A simulation on quasi-phase-matched high-harmonic generation in gas-filled hollow core waveguide

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ABSTRACT

Coherent Extreme Ultraviolet (EUV) and X-ray sources have many applications in industry and fundamental research. Although these sources are available in the large facility centres such as synchrotrons and free electron lasers, high-harmonic generation (HHG) can provide us a tabletop approach. However, coherent EUV and X-ray sources based on HHG normally suffer a very low photon flux. Quasi-phase-matching (QPM) techniques can be applied to enhance the HHG photon flux. Here, we apply multimode quasi-phase-matching for HHG in gas-filled hollow core waveguide in the simulation. The quasi-phase-matching is achieved by inducing mode beating between two waveguide modes. A pair of pulses with adjustable relative time delay is coupled into the hollow core waveguide. The mode beating happens exactly at the end of the hollow core waveguide by controlling the intermodal delay. The simulation shows that the photon flux of HHG is much enhanced by intermodal delay controlled QPM. These simulation results are very important for developing the HHG based coherent EUV and X-ray sources with higher photon flux.

Keywords: Quasi-phase-matching, high-harmonic generation, coherent EUV and X-ray sources

1. INTRODUCTION

Nowadays coherent bright EUV and X-ray radiations are generated in the large facility centres such as synchrotrons and free electron lasers. In these facility centres, the huge particle accelerators are built for light radiations. However, they are not suitable for the small research labs. The discovery of HHG provides a new approach for the tabletop coherent EUV and X-ray sources. HHG is a nonlinear process during which a sample is illuminated by an intense driving laser. Under such conditions, the sample will emit the high harmonics of the driving laser. It can be explained by the three-step model [1]. In the first step, the electron is ionized in the strong laser field. Then it is accelerated in the laser field in the second step. In the third step, the electron comes back and recombines with the parent ion when the laser field changes direction. During the recombination, a high energy photon is emitted. The driving laser and the harmonic fields propagate with different wave vectors. To achieve a high flux of HHG, the phase mismatch $\Delta k = k_q - qk_0$ needs to be minimized where k_q and k_0 are the wavevectors for the q_{th} order harmonic and the fundamental wave, respectively. In the case of hollow core waveguides, the phase mismatch can be calculated as [2]:

$$\Delta k = \frac{2\pi q}{\lambda_0}\frac{P}{P_{atm}}\delta_n(1-\eta) - q\eta N_{atm} r_e \lambda_0 \frac{P}{P_{atm}} - \frac{q u_{nm}^2 \lambda_0}{4\pi a^2} \tag{1}$$

where q is the harmonic order, λ_0 is the driving laser wavelength, P/P_{atm} is the ratio of the gas pressure relative to atmospheric pressure, δ_n is the difference in the refractive index between the driving laser and harmonic wavelengths, η

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is the ionization ratio, N_{atm} is the atomic number density at standard temperature and pressure conditions (STP), r_e is the classical electron radius, u_{nm} is the m^{th} root of the $(n-1)^{th}$ Bessel function of the first kind, and a is the inner radius of the hollow core waveguide. The three terms in Equation (1) represent the neutral gas dispersion, plasma dispersion and waveguide dispersion from left to right respectively. The phase mismatch can be minimized by tuning gas pressure when the ionization ratio η is low. However, the perfect phase matching cannot be achieved when η is larger than some critical values. In this case only quasi-phase-matching techniques can be applied to enhance the HHG flux. The quasi-phase-matching can be achieved by modulating the HHG intensity along the propagation. The harmonic generations at destructive interference locations are suppressed and the total HHG flux will gradually increase along the whole propagation distance. Since the HHG highly depends on the driving laser intensity, the HHG intensity modulation can be achieved by modulating the driving laser intensity. When the longitudinal intensity modulation of the driving laser has a period of $L_{QPM} = 2\pi/|\Delta k|$, the HHG at destructive interference locations will be suppressed and the higher HHG flux will be achieved at the propagation end.

2. MODE BEATING APPROACH

The driving laser propagates in the hollow core waveguide by grazing incidence reflections and only leaky modes are supported. These modes have been well studied in [3]. For hollow core waveguide diameters sufficiently larger than the wavelength, the hybrid modes EH_{1m} modes are linearly polarized and can be efficiently excited from a linearly polarized input driving laser. The propagation phase constant of the EH_{1m} waveguide mode can be written as [3]

$$\beta_{1m} = \frac{2\pi}{\lambda_0}\left\{1 - \frac{1}{2}\left(\frac{u_{1m}\lambda_0}{2\pi a}\right)^2\right\} \tag{2}$$

where λ_0 is the driving laser wavelength, u_{1m} is the m^{th} root of the 0^{th} Bessel function of the first kind and a is the inner radius of the hollow core waveguide. The driving laser intensity modulation can be achieved by mode beating inside the hollow core waveguide [4]. A new scheme for quasi-phase-matching of high harmonic generation via multimode beating was proposed [5]. In 2017, multimode quasi-phase-matching of high harmonic generation in a photonic crystal fiber was demonstrated for the first time [6]. As an example, the modulated laser intensity by mode beating of waveguide modes EH_{11} and EH_{110} can be calculated as

$$I_{beating} = |E_{11}\exp(i\beta_{11}z) + E_{110}\exp(i\beta_{110}z)|^2 \tag{3}$$

For a hollow core waveguide of inner diameter 150 µm, the modulated intensity due to the fundamental waveguide mode EH_{11} beating with the 10^{th} order mode EH_{110} was plotted along the propagation distance using Equation (3) in Figure 1. The beating period of two waveguide modes EH_{1n} and EH_{1v} can be calculated by $L_B = 2\pi/|\beta_{1n} - \beta_{1v}| \approx 8\pi^2 a^2/|\lambda_0(u_{1n}^2 - u_{1v}^2)|$.

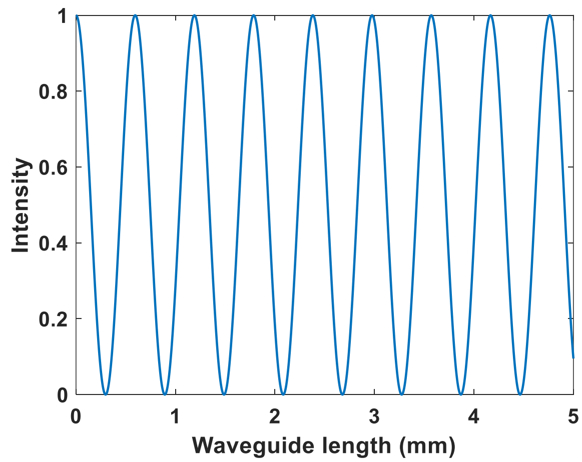

Figure 1. The modulated axial intensity as a function of waveguide length, obtained by EH_{11} and EH_{110} beating.

3. RESULTS AND DISCUSSION

Taking argon gas as an example, we calculated the values of quasi-phase-matching length L_{QPM} for different harmonic orders and gas pressures. They are plotted in the Figure 2. In the simulation, the ionization ratio is assumed to be 0.4 % at which the phase mismatch cannot be 0 by tuning the gas pressure. In this case, only quasi-phase-matching can be applied for a higher HHG yield. Figure 2 shows that the quasi-phase-matching length becomes smaller as the harmonic order and gas pressure increase. Those black lines mark the values of beating periods of the fundamental mode with the 10th, 11th, 13th, 15th, and 17th higher order modes, respectively. The optimal QPM is achieved when $L_B = L_{QPM}$. Taking the fundamental mode EH_{11} beating with the 10th order mode EH_{110} and the 15th order mode EH_{115} as examples, the beating periods of $L_{B10} = 6.0 \times 10^{-4}\ m$ and $L_{B15} = 2.6 \times 10^{-4}\ m$ are achieved, respectively. From Figure 2, we can see that high harmonics from 200 eV to 500 eV can be quasi-phase-matched by choosing the correct gas pressure and higher order mode beating.

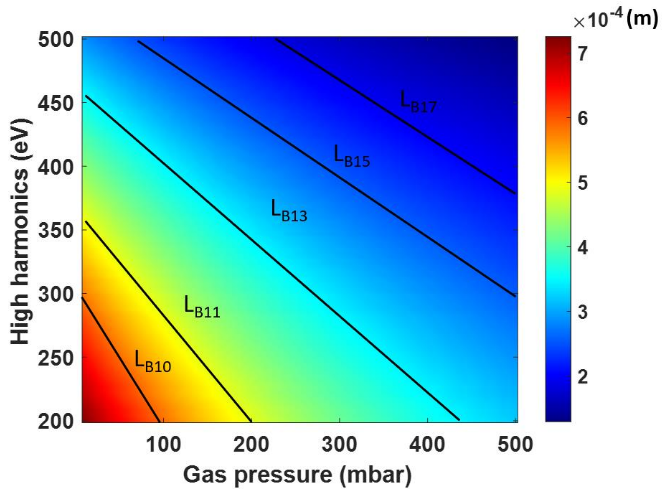

Figure 2. The calculated values of L_{QPM} for different harmonics and argon gas pressures. Those black lines show the values of beating periods of the fundamental mode with different higher order modes (10th, 11th, 13th, 15th, and 17th).

The HHG field after a nonlinear medium of length L can be calculated by

$$E_{HHG}(L) = \int_0^L E_{HHG}^0(z)\exp(i\Delta kz)\,dz \tag{4}$$

where $E_{HHG}^0(z)$ is the harmonic field generated at the propagation position z and Δk is the phase mismatch. For Δk not equal to 0, the harmonics can only be integrated through one coherent length $L_c = \pi/\Delta k$. When the HHG is quasi-phase-matched, the harmonics generated at out of phase locations are suppressed. There would be one coherent length integration of harmonics for every quasi-phase-matching length L_{QPM}. Therefore, the HHG enhancement factor of QPM comparing to no QPM can then be approximated as

$$Q = L/L_{QPM} \tag{5}$$

For high harmonics generated from 200 eV to 300 eV, $L_{QPM} = 0.6\,mm$ by reducing gas pressure from 100 to 0 mbar. The beating period of the fundamental mode with the 10^{th} order mode is also $0.6\,mm$. In this case, the quasi-phase-matching condition $L_B = L_{QPM}$ is satisfied by beating of the fundamental mode with the 10^{th} order mode. Since the fundamental mode and the higher order modes have different propagation velocities, the mode beating can be tuned to happen at the end of the hollow core waveguide by intermodal delay dispersion techniques [7]. In this way, the high harmonics generated directly go to diagnosis or application and reabsorption is avoided. Assuming the mode beating and quasi-phase-matched HHG happen at the last 5 mm of the 2 cm hollow core waveguide, i.e., $L = 5\,mm$. We get an enhancement factor of 8 for quasi-phase-matched HHG by beating of the fundamental mode with the 10^{th} order mode. Using similar calculations, the enhancement factors for quasi-phase-matched HHG by beating of the fundamental mode EH_{11} with different higher order modes are listed in Table 1.

Table 1. The enhancement factors calculated for beating of the fundamental with different higher order modes.

Beating modes	Beating period (mm)	Quasi-phase-matching length (mm)	Enhancement factor
EH_{11} and EH_{110}	0.60	0.60	8
EH_{11} and EH_{111}	0.49	0.49	10
EH_{11} and EH_{113}	0.35	0.35	14
EH_{11} and EH_{115}	0.26	0.26	19
EH_{11} and EH_{117}	0.20	0.20	25

4. CONCLUSION

In conclusion, we calculated the QPM length L_{QPM} versus the harmonic order and gas pressure for HHG in an argon filled hollow core waveguide. The mode beating between the fundamental mode and a higher order mode happens at the end of the hollow core waveguide by tuning the input pulse time delay. The quasi-phase-matched HHG can be achieved by choosing the correct gas pressure and higher order mode beating. From our simulation, the enhancement factors up to 25 can be achieved for QPM high harmonics generated at the end of the hollow core waveguide by the fundamental mode EH_{11} beating with the higher order modes. Based on these simulation results, we will conduct our experiment in the future and hope to achieve the higher HHG flux.

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REFERENCES

[1] Corkum, P. B., "Plasma perspective on strong field multiphoton ionization, " Physical Review Letters, 71, 1994 (1993).

[2] Lytle, A. L., "Phase matching and coherence of high-order harmonic generation in hollow waveguides," Doctoral Dissertation, University of Colorado at Boulder, 37 (2008).

[3] Marcatili, E. A. and Schmeltzer, R. A., "Hollow metallic and dielectric waveguides for long distance optical transmission and lasers," Bell System Technical Journal, 43, 1783-1809 (1964).

[4] Pfeifer, T. and Downer, M. C., "Direct experimental observation of periodic intensity modulation along a straight hollow-core optical waveguide," *JOSA B,* 24(5), 1025-1029 (2007).

[5] Dromey, B., Zepf, M., Landreman, M. and Hooker, S. M., "Quasi-phasematching of harmonic generation via multimode beating in waveguides," Optics Express, 15, 7894-7900 (2007).

[6] Anderson, P.N., Wiegandt, F., Yu, F., Treacher, D.J., Lloyd, D.T., Mosley, P.J., Hooker, S.M. and Walmsley, I.A., "Multimode quasi-phase-matching of high-order harmonic generation in gas-filled photonic crystal fibers," *CLEO/Europe-EQEC IEEE*, 1-1 (2017).

[7] Wiegandt, F., Anderson, P. N., Yu, F., Treacher, D. J., Lloyd, D. T., Mosley, P. J., Hooker, S. M. and Walmsley, I. A., "Quasi-phase-matched high-harmonic generation in gas-filled hollow-core photonic crystal fiber," Optica, 6, 442-447 (2019).