Dynamic Spectrum Assignment for White Space Devices with Dynamic and Heterogeneous Bandwidth Requirements

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***Abstract*—In this paper, a TV white space (TVWS) network in which there is a centralized spectrum manager (SM) is considered. The SM is connected to the geo-location database (GLDB) to periodically obtain the available spectrum fragments, and is response for assigning the available spectrum to the white space devices (WSDs) in its coverage region. The spectrum available is usually fragmented with different bandwidth, and the bandwidth required by the WSDs can also be diverse. Unlike existing works which assume complete knowledge of all WSDs' bandwidth requirements before assignment, we consider the more practical settings that WSDs request bandwidth in a sequential manner. Upon the arrival of a bandwidth request, the SM has to determine which fragment the request should be assigned to, without a prior knowledge of the bandwidth of the future requests. We are interested to design the spectrum assignment policy at the SM upon each arrival of bandwidth request. The above problem is formulated as a stochastic sequential decision-making problem. The optimal spectrum assignment policy to maximize the overall spectrum utilization of the TVWS network is computed through the value iteration method. We demonstrate the performance advantage of our proposed optimal spectrum assignment policy over two heuristic policies through numerical results.**

I. INTRODUCTION

The demand for wireless communication services has increased dramatically over recent years due to the popularity of smart mobile devices. However, the spectrum allocated for such services is limited, which is impossible to accommodate the ever increasing demand. To solve this problem, dynamic spectrum access has attracted significant attention. This spectrum access approach allows a service to opportunistically access the spectrum which belongs to but underutilized by the primary users (PUs) on a non-interfering basis. TV white space (TVWS), which refers to the temporally and geographically unused spectrum that belongs to the TV broadcasting services, is widely considered to be the first candidate to promote dynamic spectrum access.

In order to access TVWS, the current regulations and standards require a device to inquiry a geo-location database (GLDB) itself or through other devices for the list of available channels at the device's location so that it will not interfere with the incumbent users' transmissions [1]–[3]. The available spectrum fragments indicated by the GLDB for certain location or area is usually fragmented depending on the occupancy of the PUs. An illustration of such spectrum fragments is shown in Fig. 1. Due to the existence of wireless microphone users or other non-TV users, each fragment may not always occupy a bandwidth of an integer number of TV channels, i.e., the bandwidth available can be a fraction of TV channel bandwidth. In addition, white space devices (WSDs) who want to access those available spectrum fragments can also have heterogeneous bandwidth requirements due to different applications they are carrying or different access technologies they are using. Therefore, how to assign the spectrum fragments to satisfy the diverse requirements of the WSDs is an interesting and challenging problem.

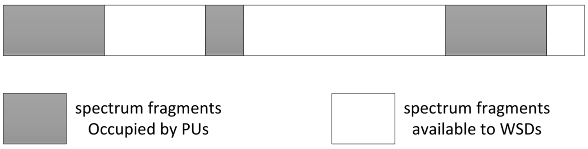

Fig. 1. An illustration of the available spectrum fragments.

In [4], the authors model the above problem to maximize the overall spectrum utilization as a multiple knapsack problem [5]. An exact solution and a heuristic solution are provided. In [6]–[8], the authors consider a few different performance metrics such as minimize spectrum fragmentation and maximize profit. A backtracking algorithm is proposed to search for the best allocation. In [9], [10], the authors consider the case that neighboring spectrum fragments can be aggregated if they are within the spectrum aggregation range imposed by hardware and investigate the associate spectrum assignment problems.

However, all these existing works assume that the spectrum requirements from the WSDs are revealed at the same time. This corresponds to dedicating a time period to collect the requests from WSDs and optimizing the resource with complete knowledge of requests. However, in practice, requests from WSDs usually arrive in a sequential manner, and the SM has to respond upon the arrival of each request. The assignment of a bandwidth request to a spectrum fragment will affect the future assignments and the ability to accommodate future requests. Therefore, it is worth investigating how the sequential behaviors and heterogenous bandwidth requirements of the WSDs affect the spectrum assignment.

In this paper, we consider a TVWS network in which there is a centralized spectrum manager (SM) to control the access of the WSDs within its coverage region. The SM is connected to the GLDB to periodically obtain the available spectrum fragments, which will then be assigned to the WSDs who request access. Similar to [4], [6]–[10], we consider that available spectrum fragments as well as the requested bandwidth from WSDs can be of different bandwidth. However, different from these works, we assume that WSDs request bandwidth in a sequential manner and the SM is able to know the distribution of bandwidth requests through past observations. Upon the arrival of a bandwidth request, the SM has to determine which fragment the request should be assigned to, without a prior knowledge of the bandwidth of the future requests. We are interested in designing the spectrum assignment policy at the SM upon each arrival of bandwidth request. We formulate the above problem as a stochastic sequential decision-making problem. The optimal spectrum assignment policy to maximize the overall spectrum utilization of the TVWS network is computed through the value iteration method. The performance advantage of our proposed optimal spectrum assignment policy over two heuristic policies is demonstrated through numerical results.

The rest of the paper is organized as follows: In Section I, we describe the system model. Section II formulates the spectrum assignment problem as a stochastic sequential decision-making problem and the optimal spectrum assignment policy is computed. We evaluate the proposed spectrum assignment policy and compare it with two heuristic policies through numerical results in Section IV. Finally, Section V draws the conclusion and points out future direction of works.

II. SYSTEM MODEL

In this paper, we consider a TVWS network in which there is a centralized SM[1] controls the access of different WSDs within its coverage area (Fig. 2). We assume that the local SM periodically[2] queries the GLDB and obtains the list of available spectrum fragments it allows to operate within its coverage area. After that, the local SM will allocate the spectrum fragments to its WSDs. Each WSD who wants to transmit must send a request to the SM. Since WSDs may carry different applications or have different access technologies, the amount of bandwidth needed by different WSDs can be different. In the request, the WSD must inform the SM the amount of bandwidth it needs.

We denote the list of available spectrum fragments assigned to the local SM after the query as $\left\{B_0^{(1)},\ldots,B_0^{(n)},\ldots,B_0^{(N)}\right\}$, where $B_0^{(n)}$ represents the bandwidth of the nth spectrum fragment assigned. We consider the case that the requests for bandwidth from different WSDs arrive one by one in a sequential manner. We denote the amount of bandwidth that the tth WSD requested as b_t and the spectrum fragments available at the time of the tth request as $\left\{B_t^{(1)},\ldots,B_t^{(n)},\ldots,B_t^{(N)}\right\}$. The SM knows exactly how much bandwidth is requested only at the time that the request arrives, but does not know a prior how much bandwidth is requested for any of the future requests. However, it is assumed that the SM knows the possible set of the requested bandwidth β and the their arrival probability distribution. The arrival probability of the different requests can be either independent or following a Markov transition process. Such information can be easily obtained based on the past observation on the arrival of the requests.

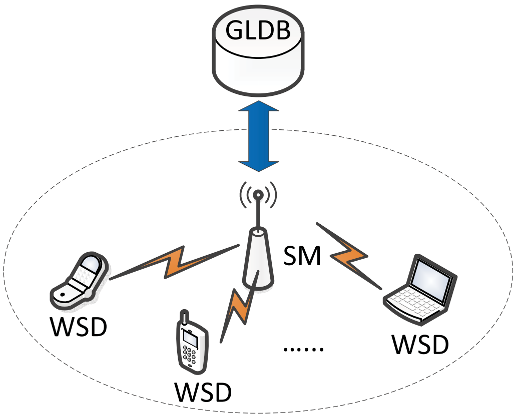

Fig. 2. The system model.

When receiving the request b_t, the local SM will decide, based on the current available spectrum fragment $\left\{B_t^{(1)},\ldots,B_t^{(n)},\ldots,B_t^{(N)}\right\}$ and the distribution information about the future requests, if the request can be accepted and if yes, which spectrum fragment n should the request be assigned to. It is considered in this paper that different WSDs do not share the same spectrum, i.e., the spectrum is assigned to a WSD on an exclusive-use basis. It is noted that when the request is accepted, to avoid spectrum fragmentation the spectrum assigned to b_t is adjacent to edge of the fragment n that is allocated. The process will repeat until there is no more request arrived or the left spectrum fragments are not able to accommodate any new request. An illustration of the operation of the TVWS network is shown in Fig. 3.

The objective of the paper is to design how spectrum is assigned at the SM considering the sequential arrival of different bandwidth requests to maximize the overall spectrum utilization of the system.

III. PROBLEM FORMULATION AND OPTIMAL SPECTRUM ASSIGNMENT POLICY

The above problem can be considered as a stochastic sequential decision-making problem and can be therefore formulated mathematically under the framework of dynamic programming [15].

At the time when the tth request arrives, the local SM observes the system state $s_t = \left\{B_t^{(1)},\ldots,B_t^{(N)},b_t\right\}$, which

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[1] The role of the SM is to coordinate the coexistence among WSDs and similar entities have been considered in a number of existing works and standards such as [11]–[14] etc.

[2] For example, FCC requires the WSDs to recheck the GLDB on a daily basis [1].

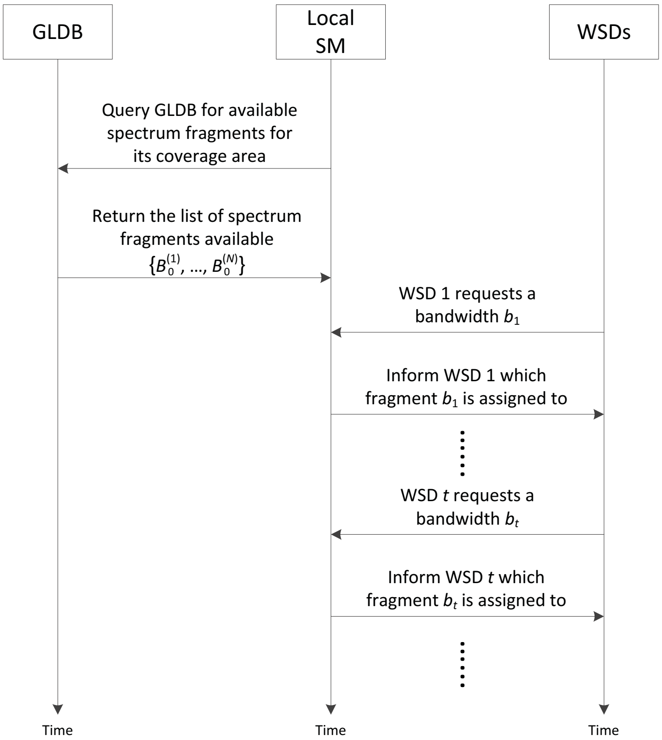

Fig. 3. Illustration of the operation of the TVWS network.

can be characterized by both the available spectrum fragments and the current requested bandwidth. With the observation, the SM has to take an action a_t, where $a_t \in \{0, 1, \dots, n, \dots, N\}$ with $a_t = 0$ representing the current requested b_t is rejected and $a_t = n$ representing b_t is assigned to the nth fragment. When the available spectrum fragment has the capacity to accommodate the incoming request b_t, i,e., $\max\{B_t^{(n)}\} \geq b_t$, the SM has to select one feasible fragment, i.e., $a_t \in \mathcal{A}(s_t)$, where $\mathcal{A}_t(s_t) = \left\{n | B_t^{(n)} \geq b_t\right\}$. Denote b_{min} as the minimum possible bandwidth of the incoming requests, i.e., $b_{\min} = \min\{b | b \in \beta\}$, which can be obtained based on past observations. When the requested bandwidth is greater than any of the available fragments, i.e., $b_{\min} \leq \max\{B^{(n)}\} < b_t$, the SM has to reject the current request, i.e., $a_t \in \mathcal{A}(s_t)$, where $\mathcal{A}(s_t) = \{0\}$.

With the action a_t taken, the available spectrum fragments at the time of next spectrum request will be given according to the following system equations:

$$B_{t+1}^{(n)} = \begin{cases} \mathcal{T}, & \max_n \left\{B_t^{(n)}\right\} < b_{\min}, \\ B_t^{(n)} - b_t, & B_t^{(n)} \geq b_t \text{ and } a_t = n, \\ B_t^{(n)}, & a_t \neq n. \end{cases} \quad (1)$$

where $\mathcal{T}$ denotes the termination state when the system cannot accommodate any new request and hence no more action is taken further.

We assume that the tth WSD can achieve a throughput which is proportional to its granted bandwidth b_t if the request is accepted by the SM, and otherwise, zero throughput is achieved:

$$r(s_t, a_t) = \begin{cases} b_t, & B_t^{(n)} \geq b_t \text{ and } a_t = n, \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

The objective of our design is to find a policy, which is basically a sequence of functions $\pi = \{\mu_0, \mu_1 \dots, \mu_t, \dots\}$, $t = 0, 1 \dots$, mapping each state s_t into control a_t, i.e., $a_t = \mu_t(s_t)$ such that overall spectrum utilization of the system is maximized

$$J^*(s) = \max_{\pi} \lim_{T \to \infty} E\left[\sum_{t=0}^{T-1} r(s_t, a_t) \mid s_0 = s\right] \quad (3)$$

where $J^*(s)$, $s = \left\{B_0^{(1)}, \dots, B_0^{(N)}, b\right\}$ is the maximum spectrum utilization associated with an initial state s.

According to [15], the maximum spectrum utilization $J^*(s)$, $s \in \mathcal{S}$, where $\mathcal{S}$ is set of all possible state and is assumed to be finite, are the unique solution satisfying the following Bellman's equation

$$J^*(s) = \max_{a \in A(s)} r(s, a) + E\left[J^*(\tilde{s})\right] \quad (4)$$

where $\tilde{s} = \left\{\tilde{B}^{(1)}, \dots, \tilde{B}^{(N)}, \tilde{b}\right\}$ is the next state following the system equation (1) and the expectation is taken with respect to (w.r.t.) the distribution of the request[3]. The Bellman's equation can be further expanded in the form as shown in (5) on the top of next page. Note that the maximum throughput is bounded by the total bandwidth of the initial spectrum fragments, i.e., $J^*(s_0) \leq \sum_{n=1}^{N} B_0^{(n)}$.

To calculate the optimal value $J^*(s)$ for $s \in \mathcal{S}$, the well-known value iteration method can be applied [15] and is described in Algorithm 1. The basic idea is to calculate the new value functions $J^{(k+1)}$ based on the value functions computed in the previous iteration $J^{(k)}$ as shown in (6). Then the value functions will converge to the optimal value functions as $k \to \infty$, i.e., $J^*(s) = \lim_{k \to \infty} J^{(k)}(s)$. Furthermore, the optimal policy is stationary, i.e., $\mu_0 = \dots = \mu_t = \dots$, and for any state $s \in \mathcal{S}$, it can be computed as (7) when the algorithm converges.

Algorithm 1 Algorithm for computing $J^*(s)$, $s \in \mathcal{S}$

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1) Initialize $J^0(s) = 0, s \in \mathcal{S}$ and set $k = 0$;
2) For each $s \in \mathcal{S}$, compute
       $J^{(k+1)}(s) = \max_{a \in A(s)} r(s, a) + E\left[J^{(k)}(\tilde{s})\right];$   (6)
3) Repeat 2 until convergence is achieved;
4) For each $s \in \mathcal{S}$, compute
       $\mu(s) = \arg\max_{a \in A(s)} r(s, a) + E\left[J^{(k)}(\tilde{s})\right].$   (7)
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[3] If the arrivals of request are independent, the expectation is w.r.t. Prob $\left\{\tilde{b}\right\}$; If the arrivals are Markovian, the expectation is taken w.r.t. Prob $\left\{\tilde{b} \mid b\right\}$.

$$J^*\left(B^{(1)},\dots,B^{(n)},\dots,B^{(N)},b\right) = \begin{cases} 0, & \max\{B^{(n)}\} < b_{\min}, \\ E\left[J^*\left(B^{(1)},\dots,B^{(n)},\dots,B^{(N)},\tilde{b}\right)\right], & b_{\min} \le \max\{B^{(n)}\} < b, \\ \max\limits_{a=n,B^{(n)}\ge b} b + E\left[J^*\left(B^{(1)},\dots,B^{(n)}-b,\dots,B^{(N)},\tilde{b}\right)\right], & \max\{B^{(n)}\} \ge b. \end{cases} \tag{5}$$

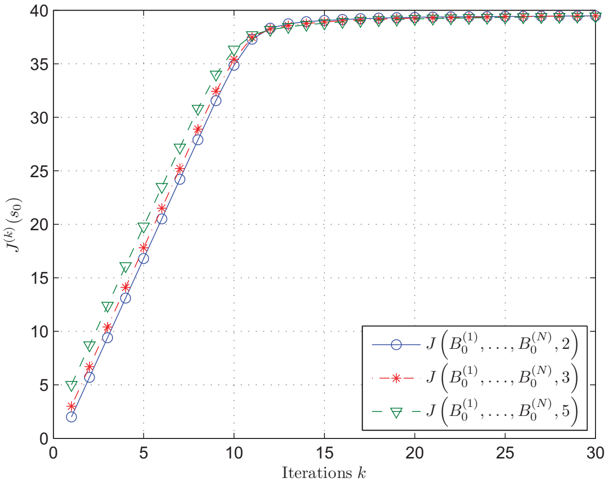

Fig. 4. The value functions for the initial state $s_0 = \left\{B_0^{(1)},\dots,B_0^{(N)},b\right\}$, $b \in \beta$ computed at different iterations.

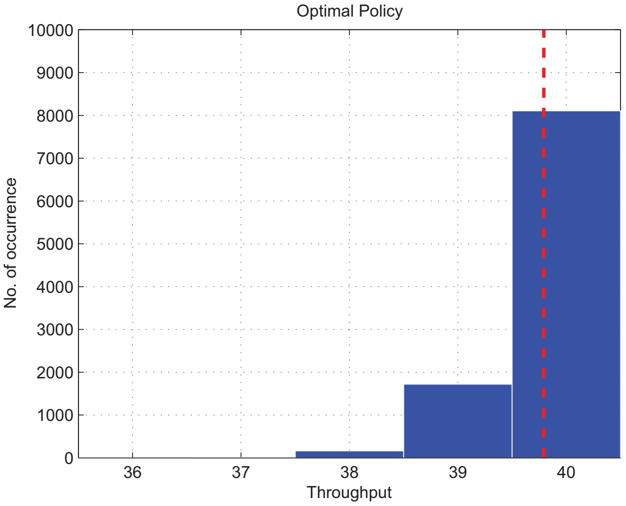

Fig. 5. Histogram of the achievable throughputs of the optimal policy.

IV. NUMERICAL RESULTS

In this section, we provide some numerical results showing the performance of the proposed solution. We consider at the beginning the spectrum fragments available to the SM is given as $\left\{B_0^{(1)},\dots,B_0^{(N)}\right\} = \{7,8,9,16\}$MHz. The set of possible requested bandwidth from WSDs is given by $\beta = \{2,3,5\}$MHz with the corresponding arrival probability given by [0.1, 0.5, 0.4].

First, we compute the optimal spectrum assignment policy using the value iteration method in Algorithm 1. We plot the value functions for the initial state $s_0 = \left\{B_0^{(1)},\dots,B_0^{(N)},b\right\}$, $b \in \beta$, computed at different iterations in Fig. 4. The convergence of the algorithm can be observed within a couple of iterations.

Next, we compare the optimal spectrum assignment policy with two heuristic policies. The first heuristic policy is to randomly assign the arrived bandwidth request to a spectrum fragment which can accommodate it. The strategy is thus termed as "Random Policy". The second one is to assign the arrived bandwidth request to the smallest fragment which can accommodate it. The strategy is thus termed as "Smallest Fragment Policy". We conduct 10000 computer simulations and in each simulation we generate the arrival process according to the considered probability distribution. We execute the optimal policy as well as the two heuristic ones.

The histograms of the achieved throughputs at the end of different simulations for the three policies are shown in Figures 5-7, respectively. Note that the maximum throughput that can be achieved by the policies is bounded by the sum of the bandwidth in the initial fragments, which is 40 MHz in our settings. It can be observed from Fig. 5 that by considering the distribution of the future arrivals, the optimal policy can better utilize the available spectrum fragments, and it can achieve the maximum throughout of 40 MHz for 81% simulations. In contrast, the random and smallest fragment policies can only fully utilize the available spectrum fragments for 11% and 9% of the simulations, respectively. In addition, we also plot the spectrum utilization of the three policies averaged over 10000 simulations. They are shown as the dashed lines in Figures 5-7. We can see that the optimal, random and the smallest fragment policies can achieve an average throughput of 39.8MHz, 38.5MHz, and 38.4MHz, respectively. We can see that the proposed spectrum assignment policy achieves the highest spectrum utilization overall.

V. CONCLUSION AND FUTURE WORKS

In this paper, we have considered a TVWS network in which the bandwidth requests from WSDs arrive in a sequential manner. A centralized SM is responsible for assigning the heterogeneous bandwidth requests from WSDs to different

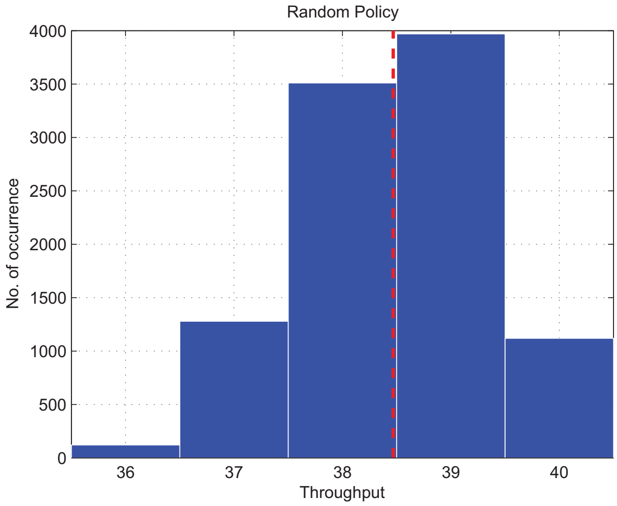

Fig. 6. Histogram of the achievable throughputs of the random policy.

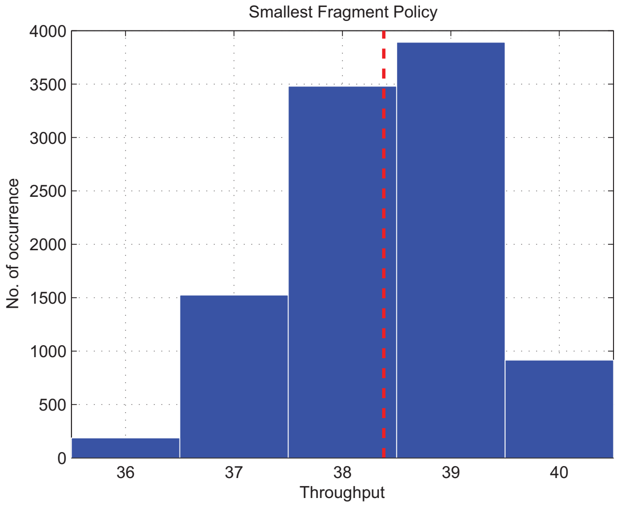

Fig. 7. Histogram of the achievable throughputs of the smallest fragment policy.

spectrum fragments, and it has to determine, upon the arrival of each bandwidth request, which fragment the requests should be assigned to, without a prior knowledge of the bandwidth of the future requests. We considered the design of the spectrum assignment policy at the SM. We formulated the above problem as a stochastic sequential decision-making problem. The optimal spectrum assignment policy to maximize the overall spectrum utilization of the TVWS network has been computed through the value iteration method. The performance advantage of our proposed optimal spectrum assignment policy over two heuristic policies has been demonstrated through numerical results.

In the current work, as the initial attempt to address the dynamic and heterogenous behaviors of the WSDs in TVWS networks, we have made a few assumptions to simplify the modeling. First, we have considered that each WSDs is allocated the spectrum on an exclusive-use basis. Future works could explore the scenario when different WSDs can share part of the spectrum. Second, the heterogeneity among WSDs only arises from the bandwidth requirements. A possible extension of the current look could be looking at other heterogeneities beyond the bandwidth requirements such as priorities, transmit power limitations, etc. Third, in the current work, we have assumed that WSDs, upon allocated the spectrum, continue to use the spectrum till the next GLDB enquiry by the SM. In practice, WSDs' access could be bursty. If WSDs can inform the SM when they finish their transmissions, the released spectrum can be assigned to other future requests. Revising the current model to include such scenarios could also be explored.

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