e}s_{22e}-s_{12e}s_{21e}) \\ &\quad +\Gamma_G s_{11b}+\Gamma_L s_{22b}-\Gamma_G\Gamma_L(s_{11b}s_{22b}-s_{12b}s_{21b})\} \end{aligned} \tag{9}$$

since the magnitudes of reflection coefficient of test port and DUT are all much smaller than 1. The limit of the mismatch error in differential phase-shift measurements may be written as

$$\begin{aligned} \sin(\lim E_{mis}) &= |\Gamma_G s_{11b}|+|\Gamma_G s_{11e}|+|\Gamma_L s_{22b}|+|\Gamma_L s_{22e}|+ \\ &\quad |\Gamma_G\Gamma_L|(|s_{11b}s_{22b}|+|s_{11e}s_{22e}|+|s_{12b}s_{21b}|+|s_{12e}s_{21e}|) \end{aligned} \tag{10}$$

The mismatch error in differential phase-shift measurements is assumed to be uniformly distributed in $[-\lim E_{mis}, \lim E_{mis}]$. Thus the standard uncertainty due to mismatch is

$$u_m = \frac{\lim E_{mis}}{\sqrt{3}} \tag{11}$$

A 10 dB attenuator is connected at each test port to reduce the mismatch uncertainty in phase-shift measurement. Based on the measured S-parameters of DUT and test ports, the maximum absolute mismatch error, $\lim E_{mis}$, can be obtained using the above equation. The maximum value of $\lim E_{mis}$ for a WR-03 rotary vane attenuator (RVA) with attenuation from 0 dB to 60 dB in 10 dB step have been used to estimate the standard mismatch uncertainty, u_m, to be 0.22° at 325 GHz.

3.3.2. Mismatch error for insertion phase-shift measurement

When a 2-port network with scattering parameters s_{11}, s_{12}, s_{22} and s_{21} is inserted between the test ports, its insertion phase-shift can be measured. The insertion phase-shift is a special case of the differential phase-shift, where the initial 2-port network is lossless, non-reflecting and has zero characteristic phase-shift (these conditions are $s_{11b} = s_{22b} = 0$ and $s_{21b} = s_{12b} = 1$). Thus the insertion phase-shift is

$$\theta_I = \arg\left(\frac{s_{21}\cdot(1-\Gamma_G\Gamma_L)}{(1-\Gamma_G s_{11})(1-\Gamma_L s_{22})-\Gamma_G\Gamma_L s_{12}s_{21}}\right) \tag{12}$$

Mismatch error for insertion phase-shift measurement has not been studied in previous literature. The mismatch error for insertion phase-shift measurement can be derived as

$$\begin{aligned} E_{mis} &= \arg\left\{\frac{(1-\Gamma_G s_{11})(1-\Gamma_L s_{22})-\Gamma_G\Gamma_L s_{12}s_{21}}{1-\Gamma_G\Gamma_L}\right\} \\ &\approx \arg\left\{1-\Gamma_G s_{11}-\Gamma_L s_{22}+\Gamma_G\Gamma_L(s_{11}s_{22}-s_{12}s_{21}+1)\right\} \end{aligned} \tag{13}$$

The approximation in the above equation is made since the reflection coefficients of the test port and DUT all have small magnitude values, which are much less than 1. Let

$$E'_d = 1-\Gamma_G s_{11}-\Gamma_L s_{22}+\Gamma_G\Gamma_L(s_{11}s_{22}-s_{12}s_{21}+1), \tag{14}$$

and

$$E'_p = -\Gamma_G s_{11}-\Gamma_L s_{22}+\Gamma_G\Gamma_L(s_{11}s_{22}-s_{12}s_{21}+1), \tag{15}$$

thus

$$E_{mis} = \arg(E'_d) = \arg(1+E'_p) \tag{16}$$

For arbitrary phases of the coefficients, the magnitude of E'_p is at its maximum value when all of the vectors have the same directions, i.e.,

$$\lim|E'_p| = |\Gamma_G s_{11}|+|\Gamma_L s_{22}|+|\Gamma_G\Gamma_L|(|s_{11}s_{22}|+|s_{12}s_{21}|+1) \tag{17}$$

Similarly, the maximum value of E_{mis} will occur when $|E'_p|$ is at its maximum and the vector E'_p is perpendicular to E'_d, as shown in Fig. 2. Thus the limit of the mismatch error can be derived as

$$\sin(\lim E_{mis}) = |\Gamma_G s_{11}|+|\Gamma_L s_{22}|+|\Gamma_G\Gamma_L|(|s_{11}s_{22}|+|s_{12}s_{21}|+1) \tag{18}$$

The mismatch error in insertion phase-shift measurements is assumed to be uniformly distributed in $[-\lim E_{mis}, \lim E_{mis}]$ and the standard uncertainty due to mismatch can be derived as

$$u_m = \frac{\lim E_{mis}}{\sqrt{3}} \quad (19)$$

Based on the measured S-parameters of DUT and test ports, $\lim E_{mis}$ can be calculated. The standard uncertainty due to mismatch error in insertion phase-shift measurements of a WR-03 waveguide shim (used as a DUT) is estimated to be 0.14° at 325 GHz.

3.4. Phase drift

The phase noise of the source signal at 325 GHz is about −16 dBc and −74 dBc at 1 Hz and 1 kHz offset of the carrier frequency, respectively. The detected phase by the main channel receiver has some drift due to the phase noises of the millimeter-wave source, the local oscillators and the multiplier chains. The LIA uses a phase reference signal produced by the reference channel receiver, which helps to stabilize the phase angle reading detected by the LIA. However, the local oscillator (LO) signal for the reference subharmonic mixer still has some random phase jitter which is not synchronized with the LO for the main subharmonic mixer. Thus some phase drifts in the receiver can be expected.

The phase drift error can be assumed to be uniformly distributed in $[-e_d, e_d]$, where e_d can be measured by the LIA in our system for DUT attenuation ranging from 0 to 60 dB. The measurement of phase-shift of a DUT can be completed within one minute. The maximum absolute phase drift in one minute, e_d, has been measured by the LIA in our system to be 0.3° when the DUT attenuation is below 20 dB, and measurements show e_d increases to 0.8° when the DUT attenuation variation is 60 dB. When the DUT attenuation is below 20 dB, the standard uncertainty due to phase drift error can be estimated as

$$u_d = \frac{e_d}{\sqrt{3}} = 0.17^\circ. \quad (20)$$

When the DUT attenuation variation is 0–60 dB, e_d is measured to be 0.8° and the standard uncertainty due to the phase drift error can be estimated as

$$u_d = \frac{e_d}{\sqrt{3}} = 0.46^\circ. \quad (21)$$

The standard uncertainties due to the phase drift for DUT attenuation variation at 0–30 dB, 0–40 dB and 0–50 dB have been estimated based on the measured value of e_d for each case, as shown in Table 1.

3.5. Leakage

A millimeter-wave leakage signal refers to the signal which arrives at the receiver via a path different from the main path which goes through the DUT. For a DUT with high attenuation, the leakage signal might cause a phase-shift measurement error. When the DUT is set to its datum position (0 dB attenuation), the signal at Port 2 is

$$\vec{x}_1 = \vec{x}_N + \vec{x}_L \tag{22}$$

where $\vec{x}_N$ is the main path signal and $\vec{x}_L$ is the leakage signal. We can assume $\vec{x}_N$ has a magnitude much higher than that of the leakage signal. Thus

$$\vec{x}_1 \approx \vec{x}_N \tag{23}$$

When the DUT is set to a high attenuation, the signal at Port 2 can be expressed as

$$\vec{x}_2 = \vec{x}_M + \vec{x}_L \tag{24}$$

where $\vec{x}_M$ is the main path signal. The angle between the two vectors $\vec{x}_2$ and $\vec{x}_M$ is referred to as θ_L. The phase-shift measurement error due to leakage, E_L, can be derived as

$$E_L \approx \theta_L = \arg(\vec{x}_2) - \arg(\vec{x}_M). \tag{25}$$

The phase difference between $\vec{x}_M$ and $\vec{x}_L$ is unknown. The maximum error in phase-shift measurement occurs when $\vec{x}_2$ is perpendicular to $\vec{x}_L$, as shown in Fig. 3. Thus the maximum of E_L is derived as

$$\max(E_L) = \sin^{-1} \frac{|\vec{x}_L|}{|\vec{x}_M|} \tag{26}$$

The DUT attenuation is

$$A_{DUT} = 20 \log_{10} \frac{|\vec{x}_1|}{|\vec{x}_M|} \quad \text{(dB)} \tag{27}$$

We define the leakage path attenuation as

$$A_L = 20 \log_{10} \frac{|\vec{x}_1|}{|\vec{x}_L|} \quad \text{(dB)} \tag{28}$$

then

$$A_L - A_{DUT} = 20\log_{10}\frac{|\vec{x}_M|}{|\vec{x}_L|} \quad \text{(dB)} \tag{29}$$

and

$$\max(E_L) = \sin^{-1}\left(10^{\frac{A_{DUT}-A_L}{20}}\right) \tag{30}$$

Fig. 4 shows the computed maximum of E_L for various values of $A_L - A_{DUT}$ ranging from 50 dB to 110 dB. To keep the maximum of E_L to be smaller than 0.00057°, the magnitude of the leakage path signal x_L has to be at least 100 dB below that of the main path signal x_M , i.e., $A_L - A_{DUT} \geq 100$ dB.

There is an internal RF leakage path which goes through the reference RF mixer, RF isolators, the directional coupler and the main RF mixer RF port. To reduce such leakage, six RF isolators are inserted in the path between LO port of two RF mixers to provide total isolation of around 180 dB. Directional couplers with high directivity are used to provide 50 dB isolation between the two channels. The isolation between the main RF mixer RF port and the LO port is about 30 dB. The total internal leakage path attenuation is more than 180 dB, thus the leakage error is negligible for measuring phase-shift of a DUT with attenuation below 60 dB.

3.6. Repeatability

The differential phase-shift of a 0–60 dB RVA at WR-03 band has been measured at 325 GHz to estimate the error due to repeatability. Fig.5 shows the precision RVA used for this measurement. A precision RVA has good repeatability in its attenuation and differential phase-shift for various dial settings [19, 20]. Six measurements have been made for each variation of attenuation step (0–10 dB, 0–20 dB, ..., 0–60 dB). According to the Guide to the expression of Uncertainty in Measurement (GUM)[15], the standard uncertainty due to measurement repeatability error is obtained as

$$u_r = \sqrt{\frac{\sum_{k=1}^{n}(y_k - \bar{y})^2}{n(n-1)}} \tag{31}$$

where

$$\bar{y} = \frac{\sum_{k=1}^{n} y_k}{n} \tag{32}$$

and y_k are the phase-shift measurement data and n is the number of measurement $(n = 6)$.

3.7. Phase-shift measurement error of gauge-block attenuator

When the phase-shift of a variable attenuator with its attenuation varies from datum setting to A_d ($A_d > 20$ dB), the level-set attenuator (LSA) is replaced by a Gauge-block Attenuator (GBA) with variable attenuation from 0 to 20 dB. The GBA is first set to 20 dB when DUT is at its datum setting, and the phase angle measured by the system is θ_1. The GBA is then set to 0 dB when DUT attenuation is set to A_d, and the phase angle measured by the system is θ_2. The differential phase-shift of DUT is then obtained from

$$\theta_{DUT} = \theta_{GBA} + \theta_2 - \theta_1, \tag{33}$$

where θ_{GBA} is the measured differential phase-shift of the GBA (from 0 to 20 dB). This is called an indirect method for phase-shift measurement.

The phase-shift of the GBA has been measured previously by our proposed system with a estimated measurement uncertainty, which should be included as one of the uncertainty components for the DUT phase-shift measurement using indirect method when its attenuation variation is more than 20 dB. The standard uncertainty due to GBA is

$$u_g = \frac{U_g}{2}, \tag{34}$$

where U_g is the expanded phase-shift measurement uncertainty of the GBA with a coverage factor of 2. In Table 1, the expanded phase-shift measurement uncertainty of the GBA is estimated to be 0.57°, hence the u_g is estimated to be 0.29° and it is included as one of the uncertainty components for phase-shift measurement of DUT when its attenuation variation is more than 20 dB.

3.8. Combined and expanded uncertainty

The measurement uncertainties due to IVD phase error, receiver noise, phase drift, mismatch error, repeatability and GBA phase-shift measurement error (only for DUT attenuation variation beyond 20 dB) are independent of each other. Hence they can be combined using root-sum-of-the-squares method [15] to yield the combined standard uncertainty of the phase-shift measurement, u_s, which is given by

$$u_s = \sqrt{u_i^2 + u_n^2 + u_d^2 + u_g^2 + u_m^2 + u_r^2} \tag{35}$$

The expanded uncertainty of the phase-shift measurement at 95% confidence level is obtained as

$$U = ku_s \tag{36}$$

where k is the coverage factor. The effective degrees of freedom are sufficiently large (>15000), thus the coverage factor $k = 2$. Table 1 shows the uncertainty budget for differential phase-shift measurement of a WR-03 RVA at 0–20 dB, 0–30 dB, 0–40 dB, 0–50 dB and 0–60 dB variation steps, respectively, at 325 GHz. The expanded uncertainties for differential phase-shift measurement of a 0–60 dB RVA are estimated to be 0.57°–1.3° at 325 GHz for various attenuation steps.

4. Conclusion

Detailed analysis of the measurement errors and uncertainty estimation of a millimeter-wave phase-shift measurement system at 325 GHz have been presented. The differential phase-shift measurement uncertainties are estimated to be 0.57°–1.3° for a WR-03 RVA with attenuation steps up to 60 dB at 325 GHz. This system can provide accurate phase-shift measurement for various devices at WR-03 band. A waveguide shim or a RVA calibrated by the proposed measurement system can be used as a phase-shift reference standard to provide phase-shift reference values for calibration of other instruments, such as a VNA. The error analysis will also be useful for other applications where accurate phase measurement is required of millimeter-wave signal.

References

[1] H. Meinel, Commercial applications of millimeter waves: history, present status, and future trends, IEEE Trans. Microw. Theory Tech. 43 (1995) 1639 – 1653.

[2] M. Kemp, Millimetre wave and terahertz technology for the detection of concealed threats: a review, in: Proc. SPIE, volume 6402, USA, pp. 64020 – 64021.

[3] H. Essen, A. Wahlen, R. Sommer, W. Johannes, R. Brauns, M. Schlechtweg, A. Tessmann, High-bandwidth 220 GHz experimental radar, Electron. Lett. 43 (2007) 1114 – 1116.

[4] C. Jastrow, K. Munter, R. Piesiewicz, T. Kurner, M. Koch, T. Kleine-Ostmann, 300 GHz transmission system, Electron. Lett. 44 (2008) 213 – 14.

[5] L. Gradinarsky, O. Nystrom, V. Belitsky, S. Folestad, V. Vassilev, A method for detection of powder materials in metallic hollow structures using microwaves, Measurement 41 (2008) 637–646.

[6] M. Pastorino, Recent inversion procedures for microwave imaging in biomedical, subsurface detection and nondestructive evaluation applications, Measurement 36 (2004) 257–269.

[7] W. P. Ernst, Universal microwave phase-measuring system, IEEE Trans. Instrum. Meas. 19 (1970) 354 –357.

[8] C. P. Eio, S. J. Protheroe, N. M. Ridler, Characterising beadless air lines as reference artefacts for S-parameter measurements at RF and microwave frequencies, IEE Proc. - Sci., Meas. and Tech. 153 (2006) 229 –234.

[9] C. Jastrow, S. Priebe, B. Spitschan, J. Hartmann, M. Jacob, T. Kurner, T. Schrader, T. Kleine-Ostmann, Wireless digital data transmission at 300 GHz, Electron. Lett. 46 (2010) 661 – 3.

[10] H.-J. Song, T. Nagatsuma, Present and future of Terahertz communications, IEEE Trans. Terahertz Science Technology 1 (2011) 256 –263.

[11] P. Goy, M. Gross, Millimeter and submillimeter wave vector measurements, in: Proc. SPIE, volume 3465, USA, pp. 288 – 303.

[12] R. Clarke, R. Pollard, N. Ridler, M. Salter, A. Wilson, Traceability to national standards for s-parameter measurements of waveguide devices from 110 GHz to 170 GHz, in: 73rd ARFTG Microwave Measurement Conference, pp. 1 –10.

[13] N. Ridler, R. Clarke, M. Salter, A. Wilson, Traceability to national standards for S-parameter measurements in waveguide at frequencies from 220 GHz to 330 GHz, in: 78th ARFTG Microwave Measurement Conference, pp. 103 –108.

[14] T. Wu, Accurate measurement of millimetre-wave phase-shift from 220 GHz to 325 GHz using a dual-channel system, Electron. Lett. 48 (2012).

[15] JCGM 100:2008, Evaluation of measurement data - Guide to the expression of uncertainty in measurement, Joint Committees for Guides in Metrology, 2008.

[16] W. Sze, Comparator for calibration of inductive voltage dividers from 1 to 10 khz, ISA Transactions 6 (1968) 263 – 7.

[17] J. J. Hill, T. A. Deacon, Voltage-ratio measurement with a precision of parts in 10E9 and performance of inductive voltage dividers, IEEE Trans. Instrum. Meas. 17 (1968) 269 –278.

[18] G. E. Schafer, Mismatch errors in microwave phase shift measurements, IRE Trans. Microwave Theory Techniques 8 (1960) 617 –622.

[19] T. Otoshi, C. Stelzried, A precision compact rotary vane attenuator, IEEE Trans. Microw. Theory Tech. MTT-19 (1971) 843 – 54.

[20] W. Larson, Rotary-Vane Attenuator as an Interlaboratory Standard, National Bureau of Standards Monograph 144, U.S. Dept. of Commerce, 1975.

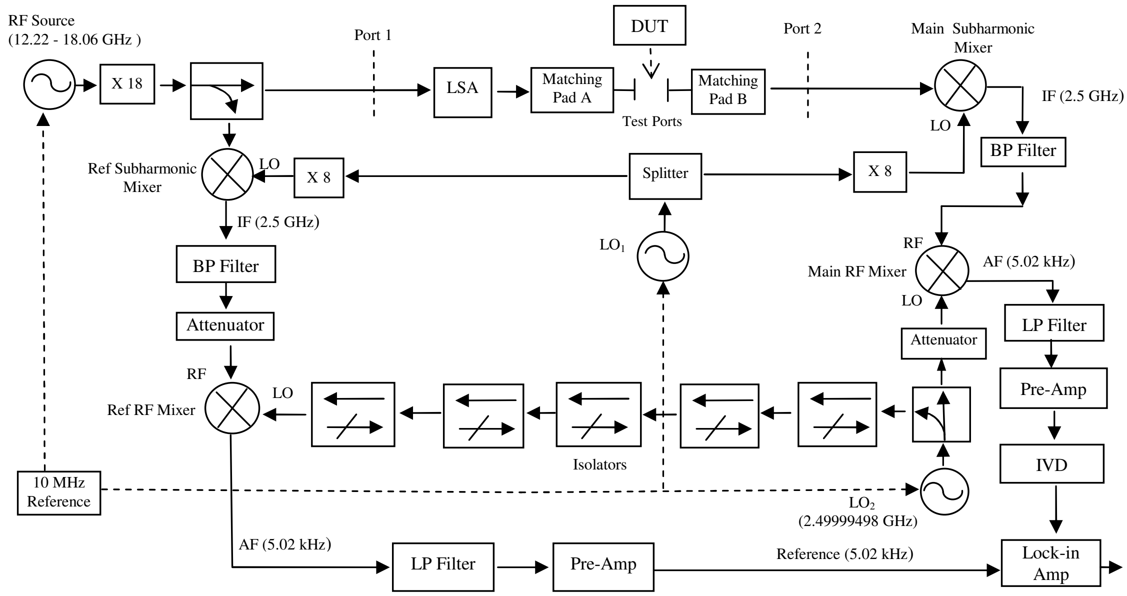

Figure 1: Block diagram of a dual-channel millimeter-wave phase-shift measurement system from 220 GHz to 325 GHz.

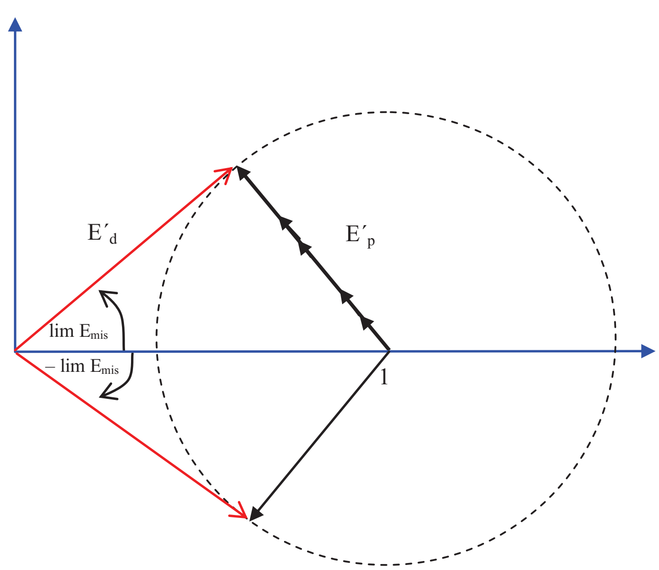

Figure 2: Illustration of maximum mismatch error, lim E_{mis}, in insertion phase-shift measurement.

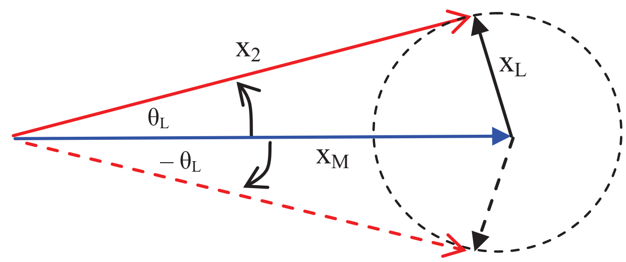

Figure 3: Illustration of maximum error in phase-shift measurement due to leakage.

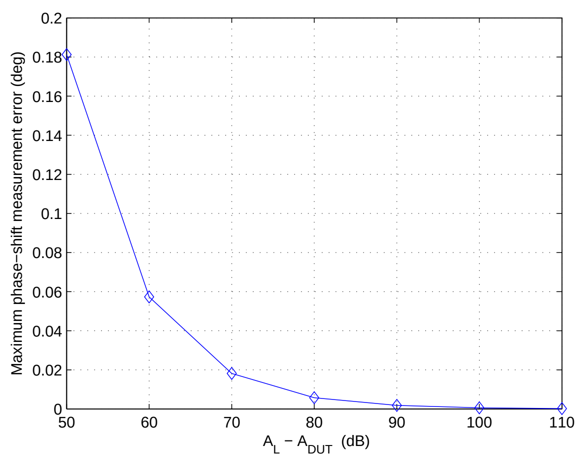

Figure 4: Maximum error in phase-shift measurement due to leakage for various differences between DUT path attenuation and leakage path attenuation.

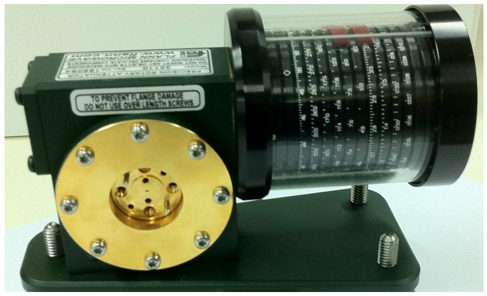

Figure 5: A WR-03 0–60 dB rotary vane attenuator used to estimate the repeatability in phase-shift measurement.

Table 1: Uncertainty budget for differential phase-shift measurement of a 0–60 dB RVA at 325 GHz (the uncertainties are given in degrees).

Uncertainty Component	Norminal Incremental Attenuation Step (dB)				
	0 - 20	**0 - 30**	**0 - 40**	**0 - 50**	**0 - 60**
Phase error due to IVD	0.009	0.009	0.009	0.019	0.024
Mismatch error	0.22	0.22	0.22	0.22	0.22
Receiver noise	0.058	0.12	0.17	0.23	0.29
Phase drift	0.17	0.26	0.35	0.40	0.46
Phase-shift measurement error of GBA	0.00	0.29	0.29	0.29	0.29
Repeatability	0.03	0.04	0.05	0.06	0.08
Combined uncertainty	0.29	0.46	0.53	0.59	0.66
Expanded uncertainty (k=2)	0.57	0.92	1.1	1.2	1.3