OPTIMAL SOLUTION TO ADAPTIVE SUBCARRIER-AND-BIT ALLOCATION IN MULTICLASS MULTIUSER OFDM SYSTEM

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Abstract Subcarrier-and-bit allocation (SBA) has been extensively investigated in the literature to improve spectral efficiency of multiuser orthogonal frequency division multiplex (OFDM) systems. However, in previous studies, only suboptimal solutions were given and only single-class case was considered. In this paper, a unified analysis of adaptive SBA for multiclass multiuser OFDM system is presented. The constrained power optimization is formulated as a mixed integer nonlinear programming (MINLP) problem. The optimal solution to this problem is derived, i.e., the instantaneous total transmit power is minimized with the quality-of-service (QoS) (data rate and bit error rate) of each class guaranteed. The optimized system performance is compared with both the fixed subcarrier and bit allocation scheme and the rate-adaptive scheme without multiuser diversity. The framework presented in this paper, based on the optimal solution, can be used as a benchmark for future developed heuristic algorithm.

1. Introduction

Next generation mobile communication (NextG) is featured by providing high rate, high quality data transmission. The system should be able to support multiclass services with the satisfaction of individual quality-of-service (QoS) requirements; this, in turn, demands more and more of the limited spectrum and calls for wise use of system resources. Therefore, adaptive resource allocation has become an essential topic in NextG system design.

OFDM technique is a very attractive candidate for nextG, not only because it can exploit frequency diversity to combat multipath fading, and thus enhance the system capacity, but also because of its capability of flexible frequency access to facilitate adaptive resource allocation. It can assign more bits on those better subcarriers which have higher signal-to-noise ratio (SNR) and less bits on worse subcarriers or simply switch it off, i.e., no data is transmitted on those subcarriers. This concept is generally called "water-filling". For multiuser OFDM, system can even dynamically distribute subcarriers among different users according to their respective channel conditions rather than allocate fixed groups of subcarriers to a certain user. In this way, multiuser diversity is exploited, which is why adaptive subcarrier-and-bit allocation (SBA) is able to achieve higher spectral efficiency in the multiuser environment, compared with the fixed subcarrier and bit allocation scheme or the rate-adaptive scheme without dynamic subcarrier selection.

Recently, many SBA algorithms for multiuser OFDM system were proposed. In [1], Wong aimed to minimize the overall transmit power with given QoS requirements. The integer constraints were relaxed and an assumption of time-sharing subcarriers was made. Then, a Lagrangian-based algorithm is proposed to solve the modified problem to give the lower bound of the minimum transmit power. In order to reduce computational complexity of this algorithm, a heuristic subcarrier allocation scheme was proposed in [2], with the assumption of fixed modulation modes. Later in [3], Zhang proposed another reduced-complexity subcarrier-bit-and-power allocation algorithm to maximize the overall system throughput. Equal power distribution over the subcarriers was assumed so that SBA problems can be decoupled, and therefore the modified problem could be easily solved through linear integer programming.

These algorithms and those reported elsewhere [4] [5], however, only gave the suboptimal solutions to the original problems with certain assumptions or relaxations. For example, some algorithm avoided integer programming by relaxing discrete integer set to real set [1] and allowing subcarrier sharing [1] [4]; some avoided nonlinear programming by converting the nonlinear objective function into a linear one based on some assumptions [2] [3]; still some used a two-step adaptation to decouple the combinatorial problem [5] [8]. Although suboptimal solutions were shown to be efficient to reduce the computational complexity for realtime implementation, we still have no idea about the gap between the suboptimal and optimal performance. This is obviously because the optimal solution has yet been given so far, and it remains a challenging topic in this area. Moreover, variety of service classes was not taken into consideration in the previous literature, which restricted the application of these algorithms in only single-class case. In this paper, we deal with transmit power optimization of adaptive SBA for multiuser OFDM system supporting multiclass services. Each class has its own QoS requirements such as target data rate and bit error rate (BER). The purpose is to conduct a unified analysis of this constrained optimization problem and then derive its exact optimal solution. The results obtained from this theoretical framework will be useful for comparing the accuracy of any developed suboptimal or heuristic algorithm.

2. Problem Definition

Here we consider a rate-adaptive downlink OFDM system supporting two classes of services. Class 1 needs a constant data rate of R_1 bits/OFDM symbol and a target BER of P_{e_1}; while Class 2 needs a minimum data rate of R_2 bits/OFDM symbol and a target BER of P_{e_2}. The number of subcarriers is N, shared by K_1 Class 1 users and K_2 Class 2 users. With careful design of OFDM signal, i.e., if the length of cyclic prefix is longer than the maximum delay of the multipath channel, intersymbol interference is mitigated; hence, each subcarrier experiences only flat fading. Here we let $g_{k,n}$ denote fading gains as seen by the kth user on the nth subcarrier. Also let N_0 denote the power spectral density (PSD) of the white Gaussian noise, and assume it is the same for all subcarriers and all users.

Further, we let $s_{k,n}$ denote the assignment indicator, i.e., if the nth subcarrier is assigned to the kth user, $s_{k,n} = 1$; otherwise $s_{k,n} = 0$. In our system, no subcarrier can be assigned to more than one user. Therefore, if $s_{k,n} = 1$, $s_{k',n} = 0$ for all $k' \neq k$. Let $c_{k,n}$ denote the number of bits of the kth user assigned onto the nth subcarrier. Assume that square signal constellations 4QAM, 16QAM and 64QAM are considered in our system model, so the number of bits within each symbol has three possible values: 2,4 and 6. Hence, the integer sets for $s_{k,n}$ and $c_{k,n}$ are respectively $S = \{0, 1\}$ and $C = \{0, 2, 4, 6\}$. $c_{k,n} = 0$ means that the kth user transmits no information bits on the nth subcarrier.

In [1], the function of the required received power at a given BER P_e and constellation of c bits/symbol for QAM signals is presented:

$$f(c) = \frac{N_0}{3}\left[Q^{-1}\left(\frac{P_e}{4}\right)\right]^2 (2^c - 1) \tag{1}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}\mathrm{d}t \tag{2}$$

Whereas, in order to maintain QoS requirements of each user, the assigned power at the transmitter for the kth user on the nth subcarrier is

$$
\begin{aligned}
P_{k,n} &= f(c_{k,n})/g_{k,n}^2 \\
&= \frac{N_0}{3}\left[Q^{-1}\left(\frac{P_e}{4}\right)\right]^2 (2^{c_{k,n}}-1)/g_{k,n}^2 \qquad (3)
\end{aligned}
$$

on condition that the nth subcarrier is assigned to the kth user, i.e., $s_{k,n}=1$; otherwise, $P_{k,n}=0$.

In regard to the adaptive SBA problem concerned in this paper, our target is to minimize the overall transmit power while satisfying all the data rate and BER constraints for both Class 1 and Class 2 users. Referring to (2) and its condition, the problem can be numerically formulated as below:

$$
\begin{aligned}
\min_{s_{k,n},c_{k,n}} \quad & \sum_{k=1}^{K_1}\sum_{n=1}^{N}\rho_1(2^{s_{k,n}c_{k,n}}-1)/g_{k,n}^2 \\
& + \sum_{k=K_1+1}^{K_1+K_2}\sum_{n=1}^{N}\rho_2(2^{s_{k,n}c_{k,n}}-1)/g_{k,n}^2 \qquad (4)
\end{aligned}
$$

subject to

$$
\begin{aligned}
\sum_{n=1}^{N} s_{k,n}c_{k,n} &= R_1, \quad k=1,2,\ldots,K_1 \qquad (5)\\
\sum_{n=1}^{N} s_{k,n}c_{k,n} &\geq R_2, \quad k=K_1+1,K_1+2,\ldots,K_1+K_2 \qquad (6)
\end{aligned}
$$

$$
\sum_{k=1}^{K_1+K_2} s_{k,n}=1, \quad n=1,2,\ldots,N \qquad (7)
$$

and

$$
s_{k,n}\in S \text{ integer}, \quad c_{k,n}\in C \text{ integer} \qquad (8)
$$

where

$$
\begin{aligned}
\rho_1 &= \frac{N_0}{3}\left[Q^{-1}\left(\frac{P_{e_1}}{4}\right)\right]^2 \\
\rho_2 &= \frac{N_0}{3}\left[Q^{-1}\left(\frac{P_{e_2}}{4}\right)\right]^2 \qquad (9)
\end{aligned}
$$

are constants related to service category.

Table 1. $K_1 = K_2 = 2$,$N = 4$,$R_1 = 2$ bits/OFDM symbol,$R_2 = 4$ bits/OFDM symbol,$P_{e_1} = 10^{-5}$, $P_{e_2} = 10^{-3}$, $N_0 = 1$.

$s_{k,n}/c_{k,n}$	$n=1$	$n=2$	$n=3$	$n=4$
$k=1$	1/2	0/0	0/2	0/6
$k=2$	0/0	0/4	1/2	0/0
$k=3$	0/0	0/2	0/0	1/4
$k=4$	0/2	1/4	0/6	0/6

Table 2. Fading gains on each subcarrier for each user.

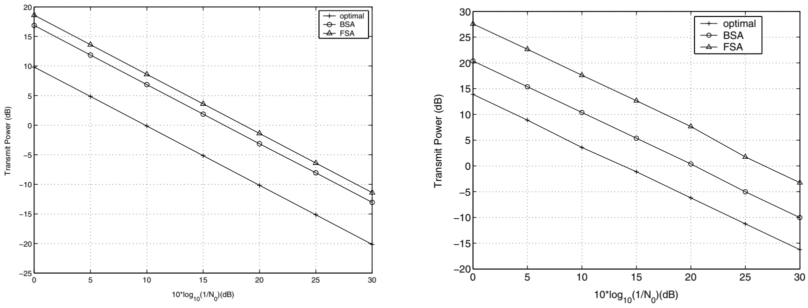
$g_{k,n}$	$n=1$	$n=2$	$n=3$	$n=4$
$k=1$	1.0332	0.386	0.77132	2.323
$k=2$	0.94289	1.2926	1.5579	0.92533
$k=3$	0.65558	1.1125	0.94374	2.0745
$k=4$	0.30256	2.0287	1.6813	1.1541

This formulation, as defined by (3)-(8), is a mixed-integer nonlinear programming (MINLP) problem with a nonlinear objective function (3) and $2(K_1+K_2)N$ integer optimization variables on discrete set. It has K_1 nonlinear equality constraints in (5), K_2 nonlinear inequality constraints in (6) and N linear constraints in (7).

3. Solution and Results

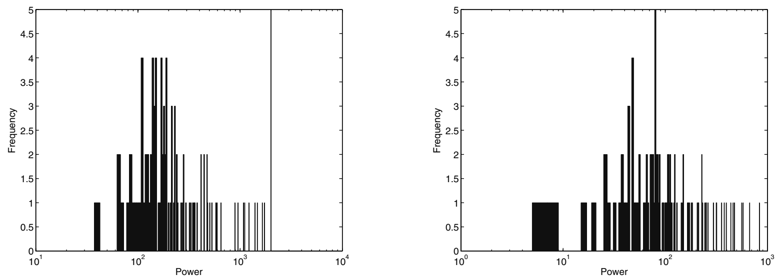
The gradient matrix and Hessian matrix for both the nonlinear objective function and the constraints are obtained. Nonlinear branch and bound algorithm [6] is used to search for the optimum solution for our problem. No relaxations or approximations are made in this process. The detail of the algorithm can be found in [7].

The optimum solution can be achieved for any given number of users and subcarriers. A simple case study is shown in Table 1-2, where the optimum solution is obtained at a snapshot of channel gains on each subcarrier for each user. The assignment results when the minimum power are achieved is presented in Table 1, where we can see the optimal subcarrier allocation and constellation selection on each subcarrier for each user with every constraints fulfilled. The channel gains on each subcarrier as seen by each user are also tabulated in Table 2 for this case. The minimized total transmit power is 56.9 unit. In order to further explain the result, if we change $c_{3,1} = 4$ and $c_{4,4} = 4$, and accordingly assign $s_{1,3}$, $s_{2,2}$, $s_{3,1}$ and $s_{4,4}$ equal to 1, the resulted total

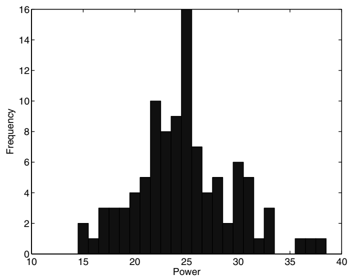


(a) $N = 4$. (b) $N = 8$.

Figure 1. Performance comparisons between the optimal solution and other schemes. $K_1 = 2$, $K_2 = 1$, $R_1 = 2$ bits/OFDM symbol, $R_2 = 6$ bits/OFDM symbol, $P_{e_1} = 10^{-2}$, $P_{e_2} = 10^{-4}$.



(a) FSA solution. (b) BSA solution.



(c) Optimal Solution.

Figure 2. Probability density distributions for minimized transmit power of the 3 SBA schemes with a number of channel gain observations. $K_1 = 2$, $K_2 = 1$, $N = 8$, $R_1 = 2$ bits/OFDM symbol, $R_2 = 6$ bits/OFDM symbol, $P_{e_1} = 10^{-2}$, $P_{e_2} = 10^{-4}$, $N_0 = 1$.

transmit power needed for the new assignment is 233.9 unit, higher than the optimal value by 6.14dB.

The performance of our optimal SBA scheme is compared with the rate-adaptive scheme without multiuser diversity as well as a simple subcarrier allocation scheme discussed in [8]. In fixed subcarrier allocation scheme like traditional FDMA scheme, subcarriers are allocated to the users according to certain predetermined mapping rule. In other words, $\{s_{k,n}\}$ are known before the optimization process. The optimal constellations on each subcarrier for each user $\{c_{k,n}\}$ are searched to achieve the maximum revenue. Moreover, a simple subcarrier allocation scheme is also proposed in [8] for multiuser OFDM systems, in which each subcarrier is allocated to the user who sees the highest channel gain. In this way, the subcarrier and bit allocations are decoupled. $\{s_{k,n}\}$ are firstly decided according to the ranking of channel gains, and then $\{c_{k,n}\}$ can be easily obtained to maximize the system revenue while all the constraints are satisfied. Hereafter, we address the two schemes mentioned above FSA and BSA, respectively, for the convenience of future reference.

The transmission power of the multiclass multiuser OFDM system with different number of subcarriers is presented in Fig. 1 for $N = 4$ and $N = 8$, where the performance is also compared with FSA and BSA schemes. The curves are generated based on 100 observations when the channel changes and the average values are plotted against the noise spectral density N_0. The required transmission power of our presented optimal solution is always well under the other two schemes, as we can see from the gap between the optimal curve and the FSA, BSA curves in both (a) and (b). Furthermore, when the number of subcarriers increases from 4 to 8, the power difference between our optimal solution and FSA goes from 8dB to 13dB. The wider gap indicates that the performance degradation of fixed subcarrier allocation becomes worse as the system complexity increases in the multiclass multiuser environment. For BSA scheme, although the gap closes up by less than 1dB when N increases, the required transmit power is still 6dB more than that of the optimal scheme. This manifests large potential for performance enhancement of the current heuristic algorithms and imposes great need to explore more efficient suboptimal subcarrier allocation schemes to close the wide gap. Thus, the presented theoretical framework can be used as the benchmark for current and future developed heuristic SBA schemes for multiclass multiuser OFDM systems.

In order to study the power variations with changing channel conditions, the probability density distributions for the minimum required transmit power of the 3 schemes concerned in this paper is also presented in Fig. 2. The histograms for the minimum power at the 100 snapshots of channel gains are drawn for different SBA schemes, where we can see that not only the values of power by the optimal scheme are generally much smaller than the other two schemes, the variance of the optimal solution at different channel gains is much smaller than the other two schemes. The smaller variance shows the advantage of the optimal scheme as it reduces the dynamic range of the transmit power which leads to an easier and less expensive transmitter design.

4. Conclusions

In this paper, we present the theoretical analysis for adaptive SBA algorithm in multiclass multiuser OFDM systems. The downlink transmission supporting 2 user classes is examined. The transmission power is minimized with fulfillment of the QoS constraints of each class. The exact optimal solution is obtained using MINLP. Some simple case studies are presented and the performance curves are presented and compared with rate-adaptive OFDM with fixed subcarrier allocation and also OFDM with only a simple subcarrier allocation scheme. Our preliminary results gain an insight into the gap between the system performance of the heuristic schemes and that of the optimal solution. The proposed optimal scheme, therefore, can provide the benchmark for the current and future research of SBA algorithms for multiclass rate-adaptive systems.

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