

Air-Ground Collaborative Control for Angle-Specified Heterogeneous Formations

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Abstract—The need for better search and rescue capabilities has led to the increased demand for collaborative aerial-ground multi-robot deployments. However, most existing solutions require high communication bandwidth or bulky sensor equipment that is not suitable especially for aerial robots. To make the execution of these tasks more efficient, this paper proposes an angle-specified heterogeneous leader-follower formation framework consisting of unmanned aerial vehicles (UAVs) flying in a 3D space and ground robots moving in a 2D plane. The UAVs are controlled to maintain a desired angle-specified formation with the first three UAVs as the leaders forming a triangular shape and the remaining UAVs as the followers. The follower UAVs only need direction measurements to track the leader UAVs. For the ground robots, two leader robots track the UAV group and determine the orientation and scale of the ground formation, while the follower robots track the leader robots using only direction measurements. The proposed heterogeneous formation framework is energy-efficient as most robots only require low-cost and lightweight direction measurements. The stability of the proposed formation control algorithms is proved and validated through various physical application experiments.

Index Terms—Multi-agent systems, formation control, collaborative formations, angle measurements, cooperative robots.

I. INTRODUCTION

The multi-agent formation control problem has been extensively studied in recent years due to its wide applications, such as cooperative transportation of ground robots in a 2D space [1], search and rescue of UAVs in a 3D space [2], and synthetic aperture radar of formation flying satellites in 3D [3]. Different formation control approaches are proposed when the formation shape is specified by different variables and each agent is equipped with different sensors [4]. For instance, when each agent is equipped with a global positioning

system (GPS) to measure its absolute position with respect to the global coordinate frame, the position-based approach can be used to achieve the desired formation [5]. To cope with the GPS-denied environment, another two local sensing-based approaches, i.e., displacement-based and distance-based formation approaches, are proposed, where relative position measurements between two neighboring agents are usually required [6], [7]. Relative position information consists of distance information and direction information, which can be acquired from, e.g., a radar combining with a monocular camera or stereo cameras, respectively. In contrast, direction measurements can be obtained from a monocular camera, which are low-cost, lightweight, and low-power, and hence the pure-direction formation approach is more favorable in engineering practices [8], [9]. With the help of the recently established bearing rigidity theory [10], [11] and angle rigidity theory [12], [13], two types of direction-only formation approaches are proposed, where the former requires the agents' coordinate frames to have the same orientation, while the latter one does not. The strict alignment of all agents' coordinate frames is hard to meet in practice due to the existence of sensing noise and disturbance [14]. Thus, it is preferable that all agents' coordinate frames are allowed to have different orientations, in which each agent only needs to measure directions in its local coordinate frame [15]. Motivated by this, we aim at achieving the desired formation using local direction measurements.

Many formation control problems focus on the control design for a particular group of mobile agents or the case that all agents lie in the same moving space, such as the formation control algorithms designed in [5]–[7], [11], [12], [15]–[18] for mobile agents moving in $\mathbb{R}^2$ or $\mathbb{R}^3$. However, in many practical tasks, when a formation consists of a heterogeneous group of aerial and ground vehicles, the overall working efficiency can be improved significantly. For instance, during a search and rescue mission, a UAV flying in 3D has the ability to obtain a much wider view of an unknown environment, which can be used to search for the target to be rescued. Although ground robots in 2D only have a limited view of the environment, they have the capability to carry heavier supplies or necessities. Therefore, to achieve the search and rescue task, employing a collaborative group of UAVs and ground robots is more efficient than using a homogeneous group of UAVs or ground robots [19], [20].

Due to the stated advantage, a collaborative team of aerial vehicles and ground robots is employed in [21] to achieve aerial manipulation tasks where the ground robots aid and

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assist the aerial vehicles. In [22], a heterogeneous team of fixed-wing UAVs and unmanned ground vehicles (UGVs) is used to achieve the tasks of cooperative search, identification, and localization of targets. Under the cooperation of aerial and ground robots, [23] presents a lightweight solution for the tasks of localization and target tracking. To conduct semi-autonomous off-road navigation, an integrated aerial and ground vehicle system is developed in [24]. By using a collaborative group of unmanned aerial and ground vehicles, a cooperative path planning approach is developed in [25] for target tracking in urban environments. To achieve cooperation between UAVs and robots, a specific shape, i.e., a desired spatial configuration among the vehicles, is required to be formed for most of the aforementioned tasks. Motivated by this requirement, we first propose a two-layer leader-follower framework in this paper to describe the working pattern among aerial vehicles and ground robots. Specifically, the two layers refer to the swarm group of M UAVs and the swarm group of $N - M$ ground robots, of which both have a leader-follower framework. The information flow between the two layers is directed from the UAV layer to the layer of the ground robots. Next, we use lightweight and low-power direction measurements for vehicles to achieve the prescribed formation shape. Moreover, we allow all the vehicles to have their own local coordinate frames to measure directions, providing more robustness for the system.

The rest of this paper is organized as follows. Section II gives the problem formulation and preliminaries. Section III introduces formation control of UAVs by splitting them into leaders and followers. Then, Section IV presents the formation control for ground robots. Experiments and discussions are provided in Section V.

II. PRELIMINARIES

The multi-robot collaborative formation consists of $M > 3$ UAVs labeled from 1 to M , and $(N - M) \geq 3$ ground robots labeled from $M + 1$ to N . Denote by $\mathcal{V}_d = \{1, 2, \dots, M\}$ the UAV set and $\mathcal{V}_r = \{M + 1, \dots, N\}$ the robot set, respectively. In the UAV group, we label the leader UAVs as 1, 2, 3, while the other UAVs as follower UAVs. In the robot group, we label the leader robots as $M + 1, M + 2$ and the other robots as follower robots. These two groups form the two-layer leader-follower formation control framework. For notation conciseness, we denote robots $[i] = M + i, i \in \{1, 2, \dots, N - M\}$ as illustrated in Fig. 1.

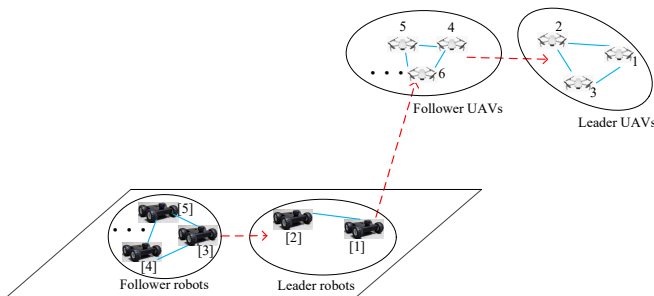


Fig. 1: A heterogeneous group of UAVs and ground robots.

A. Position Dynamics of UAVs

Consider an Earth-fixed inertial coordinate frame $\sum_I - X_I Y_I Z_I$ and a body-fixed coordinate frame $\sum_B^i - X_B^i Y_B^i Z_B^i$ rigidly attaching in UAV $i, i = 1, \dots, M$. Define O_I as the origin of the fixed inertial frame, O_B^i as the mass center of the i^{th} quadrotor UAV, and $p_i = [x_i, y_i, z_i]^T \in \mathbb{R}^3$ as the position of O_B^i described in $\sum_I$. The position dynamics for $i^{\text{th}}, 1 \leq i \leq M$ quadrotor UAV can be mathematically modeled as [26], [27]

$$m_i \ddot{p}_i + m_i g z_G = u_i, i \in \mathcal{V}_d = \{1, \dots, M\} \quad (1)$$

where $m_i \in \mathbb{R}^+$ is the constant mass of UAV i , g represents the gravity of Earth, $z_G = [0, 0, 1]^T$, $u_i \in \mathbb{R}^{3 \times 1}$ is the total translational control force applied in the UAV, which can be produced by the rotation of the four rotors [27]. Since spatial formation is only related to the position configuration of all vehicles, we only focus on the control of position dynamics of UAVs in this paper. More details about the control allocation for an under-actuated UAV can be found in [26] and [28], [29], which directly use the desired force as a control input for the quadrotors. It is worth mentioning the difference between the desired acceleration and the desired force lies in the quadrotors' gravity, which is a constant vector in the inertial frame. Therefore, the design of the desired force and the design of the desired acceleration are very similar.

Remark 1. Generally, the motors controlling the propellers' rotation have a response model, which will cause delay between the scheduled desired force and the real produced propeller thrust. Since the response model usually is a first-order inertial system $\frac{1}{Ts+1}$ with a time constant T , the response speed of the motor is usually higher than the change/bandwidth of the desired force, for which the response model is usually neglected in those tasks with not only one quadrotor but a group of quadrotors.

B. Position Dynamics of Ground Robots

Suppose that robot $j, j = M + 1, \dots, N$ lies in the $X_I O_I Y_I$ plane. Define the position of the mass center of robot j with respect to $X_I O_I Y_I$ as $p_{rj} = [x_{rj}, y_{rj}, 0]^T \in \mathbb{R}^3$. Then, the motion dynamics of robot j can be described by the unicycle model:

$$\dot{x}_{rj} = v_j \cos \theta_j, \quad \dot{y}_{rj} = v_j \sin \theta_j, \quad \dot{\theta}_j = w_j \quad (2)$$

where v_j, w_j, θ_j are the translational moving speed, rotational angular speed, and heading angle, respectively. Now, we utilize feedback linearization for dynamics (2) to conduct the control analysis. Consider another offset point $p_j = [x_j, y_j, 0]^T$ on the robot which is shifted from the center point p_{rj} by $d \in \mathbb{R}^+$ (see Fig. 2). Then, one has the relationship $x_j = x_{rj} + d \cos \theta$; $y_j = y_{rj} + d \sin \theta$ and the transferred dynamics [30]

$$\dot{p}_j = u_j = \begin{bmatrix} \cos \theta & -d \sin \theta & 0 \\ \sin \theta & d \cos \theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ w \\ 0 \end{bmatrix}, j \in \mathcal{V}_r \quad (3)$$

where $u_j \in \mathbb{R}^3$ is the control input transformed to point p_j . Since the matrix $\begin{bmatrix} \cos \theta & -d \sin \theta \\ \sin \theta & d \cos \theta \end{bmatrix}$ is always nonsingular, v

and w can be uniquely calculated from the proposed algorithm u_j . In the follow-up sections, we first focus on the formation

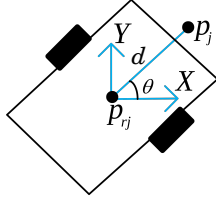


Fig. 2: Ground robot j 's position.

control for aerial vehicles, then on the ground robots.

III. FORMATION CONTROL FOR AERIAL VEHICLES

In this section, we first design formation control laws for the three leader UAVs, then for the remaining follower UAVs.

A. Formation Control Design for Leader UAVs

The objective is to design u_i for (1) such that

$$\lim_{t \rightarrow \infty} e_1(t) = \lim_{t \rightarrow \infty} (p_1(t) - p_1^*) = 0 \quad (4)$$

$$\lim_{t \rightarrow \infty} e_2(t) = \lim_{t \rightarrow \infty} (p_2(t) - p_1(t) - \delta_{12}) = 0 \quad (5)$$

$$\lim_{t \rightarrow \infty} e_3(t) = \lim_{t \rightarrow \infty} (p_3(t) - p_1(t) - \delta_{13}) = 0 \quad (6)$$

where p_1^* denotes the desired position for UAV 1, and $\delta_{ji} \in \mathbb{R}^3$ is UAV i 's desired relative position with respect to UAV j . Note that p_1^* determines the desired absolute position of the UAV formation, and δ_{ji} determines the orientation and scale of the UAV formation. Therefore, given the three constraints $p_1^*, \delta_{12}, \delta_{13}$, the three leader UAVs' desired positions are uniquely determined.

Now, we design the control laws for UAVs 1 to 3 as follows,

$$u_1 = m_1 g z_G - k_v \dot{p}_1 - k_p (p_1 - p_1^*) \quad (7)$$

$$u_2 = m_2 g z_G - k_v \dot{p}_2 - k_p (p_2 - p_1 - \delta_{12}) \quad (8)$$

$$u_3 = m_3 g z_G - k_v \dot{p}_3 - k_p (p_3 - p_1 - \delta_{13}) \quad (9)$$

where k_p, k_v are two positive scalars, UAV 1 measures its absolute position p_1 and velocity $\dot{p}_1$, UAV $i, i = 2, 3$ will measure the relative position $p_i - p_1$ with respect to UAV 1 and velocity $\dot{p}_i$.

Proposition 1. *If the first three UAVs governed by (1) are controlled by (7)-(9), the objectives (4)-(6) can be achieved globally and exponentially.*

Proof. Substituting (7)-(9) into (1) yields

$$\ddot{e}_1 = -\frac{k_v}{m_1} \dot{e}_1 - \frac{k_p}{m_1} e_1 \quad (10)$$

$$\ddot{e}_2 = -\frac{k_v}{m_2} \dot{e}_2 - \frac{k_p}{m_2} e_2 - \frac{k_v}{m_2} \dot{e}_1 - \ddot{e}_1 \quad (11)$$

$$\ddot{e}_3 = -\frac{k_v}{m_3} \dot{e}_3 - \frac{k_p}{m_3} e_3 - \frac{k_v}{m_3} \dot{e}_1 - \ddot{e}_1 \quad (12)$$

The polynomial characteristic of the linear closed-loop dynamics (10) can be written as

$$\lambda^2 + \frac{k_v}{m_1} \lambda + \frac{k_p}{m_1} = 0 \quad (13)$$

where $\lambda \in \mathbb{C}$ is the eigenvalue of (10). Then, one has that the two eigenvalues of (13) have negative real parts. Thus $e_1(t) \rightarrow 0$, $\dot{e}_1(t) \rightarrow 0$ and $\ddot{e}_1(t) \rightarrow 0$ at an exponential rate as $t \rightarrow \infty$. Regarding $\dot{e}_1$ and $\ddot{e}_1$ as external inputs and e_2 as the state, (11)'s polynomial characteristic is similar to (10). Then, using ISS theorem [31], one has that $e_2(t) \rightarrow 0$ and $e_3(t) \rightarrow 0$ exponentially and globally as $t \rightarrow \infty$. It follows that $\dot{p}_i, i = 1, 2, 3$ also exponentially converge to zero and p_i will converge to fixed points. $\square$

B. Formation Control Design for Follower UAVs

To achieve the desired overall formation, we add the remaining UAVs from 4 to M step by step into the existing formation in a cascading way. First, we illustrate how to add UAV 4 into the first triangular formation formed by UAVs 1, 2, 3. Then the remaining UAVs can be similarly added. Denote by $p_4^* \in \mathbb{R}^3$ the desired position of UAV 4 which is unknown for UAV 4 due to the absence of global position measurement. Since each UAV has three degrees of freedom in a 3D space, we now use three angle constraints to uniquely determine the desired position of UAV 4 such that the overall tetrahedral formation formed by UAVs 1-4 is unique. Towards this end, we add UAV 4 into the triangular formation using the three angle constraints $\alpha_{243}^*, \alpha_{431}^*, \alpha_{432}^*$ as shown in Fig. 3. It can be checked that if the magnitude of these three angle constraints belongs to $(0, \pi)$, then there exist two desired positions for UAV 4, i.e., p_4^* is only locally uniquely determined. To guarantee the desired position p_4^* to be globally unique, we need to define the signed angle for $\alpha_{243} \in [0, 2\pi)$

$$\alpha_{243} = \begin{cases} \arccos(b_{42}^\top b_{43}), & \text{if } b_{41}^\top (b_{42} \times b_{43}) > 0 \\ 2\pi - \arccos(b_{42}^\top b_{43}), & \text{otherwise} \end{cases} \quad (14)$$

where $b_{ij} = \frac{p_j - p_i}{\|p_j - p_i\|} \in \mathbb{R}^3$ denotes the direction starting from p_i and pointing towards p_j , and $\times$ denotes the cross product of two vectors. Since $b_{43}^\top (b_{41} \times b_{42})$ is a scalar, it has the same value in both the global coordinate frame and UAV 4's local coordinate frame, which can thus be obtained by UAV 4's local direction measurements. While for the other two angles, we can use normal angle definition for them, i.e., $\alpha_{431} = \arccos(b_{34}^\top b_{31}) \in [0, \pi]$, $\alpha_{432} = \arccos(b_{34}^\top b_{32}) \in [0, \pi]$.

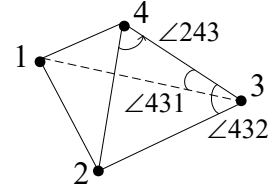


Fig. 3: Add follower UAV 4 into triangular formation $\triangle 123$.

Lemma 1. *Given fixed non-collinear UAVs 1, 2, 3 and desired angles $\alpha_{243}^* \in (0, 2\pi)$, $\alpha_{431}^* \in (0, \pi)$, $\alpha_{432}^* \in (0, \pi)$, the desired position of UAV 4 is globally and uniquely localized.*

Proof. Since the positions of UAVs 1, 2, 3 are fixed, α_{243}^* and α_{432}^* allow p_4^* to lie in the circle C_1 whose origin O_1 is in line 32, see Fig. 4. Since p_2 and p_3 are fixed, angle constraint

α_{431}^* will allow point 4 to lie in the cone C_2 whose apex is p_3 and rotation axis is $\vec{31}$. Note that C_1 and C_2 have at

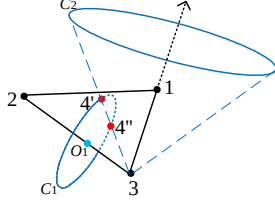


Fig. 4: Uniquely localizing UAV 4 by three angle constraints.

most two intersections $4'$ and $4''$ which are on different sides with respect to the plane 123. Since $b_{4'1}^\top(b_{4'2} \times b_{4'3})$ and $b_{4''1}^\top(b_{4''2} \times b_{4''3})$ have different sign, $\alpha_{24'3}^*$ is different from $\alpha_{24''3}^*$. Therefore, under the angle constraints $\alpha_{243}^*, \alpha_{431}^*, \alpha_{432}^*$, the desired position of UAV 4 is globally and uniquely determined. $\square$

According to Proposition 1 and Lemma 1, to manipulate UAV 4 to reach the desired position, the control objective for UAV 4 can be described as

$$\lim_{t \rightarrow \infty} e_{41}(t) = \lim_{t \rightarrow \infty} (\alpha_{243}(t) - \alpha_{243}^*) = 0 \quad (15)$$

$$\lim_{t \rightarrow \infty} e_{42}(t) = \lim_{t \rightarrow \infty} (\alpha_{431}(t) - \alpha_{431}^*) = 0 \quad (16)$$

$$\lim_{t \rightarrow \infty} e_{43}(t) = \lim_{t \rightarrow \infty} (\alpha_{432}(t) - \alpha_{432}^*) = 0 \quad (17)$$

To achieve (15)-(17), we design the formation control law as

$$u_4 = \begin{cases} m_4 g z_G - k_v \dot{p}_4 + b_{41} + b_{42}, & \text{if } (\alpha_{243}(t) - \pi)(\alpha_{243}^* - \pi) < 0 \\ m_4 g z_G - k_v \dot{p}_4 + \text{sgn}(\alpha_{243}^* - \pi)(b_{42} \times b_{43}), & \text{if } \alpha_{142}(t) = \pi \\ \bar{u}_4, & \text{if } (\alpha_{243}(t) - \pi)(\alpha_{243}^* - \pi) > 0 \end{cases} \quad (18)$$

where

$$\bar{u}_4 = m_4 g z_G - k_v \dot{p}_4 - e_{41} b_{43} + e_{42} b_{41} + e_{43} b_{42}, \quad (19)$$

and k_v is a positive scalar. The control law (18) for UAV 4 needs the knowledge of the gravity $m_4 g z_G$, the velocity $\dot{p}_4$, the directions b_{4i} , $i = 1, 2, 3$, and the angle information $\alpha_{432}, \alpha_{431}$. Note that $\alpha_{432}, \alpha_{431}$ are measured by UAV 3, which can be obtained by UAV 4 through wireless communication since $\alpha_{431} = \arccos(b_{31}^\top b_{34})$, $\alpha_{432} = \arccos(b_{34}^\top b_{32})$. Note that no absolute position or relative position measurement is required in (18). The reason for designing the switching control law (18) for UAV 4 is to avoid the scenario where UAV 4 converges to an ambiguous or undesired position.

Lemma 2. Suppose that UAVs 1, 2 and 3 are static and $\pi < \alpha_{243}^* < 2\pi$. If UAV 4's initial states satisfy $0 < \alpha_{243}(0) < \pi$ and $\dot{p}_4(0) = 0$, then under the control (18), UAV 4 will reach the region satisfying $\pi < \alpha_{243} < 2\pi$.

Proof. If $\pi < \alpha_{243}^* < 2\pi$ and $0 < \alpha_{243}(0) < \pi$, then $u_4(0) = m_4 g z_G - k_v \dot{p}_4(0) + b_{41}(0) + b_{42}(0)$, which will drive UAV 4 to move towards the segment $\vec{12}$. After UAV 4 reaches the segment $\vec{12}$, $\alpha_{142} = \pi$, under which the term $(b_{42} \times b_{43})$ is perpendicular to the plane formed by UAVs 1, 2 and 3. Then, UAV 4 will move from the segment $\vec{12}$ to the correct region with respect to UAVs 1, 2, and 3, i.e., $\pi < \alpha_{243} < 2\pi$. $\square$

Since UAV 4 will always converge to the correct region with respect to UAVs 1, 2 and 3 under (18), we now present the results for UAV 4's convergence under (19).

Theorem 2. For the four-UAV formation (1) under the control laws (7)-(9) and (19), if $\frac{k_v^2}{m_4} > \frac{2 \sin \alpha_{431}^* \sin \alpha_{432}^*}{l_{43}^2 (\sin \alpha_{431}^* + \sin \alpha_{432}^*)}$, then the objectives (15)-(17) can be achieved locally and exponentially.

Proof. To obtain the convergence of e_{4i} , $i = 1, 2, 3$, we need to derive the dynamics of e_{4i} , $i = 1, 2, 3$. According to [12], the time-derivative of the angle α_{ijk} can be calculated by

$$\dot{\alpha}_{ijk} = N_{ijk} \dot{p}_k - (N_{ijk} + N_{kji}) \dot{p}_j + N_{kji} \dot{p}_i \quad (20)$$

where $N_{jik} = -\frac{b_{ij}^\top P_{b_{ik}}}{l_{ik} \sin \alpha_{jik}} \in \mathbb{R}^{1 \times 3}$ and $P_{b_{ij}} = (I_3 - b_{ij} b_{ij}^\top) \in \mathbb{R}^{3 \times 3}$. Then, it follows that

$$\dot{e}_{41} = \dot{\alpha}_{243} = N_{243} \dot{p}_3 - (N_{243} + N_{342}) \dot{p}_4 + N_{342} \dot{p}_2 \quad (21)$$

$$\dot{e}_{42} = \dot{\alpha}_{431} = N_{431} \dot{p}_1 - (N_{431} + N_{134}) \dot{p}_3 + N_{431} \dot{p}_4 \quad (22)$$

$$\dot{e}_{43} = \dot{\alpha}_{432} = N_{432} \dot{p}_2 - (N_{432} + N_{234}) \dot{p}_3 + N_{234} \dot{p}_4 \quad (23)$$

Note that (21)-(23) implies that the dynamics of e_{4i} depends on $\dot{p}_i$. Thus, we still need to derive the dynamics of $\dot{p}_i$. Substituting (19) into (1) yields

$$\ddot{p}_4 = \frac{-k_v v_4 - e_{41} b_{43} + e_{42} b_{41} + e_{43} b_{42}}{m_4} \quad (24)$$

According to [32], combining (21)-(23) and (24), one has the overall closed-loop error dynamics

$$\begin{aligned} \dot{e}_4 &= [\dot{e}_{41} \quad \dot{e}_{42} \quad \dot{e}_{43} \quad \dot{v}_4]^\top = B_4(e_f, e_4) e_4 + G_4(e_f, e_4) \dot{e}_f \\ &= \begin{bmatrix} 0 & 0 & 0 & -(N_{243} + N_{342}) \\ 0 & 0 & 0 & N_{431} \\ 0 & 0 & 0 & N_{234} \\ -\frac{b_{43}}{m_4} & \frac{b_{41}}{m_4} & \frac{b_{42}}{m_4} & -\frac{k_v}{m_4} I_3 \end{bmatrix} \begin{bmatrix} e_{41} \\ e_{42} \\ e_{43} \\ v_4 \end{bmatrix} \\ &\quad + \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \end{bmatrix} \\ &= \begin{bmatrix} 0_{3 \times 3} & C_2(e_f, e_4) \\ D_2(e_f, e_4) & -k_v/m_4 I_3 \end{bmatrix} e_4 + G_4(e_f, e_4) \dot{e}_f \end{aligned} \quad (25)$$

where $e_4 \in \mathbb{R}^{6 \times 1}$, $e_f = [\dot{p}_1, \dot{p}_2, \dot{p}_3]^\top$, $g_{11} = 0$, $g_{12} = N_{243}$, $g_{13} = N_{342}$, $g_{21} = N_{431}$, $g_{22} = 0$, $g_{23} = -(N_{431} + N_{134})$, $g_{31} = 0$, $g_{32} = N_{432}$ and $g_{33} = -(N_{432} + N_{234})$, and we used the fact $N_{ijk} b_{jk} = 0$. Because $B_4(e_f, e_4), G_4(e_f, e_4)$ are time-varying, (25) is a nonlinear system, for which we linearize (25) around the desired equilibrium $\{e_f = 0, e_4 = 0\}$ to obtain [32]

$$\begin{aligned} \dot{e}_4 &= \left(\frac{\partial (B_4(e_f, e_4) e_4 + G_4(e_f, e_4) \dot{e}_f)}{\partial e_4} \Big|_{e_f=0, e_4=0} \right) e_4 \\ &\quad + \left(\frac{\partial (B_4(e_f, e_4) e_4 + G_4(e_f, e_4) \dot{e}_f)}{\partial e_f} \Big|_{e_f=0, e_4=0} \right) \dot{e}_f \\ &= B_4^* e_4 + G_4^* \dot{e}_f \end{aligned} \quad (26)$$

where $B_4^* = B_4(e_f, e_4)|_{e_f=0, e_4=0}$, $G_4^* = G_4(e_f, e_4)|_{e_f=0, e_4=0}$ are constant matrices. As $\lim_{t \rightarrow \infty} e_f(t) = 0$, to obtain $\lim_{t \rightarrow \infty} e_4(t) = 0$, we

1 need to check the real parts of the six eigenvalues of B_4^* . The
 2 characteristic polynomial of B_4^* is [32]

$$\begin{aligned} |\lambda I_6 - B_4^*| &= \begin{vmatrix} \lambda I_3 & -C_2^* \\ -D_2^* & (\lambda + k_v/m_4)I_3 \end{vmatrix} \\ &= \det[\lambda(\lambda + \frac{k_v}{m_4})I_3 - C_2^* D_2^*] \\ &= \begin{vmatrix} \lambda(\lambda + \frac{k_v}{m_4}) + \frac{\sin \alpha_{243}^*}{l_{43}^* m_4} & a_1 & d_1 \\ 0 & \lambda(\lambda + \frac{k_v}{m_4}) + \frac{\sin \alpha_{431}^*}{m_4 l_{43}^*} & \frac{b_{43}^{*T} P_{b_{41}^*} b_{42}^*}{m_4 l_{34} \sin \alpha_{341}^*} \\ 0 & \frac{b_{34}^{*T} P_{b_{41}^*} b_{41}^*}{m_4 l_{43}^* \sin \alpha_{432}^*} & \lambda(\lambda + \frac{k_v}{m_4}) + \frac{\sin \alpha_{432}^*}{m_4 l_{43}^*} \end{vmatrix} \\ &= \left[\lambda(\lambda + \frac{k_v}{m_4}) + \frac{\sin \alpha_{243}^*}{m_4 l_{43}^*} \right] \left[\left(\lambda(\lambda + \frac{k_v}{m_4}) + \frac{\sin \alpha_{431}^*}{m_4 l_{43}^*} \right) \right. \\ &\quad \left. \times \left(\lambda(\lambda + \frac{k_v}{m_4}) + \frac{\sin \alpha_{432}^*}{m_4 l_{43}^*} \right) - \frac{b_{43}^{*T} P_{b_{41}^*} b_{42}^*}{m_4^2 l_{34} \sin \alpha_{341}^*} - \frac{b_{34}^{*T} P_{b_{41}^*} b_{41}^*}{l_{43}^* \sin \alpha_{432}^*} \right] \end{aligned} \quad (27)$$

3 where $\det(\cdot)$ represents a real square matrix's determinate,
 4 $C_2^* = C_2(e_f, e_4)|_{e_f=0, e_4=0}$, $D_2^* = D_2(e_f, e_4)|_{e_f=0, e_4=0}$,
 5 $a_1 = -(N_{243}^* + N_{342}^*)b_{41}^*$, and $d_1 = -N_{342}^*b_{42}^*$. Supposing
 6 $|\lambda I_6 - B_4^*| = 0$, one has that

$$\lambda(\lambda + \frac{k_v}{m_4}) + \frac{\sin \alpha_{243}^*}{l_{43}^* m_4} = 0 \quad (28)$$

7 will give two stable solutions with negative real parts. Now,
 8 we check the remaining four eigenvalues lying in

$$\left(\lambda(\lambda + \frac{k_v}{m_4}) + \frac{a_1}{m_4} \right) \left(\lambda(\lambda + \frac{k_v}{m_4}) + \frac{b_1}{m_4} \right) - \frac{c_1}{m_4^2} = 0 \quad (29)$$

9 where $a_1 = \frac{\sin \alpha_{431}^*}{l_{43}^*} > 0$, $b_1 = \frac{\sin \alpha_{432}^*}{l_{43}^*} > 0$, $c_1 =$
 $\frac{b_{43}^{*T} P_{b_{41}^*} b_{42}^*}{l_{34} \sin \alpha_{341}^*} - \frac{b_{34}^{*T} P_{b_{41}^*} b_{41}^*}{l_{43}^* \sin \alpha_{432}^*}$. According to [33, Theorem 6], one
 11 can similarly have $a_1 b_1 > |c_1|$. By using Routh-Hurwitz
 12 criterion [34], all the four solutions of $(\lambda(\lambda + \frac{k_v}{m_4}) +$
 $\frac{a_1}{m_4})(\lambda(\lambda + \frac{k_v}{m_4}) + \frac{b_1}{m_4}) - \frac{c_1}{m_4^2} = 0$ have negative real parts
 14 if the following condition holds [32]

$$\begin{aligned} &2(\frac{k_v}{m_4})^2(\frac{a_1}{m_4} + \frac{b_1}{m_4}) + (\frac{k_v}{m_4})^2(\frac{a_1}{m_4} + \frac{b_1}{m_4}) \\ &- 4(\frac{k_v}{m_4})^2(\frac{a_1 b_1}{m_4^2} - \frac{c_1}{m_4^2}) - (\frac{a_1}{m_4} \frac{k_v}{m_4} + \frac{b_1}{m_4} \frac{k_v}{m_4})^2 \\ &= (\frac{k_v}{m_4})^2[(\frac{a_1}{m_4} - \frac{b_1}{m_4})^2 + 2(\frac{k_v}{m_4})^2(\frac{a_1}{m_4} + \frac{b_1}{m_4}) + 4\frac{c_1}{m_4^2}] > 0 \end{aligned} \quad (30)$$

15 To meet (30), we need to have

$$k_v^2(a_1 + b_1) > 2m_4|c_1| \quad (31)$$

16 Using the fact that $2|c_1| < 2a_1 b_1$, one has that if $\frac{k_v^2}{m_4} > \frac{2a_1 b_1}{a_1 + b_1}$,
 17 then one has $\frac{k_v^2}{m_4} > \frac{2|c_1|}{a_1 + b_1}$, i.e., (30) holds. Therefore,
 18 all the six eigenvalues of B_4^* have negative real parts if
 $\frac{k_v^2}{m_4} > \frac{2 \sin \alpha_{431}^* \sin \alpha_{432}^*}{l_{43}^* (\sin \alpha_{431}^* + \sin \alpha_{432}^*)}$ which can be satisfied by choosing
 20 a sufficient large k_v . After having that B_4^* is Hurwitz and
 21 $\lim_{t \rightarrow \infty} e_f(t) = 0$, according to the input-to-state stability
 22 theorem [31, Lemma 4.7], the dynamical system (26) is
 23 exponentially stable. This implies that the nonlinear system
 24 (25) is locally stable and the control objective (15)-(17) is
 25 achieved for UAV 4. Using Lemma (1), it follows that $p_4(t)$
 26 will converge to the desired point. $\square$

Remark 2. In Theorem 2, local stability is obtained for
 the designed control law (19), which requires that the initial
 formation should be close to the desired formation. Note
 that if $p_4(0)$ locates at the correct side with respect to
 $\triangle 123$ and $\|B_4(e_f, e_4)\|$ is upper bounded and lower bounded,
 then $e_4(t)$ will converge to zero according to the persistent
 excitation theorem [35]. However, if $p_4(0)$ locates at the wrong
 side with respect to $\triangle 123$, then $p_4(t)$ will converge to the
 other ambiguous desired position according to Proposition 1.
 Therefore, if $p_4(0)$ locates at the wrong side, one can use the
 control law $u_4 = m_4 g z_G - k_v \dot{p}_4 + b_{41} + b_{42} + b_{43}$ to move
 from the wrong side to the correct side, then switch back to
 (19).

Now, we add the remaining UAVs to the existing formation.
 For each UAV i , $4 \leq i \leq N$, the formation control law can be
 described in a unified form

$$\begin{aligned} u_i &= m_i g z_G - k_v \dot{p}_i - (\alpha_{j_1 j_2} - \alpha_{j_1 j_2}^*) b_{i j_2} \\ &\quad + (\alpha_{j_1 j_2 i} - \alpha_{j_1 j_2 i}^*) b_{i j_1} + (\alpha_{j_3 j_2 i} - \alpha_{j_3 j_2 i}^*) b_{i j_3} \end{aligned} \quad (32)$$

where $j_1, j_2, j_3 \in \{1, 2, \dots, i-1\}$ are UAV i 's three neighboring
 UAVs. Define the corresponding angle errors as

$$e_{i j_1}(t) = \alpha_{j_1 j_2}(t) - \alpha_{j_1 j_2}^* \quad (33)$$

$$e_{i j_2}(t) = \alpha_{j_1 j_2 i}(t) - \alpha_{j_1 j_2 i}^* \quad (34)$$

$$e_{i j_3}(t) = \alpha_{j_3 j_2 i}(t) - \alpha_{j_3 j_2 i}^* \quad (35)$$

Then, one has the results for the N -UAV case.

Theorem 3. For the N -UAV formation under the control laws
 (7)-(9) and (32), if $\frac{k_v^2}{m_i} > \frac{2 \sin \alpha_{j_1 j_2}^* \sin \alpha_{j_2 i j_3}^*}{l_{j_2 i}^* (\sin \alpha_{j_1 j_2}^* + \sin \alpha_{j_2 i j_3}^*)}$, $\forall N \geq i \geq$
 4, then all the errors $e_1, e_2, e_3, e_{i j_1}(t), e_{i j_2}(t), e_{i j_3}(t)$ expo-
 nentially converge to zero and the desired N -UAV formation
 is achieved.

Proof. By following the same steps as (20)-(25), one has UAV
 i 's errors' dynamics

$$\begin{aligned} \dot{e}_i &= [\dot{e}_{i j_1} \quad \dot{e}_{i j_2} \quad \dot{e}_{i j_3} \quad \dot{v}_i]^\top \\ &= B_i(e_1, \dots, e_i) e_i + G_i(e_1, \dots, e_i) [e_1, e_2, \dots, e_{i-1}]^\top \end{aligned} \quad (36)$$

where $B_i(e_1, \dots, e_i) \in \mathbb{R}^{6 \times 6}$ and $G_i(e_1, \dots, e_i) \in \mathbb{R}^{6 \times 3}$. We
 use the induction method to prove this. Suppose that $e_1(t) \rightarrow$
 $0, \dots, e_{i-1}(t) \rightarrow 0$ holds and we aim at proving $e_i(t) \rightarrow 0$
 as $t \rightarrow \infty$. By using the linearization technique as (26), the
 linearized dynamics of (36) at the desired equilibrium $\{e_1 =$
 $0, \dots, e_i = 0\}$ can be written as

$$\dot{e}_i = B_i^* e_i + G_i^* [e_1, e_2, \dots, e_{i-1}]^\top \quad (37)$$

where $B_i^* = B_i(e_1, \dots, e_i)|_{e_1=0, \dots, e_i=0}$, $G_i^* =$
 $G_i(e_1, \dots, e_i)|_{e_1=0, \dots, e_i=0}$. Following (27)-(30), one also
 has that B_i^* is Hurwitz, which implies the local exponential
 stability of e_i . After having $e_i(t) \rightarrow 0$, one has that $p_i(t)$ will
 converge to a fixed point. $\square$

Remark 3. The condition $\frac{k_v^2}{m_i} > \frac{2 \sin \alpha_{j_1 j_2}^* \sin \alpha_{j_2 i j_3}^*}{l_{j_2 i}^* (\sin \alpha_{j_1 j_2}^* + \sin \alpha_{j_2 i j_3}^*)}$ also
 implies that given larger formation scale, less control gain k_v
 will be needed. In addition, the direction vectors b_{41}, b_{42}, b_{43}
 are well-defined if there is no collision among UAV 4 and

UAVs 1, 2, and 3. Since each UAV usually is equipped with a proximity sensor, inter-UAV collision can also be physically avoided. In practical applications, collision avoidance is very important for UAV formations [36], which, however, is challenging for the task of direction-based formation control. Specifically, collision avoidance under direction-based formation control can be achieved via the estimation/localization of inter-quadrotor relative positions by using the direction measurements. Then, the obtained estimated relative positions can be used for collision avoidance.

Remark 4. Different from the previous work [32] where each UAV's mass is needed in the designed angle-only control law, the angle-only control laws designed in (7)-(9) and (18) do not need to use the information of the UAVs' mass. Also, compared with [32] the control law (18) has a switching term such that the UAV can move to the desired position from any initial position, for which system convergence is improved.

IV. FORMATION CONTROL FOR GROUND ROBOTS

We consider in this section that the ground robots are governed by (3) and they always lie in the XOY plane, i.e., $p_i(3) = 0$ and $u_i(3) = 0, \forall i = M+1, \dots, N$. In the following, we design control laws first for the two leader robots, then for the remaining follower robots.

A. Formation Control for Leader Robots

Denote by $p_{[1]}^*$ the desired position of robot [1] which is related to the desired formation configuration but is unknown for robot [1]. To achieve the desired position using bearing measurements, as illustrated in Fig. 5, we define signed angle constraints between ground robot [1] = $M+1$ and two of the selected UAVs $j, k \in \{1, \dots, N\}$

$$\alpha_{r1} = \begin{cases} \arccos(b_{[1]j}^\top b_{[1]j'}), & \text{if } z_G^\top(b_{[1]j'} \times b_{[1]k'}) > 0 \\ \pi - \arccos(b_{[1]j}^\top b_{[1]j'}), & \text{otherwise} \end{cases} \quad (38)$$

$$\alpha_{r2} = \begin{cases} \arccos(b_{[1]k}^\top b_{[1]k'}), & \text{if } z_G^\top(b_{[1]j'} \times b_{[1]k'}) > 0 \\ \pi - \arccos(b_{[1]k}^\top b_{[1]k'}), & \text{otherwise} \end{cases} \quad (39)$$

where $b_{j'j}^\top b_{j'[1]} = 0, b_{k'k}^\top b_{k'[1]} = 0, j', k'$ are the projected points of j and k towards the XOY plane, and $\alpha_{r1} \in (0, \pi), \alpha_{r2} \in (0, \pi)$. Now, we use these two angle constraints

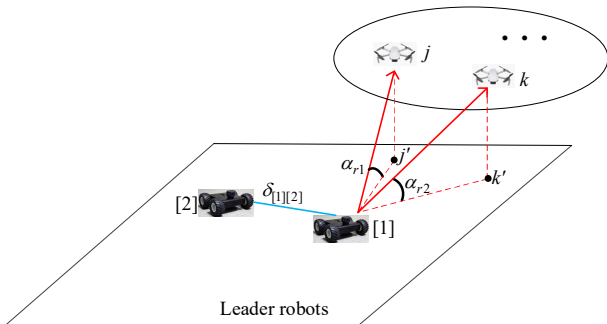


Fig. 5: Add leader robots into the UAV formation.

to uniquely localize the position of the robot [1].

Lemma 3. Given fixed $p_j^*, p_k^*, p_j^*(1) \neq p_k^*(1)$ and $\alpha_{r1}^*, \alpha_{r2}^*$, the desired position of [1] is globally and uniquely localized.

Proof. Since p_j^*, p_k^* are fixed, angle constraint α_{r1}^* allows $p_{[1]}^*$ to be in the circle C_3 whose origin is j' and radius is $p_j^*(3)/\tan \alpha_{r1}^*$. Similarly, α_{r2}^* allows $p_{[1]}^*$ to be in the circle C_4 whose origin is k' and radius is $p_k^*(3)/\tan \alpha_{r2}^*$. Since $p_{j'} \neq p_{k'}$, these two circles C_3 and C_4 have at most two intersections $[1']$ and $[1'']$. According to the geometric relationship in Fig. 6, one has $j'k' \perp [1'] [1'']$. Note that the sign of $z_G^\top(b_{[1]j'} \times b_{[1]k'})$ is different from the sign of $z_G^\top(b_{[1]j''} \times b_{[1]k''})$, which implies that α_{r1}, α_{r2} are different at the two intersections $[1']$ and $[1'']$. Therefore, under angle constraints $\alpha_{r1}^* \in (0, \pi), \alpha_{r2}^* \in (0, \pi)$, $p_{[1]}^*$ is globally and uniquely localized. $\square$

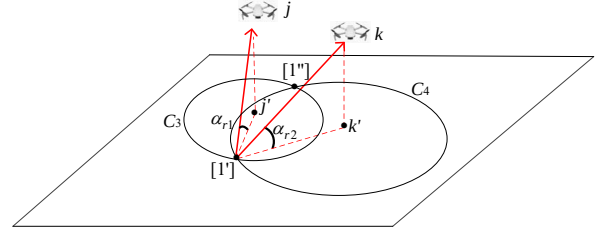


Fig. 6: Globally localize ground leader robots.

Based on Theorem 3 and Lemma 3, to manipulate the first two leader robots to be attached to the UAV formation, the objective for the two leader robots is to design $u_i, i = [1], [2]$ such that

$$\lim_{t \rightarrow \infty} (\alpha_{r1}(t) - \alpha_{r1}^*) = 0 \quad (40)$$

$$\lim_{t \rightarrow \infty} (\alpha_{r2}(t) - \alpha_{r2}^*) = 0 \quad (41)$$

$$\lim_{t \rightarrow \infty} (p_{[2]}(t) - p_{[1]}(t) - \delta_{[1][2]}) = 0 \quad (42)$$

where $\alpha_{r1}^* \in (0, \pi), \alpha_{r2}^* \in (0, \pi)$ and $\delta_{[1][2]} \in \mathbb{R}^3$ is the desired relative position of robot [2] with respect to robot [1], $\delta_{[1][2]}(3) = 0$. To achieve the objectives (40)-(42), we design the control laws as

$$u_{[1]} = -(\tan \alpha_{r1} - \tan \alpha_{r1}^*) b_{[1]j'} - (\tan \alpha_{r2} - \tan \alpha_{r2}^*) b_{[1]k'} \quad (43)$$

$$u_{[2]} = -(p_{[1][2]} - \delta_{[1][2]}) \quad (44)$$

where robot [1] will measure the bearings $b_{[1]j}, b_{[1]j'}, b_{[1]k}, b_{[1]k'}$, and robot $[i], i = 2$ will measure the relative position $p_{[1][i]}$.

Theorem 4. Suppose that two selected UAVs are controlled by (7)-(9), (32) and $p_j^*(1) \neq p_k^*(1)$. Under the controller (43)-(44), the first two leader robots will achieve (40)-(42) locally and asymptotically.

Proof. First, we define the error variables

$$\begin{aligned} e_{r1} &= \tan \alpha_{r1} - \tan \alpha_{r1}^* \\ e_{r2} &= \tan \alpha_{r2} - \tan \alpha_{r2}^* \end{aligned} \quad (45)$$

Then, we aim at obtaining the dynamics of e_{r1} and e_{r2} . Since $\tan \alpha_{r1} = \frac{l_{jj'}}{l_{[1]j'}}$, one has

$$\begin{aligned} \dot{e}_{r1} &= \frac{d(\tan \alpha_{r1} - \tan \alpha_{r1}^*)}{dt} = \frac{\dot{l}_{jj'} l_{[1]j'} - l_{jj'} \dot{l}_{[1]j'}}{l_{[1]j'}^2} \\ &= \frac{\dot{l}_{jj'}}{l_{[1]j'}} - \frac{l_{jj'} b_{[1]j'}^\top (\dot{p}_{j'} - \dot{p}_{[1]})}{l_{[1]j'}^2} \\ &= \frac{\dot{l}_{jj'}}{l_{[1]j'}} - \frac{l_{jj'} b_{[1]j'}^\top \dot{p}_{j'}}{l_{[1]j'}^2} - \frac{l_{jj'} (e_{r1} + \cos \alpha_{j'[1]k'} e_{r2})}{l_{[1]j'}^2} \end{aligned} \quad (46)$$

Similarly, one also has

$$\begin{aligned} \dot{e}_{r2} &= \frac{d(\cot \alpha_{r2} - \cot \alpha_{r2}^*)}{dt} = \frac{\dot{l}_{kk'} l_{[1]k'} - l_{kk'} \dot{l}_{[1]k'}}{l_{[1]k'}^2} \\ &= \frac{\dot{l}_{kk'}}{l_{[1]k'}} - \frac{l_{kk'} b_{[1]k'}^\top (\dot{p}_{k'} - \dot{p}_{[1]})}{l_{[1]k'}^2} \\ &= \frac{\dot{l}_{kk'}}{l_{[1]k'}} - \frac{l_{kk'} b_{[1]k'}^\top \dot{p}_{k'}}{l_{[1]k'}^2} - \frac{l_{kk'} (e_{r2} + \cos \alpha_{j'[1]k'} e_{r1})}{l_{[1]k'}^2} \end{aligned} \quad (47)$$

By defining $e_{[12]} = p_2 - p_1 - \delta_{[1][2]}$, one has

$$\dot{e}_{[12]} = -e_{[12]} + e_{r1} b_{[1]j'} + e_{r2} b_{[1]k'} \quad (48)$$

Writing (46)-(48) into a compact form yields

$$\begin{aligned} \begin{bmatrix} \dot{e}_{r1} \\ \dot{e}_{r2} \\ \dot{e}_{[12]} \end{bmatrix} &= \begin{bmatrix} -\frac{l_{jj'}}{l_{[1]j'}^2} & -\frac{l_{jj'} \cos \alpha_{j'[1]k'}}{l_{[1]j'}^2} & 0 \\ -\frac{l_{kk'} \cos \alpha_{j'[1]k'}}{l_{[1]k'}^2} & -\frac{l_{kk'}}{l_{[1]k'}^2} & 0 \\ b_{[1]j'} & b_{[1]k'} & -1 \end{bmatrix} \begin{bmatrix} e_{r1} \\ e_{r2} \\ e_{[12]} \end{bmatrix} \\ &+ \begin{bmatrix} \frac{\dot{l}_{jj'}}{l_{[1]j'}} - \frac{l_{jj'} b_{[1]j'}^\top \dot{p}_{j'}}{l_{[1]j'}^2} & \frac{\dot{l}_{kk'}}{l_{[1]k'}} - \frac{l_{kk'} b_{[1]k'}^\top \dot{p}_{k'}}{l_{[1]k'}^2} & 0 \end{bmatrix}^T \\ &= \bar{B}_5(e_r, e_j, e_k) e_r + \bar{G}_5(e_r, e_j, e_k) \end{aligned} \quad (49)$$

Since (49) is a nonlinear system, we use the linearization technique to analyze its local stability. Considering the desired equilibrium as $\{e_1 = 0, \dots, e_N = 0, e_r = 0\}$, one has $\bar{G}_5(e_r, e_j, e_k)|_{e_1=0, \dots, e_N=0, e_r=0} = 0$ since $\dot{p}_{j'} = 0, \dot{p}_{k'} = 0, \dot{p}_j = 0$ at the desired equilibrium. Linearizing nonlinear dynamics (49) around the desired equilibrium gives

$$\dot{e}_r = \bar{B}_5^* e_r \quad (50)$$

where $\bar{B}_5^* = \bar{B}_5(e_r, e_j, e_k)|_{e_1=0, \dots, e_N=0, e_r=0}$. To analyze the local stability of (50), one needs to check the real parts of the four eigenvalues of matrix $\bar{B}_5^*$. It is obvious that $\bar{B}_5^*$ has two eigenvalues -1, -1 which are stable. For the remaining two eigenvalues of $\bar{B}_5^*$, one needs to check the eigenvalues of its first block

$$A_1 = \begin{bmatrix} -\frac{l_{jj'}^*}{l_{[1]j'}^{*2}} & -\frac{l_{jj'}^* \cos \alpha_{j'[1]k'}^*}{l_{[1]j'}^{*2}} \\ -\frac{l_{kk'}^* \cos \alpha_{j'[1]k'}^*}{l_{[1]k'}^{*2}} & -\frac{l_{kk'}^*}{l_{[1]k'}^{*2}} \end{bmatrix} \quad (51)$$

Note that

$$\text{tr}(A_1) = -\frac{l_{jj'}^*}{l_{[1]j'}^{*2}} - \frac{l_{kk'}^*}{l_{[1]k'}^{*2}} < 0 \quad (52)$$

$$\begin{aligned} \det(A_1) &= \frac{l_{jj'}^*}{l_{[1]j'}^{*2}} \frac{l_{kk'}^*}{l_{[1]k'}^{*2}} - \frac{l_{jj'}^*}{l_{[1]j'}^{*2}} \frac{l_{kk'}^*}{l_{[1]k'}^{*2}} \cos^2 \alpha_{j'[1]k'}^* \\ &= \frac{l_{jj'}^*}{l_{[1]j'}^{*2}} \frac{l_{kk'}^*}{l_{[1]k'}^{*2}} \sin^2 \alpha_{j'[1]k'}^* > 0 \end{aligned} \quad (53)$$

where $\text{tr}(\cdot)$ denotes a real square matrix's trace. It follows those two eigenvalues of A_1 have negative real parts. Hence, $\bar{B}_5^*$ is Hurwitz which implies the asymptotic stability of (50). After having the asymptotic convergence of $e_{[12]}, e_{[13]}, e_{r1}$ and e_{r2} , one also has the asymptotical convergence of $\alpha_{r1} - \alpha_{r1}^*$ and $\alpha_{r2} - \alpha_{r2}^*$. $\square$

Remark 5. Similar to UAV 4 as mentioned in Remark 2, if robot [1] initially locates at the wrong side with respect to UAVs j and k , then robot [1] can first use the control law $u_{[1]} = b_{[1]j'} + b_{[1]k'}$ to move to the right side with respect to UAVs j and k , and then execute (43).

B. Formation Control for Follower Robots

Now, we add the remaining robots into the existing formation step by step in a cascading way. Similar to the case of controlling the follower UAVs, each robot in the 2D plane has two degrees of freedom. Thus, at least two angle constraints are needed to globally localize each robot. For robot [3], the two angle constraints are selected as $\alpha_{[3][1][2]}^*$ and $\alpha_{[1][3][2]}^*$ as illustrated in Fig. 7.

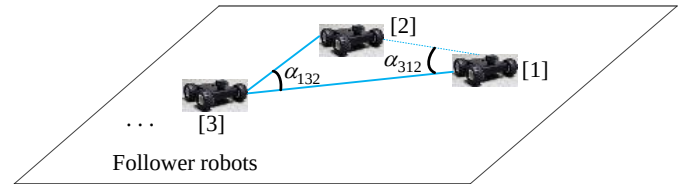


Fig. 7: Add follower robots into the existing formation.

To guarantee $p_{[3]}^*$ can be globally localized under $\alpha_{[3][1][2]}^*$ and $\alpha_{[1][3][2]}^*$, we define signed angle $\alpha_{[1][3][2]}$

$$\alpha_{[1][3][2]} = \begin{cases} \arccos(b_{[3][1]}^\top b_{[3][2]}), & \text{if } z_G^\top (b_{[3][1]} \times b_{[3][2]}) > 0 \\ 2\pi - \arccos(b_{[3][1]}^\top b_{[3][2]}), & \text{otherwise} \end{cases} \quad (54)$$

While for $\alpha_{[3][1][2]}$, we still use $\alpha_{[3][1][2]} = \arccos(b_{[13]}^\top b_{[12]})$.

Lemma 4. Given fixed $p_{[1]}^*, p_{[2]}^*, p_{[1]}^* \neq p_{[2]}^*$ and angle constraints $\alpha_{[3][1][2]}^* \in (0, \pi)$ and $\alpha_{[1][3][2]}^* \in (0, 2\pi)$, the desired position of the robot [3] is globally and uniquely localized.

Proof. Since $p_{[1]}^*, p_{[2]}^*$ are fixed, angle constraint $\alpha_{[3][1][2]}^*$ allows robot [3] to be in ray $[1][3']$ or ray $[1][3'']$, see Fig. 8. While angle $\alpha_{[1][3][2]}^*$ allows robot [3] to be in an arc C_5 . Since the sign of $\alpha_{[1][3][2]}^*$ is different from the sign of $\alpha_{[1][3'']}[2]$, only one intersection is possible under the given two angle constraints $\alpha_{[3][1][2]}^* \in (0, \pi)$ and $\alpha_{[1][3][2]}^* \in (0, 2\pi)$, i.e., the desired position of the robot [4], i.e., $p_{[3]}^*$ is globally localized. $\square$

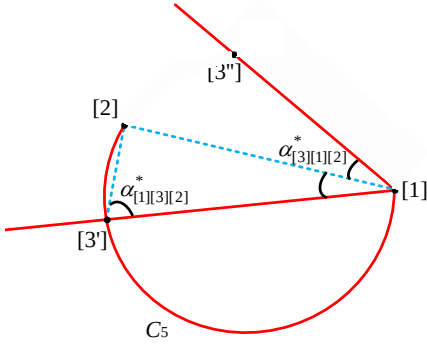


Fig. 8: Globally localize robot [3] using two angle constraints.

Substituting (59) and (60) into (58) yields [12]

$$\begin{aligned}\dot{e}_{[3]1} &= \frac{\dot{b}_{[1][3]}^T b_{[1][2]} + b_{[1][3]}^T \dot{b}_{[1][2]}}{-\sin \alpha_{312}} \\ &= -\frac{\sin \alpha_{[1][3][2]}}{l_{[1][3]}} e_{[3]1} + \frac{b_{[1][2]}^T P_{b_{[1][3]}} \dot{p}_{[1]}}{l_{[1][3]} \sin \alpha_{[3][1][2]}} \\ &\quad - \frac{b_{[1][3]}^T P_{b_{[1][2]}} (\dot{p}_{[2]} - \dot{p}_{[1]})}{l_{[1][2]} \sin \alpha_{[3][1][2]}}\end{aligned}\quad (61)$$

Similarly, one also has

$$\dot{e}_{[3]2} = \frac{\dot{b}_{[3][1]}^T b_{[3][2]} + b_{[3][1]}^T \dot{b}_{[3][2]}}{-\sin \alpha_{[1][3][2]}} \quad (62)$$

Then, one calculates

$$\begin{aligned}\dot{b}_{[3][1]}^T b_{[3][2]} &= b_{[3][2]}^T \frac{P_{b_{[3][1]}}}{l_{[3][1]}} (\dot{p}_{[1]} - \dot{p}_{[3]}) \\ &= \frac{\sin^2 \alpha_{[1][3][2]}}{l_{[3][1]}} e_{[3]1} - b_{[3][2]}^T \frac{P_{b_{[3][1]}}}{l_{[3][1]}} \dot{p}_{[1]}\end{aligned}\quad (63)$$

$$\begin{aligned}b_{[3][1]}^T \dot{b}_{[3][2]} &= b_{[3][1]}^T \frac{P_{b_{[3][2]}}}{l_{[3][2]}} (\dot{p}_{[2]} - \dot{p}_{[3]}) \\ &= \frac{\sin^2 \alpha_{[1][3][2]}}{l_{[3][2]}} e_{[3]2} + b_{[3][1]}^T \frac{P_{b_{[3][2]}}}{l_{[3][2]}} \dot{p}_{[2]}\end{aligned}\quad (64)$$

Then, it follows that [12]

$$\begin{aligned}\dot{e}_{[3]2} &= -\frac{\sin \alpha_{[1][3][2]}}{l_{[3][2]}} e_{[3]2} - \frac{\sin \alpha_{[1][3][2]}}{l_{[3][1]}} e_{[3]1} \\ &\quad + b_{[3][2]}^T \frac{P_{b_{[3][1]}}}{l_{[3][1]} \sin \alpha_{[1][3][2]}} \dot{p}_{[1]} - b_{[3][1]}^T \frac{P_{b_{[3][2]}}}{l_{[3][2]} \sin \alpha_{[1][3][2]}} b_{[2][1]} \dot{p}_{[2]}\end{aligned}\quad (65)$$

Combining (61) and (65) yields the angle error dynamics

$$\begin{aligned}\begin{bmatrix} \dot{e}_{[3]1} \\ \dot{e}_{[3]2} \end{bmatrix} &= \begin{bmatrix} -\frac{\sin \alpha_{[1][3][2]}}{l_{[1][3]}} & 0 \\ -\frac{\sin \alpha_{[1][3][2]}}{l_{[1][3]}} & -\frac{\sin \alpha_{[1][3][2]}}{l_{[2][3]}} \end{bmatrix} \begin{bmatrix} e_{[3]1} \\ e_{[3]2} \end{bmatrix} + \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \dot{p}_{[1]} \\ \dot{p}_{[2]} \end{bmatrix} \\ &= B_6(e_{r1}, e_{r2}, e_{[3]}) e_{[3]} + G_6(e_{r1}, e_{r2}, e_{[3]}) [\dot{p}_{[1]}, \dot{p}_{[2]}]^T\end{aligned}\quad (66)$$

Then, one needs to use the linearization technique to check the local stability of (66). Linearizing (66) towards the desired equilibrium $\{e_{r1} = 0, e_{r2} = 0, e_{[3]} = 0\}$ yields

$$\dot{e}_{[3]} = B_6^* e_{[3]} \quad (67)$$

where $B_6^* = B_6(e_{r1}, e_{r2}, e_{[3]})|_{e_{r1}=0, e_{r2}=0, e_{[3]}=0}$ is Hurwitz, which implies that the closed-loop dynamics (66) are locally and exponentially stable. $\square$

Now, we add the remaining robots to the existing formation. For each robot $[i]$, $4 \leq i \leq N - M$, the formation control law can be described in a unified form

$$\begin{aligned}u_{[i]} &= (\alpha_{[i][j_1][j_2]} - \alpha_{[i][j_1][j_2]}^*) b_{[i][j_2]} \\ &\quad - (\alpha_{[j_1][i][j_2]} - \alpha_{[j_1][i][j_2]}^*) b_{[i][j_1]}\end{aligned}\quad (68)$$

After using $\alpha_{[3][1][2]}^* \in (0, \pi)$ and $\alpha_{[1][3][2]}^* \in (0, 2\pi)$ to globally determine $p_{[3]}^*$, we now study how to achieve $\alpha_{[3][1][2]}^*$ and $\alpha_{[1][3][2]}^*$ using only local bearing measurements. For the robot [3], the control objective can be described as

$$\lim_{t \rightarrow \infty} e_{[3]1}(t) = \lim_{t \rightarrow \infty} (\alpha_{[3][1][2]}(t) - \alpha_{[3][1][2]}^*) = 0 \quad (55)$$

$$\lim_{t \rightarrow \infty} e_{[3]2}(t) = \lim_{t \rightarrow \infty} (\alpha_{[1][3][2]}(t) - \alpha_{[1][3][2]}^*) = 0 \quad (56)$$

To achieve the desired two angles $\alpha_{[1][3][2]}^*$ and $\alpha_{[3][1][2]}^*$, we design

$$u_{[3]} = (\alpha_{[3][1][2]} - \alpha_{[3][1][2]}^*) b_{[3][2]} - (\alpha_{[1][3][2]} - \alpha_{[1][3][2]}^*) b_{[3][1]} \quad (57)$$

where robot [3] will measure the bearings $b_{[3][1]}$, $b_{[3][2]}$ and the angle information $\alpha_{[3][1][2]}$ can be received from robot [2] by wireless communication.

Theorem 5. For a formation consisting of M UAVs and 3 robots, if the UAVs are controlled by (19), (32) and the robots are controlled by (43)-(44), (57), then robot [3] asymptotically converges to its desired position and (55)-(56) achieves.

Proof. To obtain (55)-(56), we first derive the angle error dynamics. According to [12], for $\alpha_{[3][1][2]}$, one has

$$\begin{aligned}\frac{d(\cos \alpha_{[3][1][2]})}{dt} &= -(\sin \alpha_{[3][1][2]}) \dot{\alpha}_{[3][1][2]} = \frac{d(b_{[1][3]}^T b_{[1][2]})}{dt} \\ &= \dot{b}_{[1][3]}^T b_{[1][2]} + b_{[1][3]}^T \dot{b}_{[1][2]},\end{aligned}\quad (58)$$

Then, one has

$$\begin{aligned}\dot{b}_{[1][3]}^T b_{[1][2]} &= b_{[1][2]}^T \frac{P_{b_{[1][3]}}}{l_{[1][3]}} (\dot{p}_{[3]} - \dot{p}_{[1]}) \\ &= e_{[3]1} b_{[1][2]}^T \frac{P_{b_{[1][3]}}}{l_{[1][3]}} b_{[3][2]} - b_{[1][2]}^T \frac{P_{b_{[1][3]}}}{l_{[1][3]}} \dot{p}_{[1]} \\ &= e_{[3]1} \frac{\sin \alpha_{[3][1][2]} \sin \alpha_{[1][3][2]}}{l_{[1][3]}} - b_{[1][2]}^T \frac{P_{b_{[1][3]}}}{l_{[1][3]}} \dot{p}_{[1]}\end{aligned}\quad (59)$$

Similarly, one has

$$b_{[1][3]}^T \dot{b}_{[1][2]} = b_{[1][3]}^T \frac{P_{b_{[1][2]}}}{l_{[1][2]}} (\dot{p}_{[2]} - \dot{p}_{[1]}) \quad (60)$$

where $j_1, j_2 \in \{1, 2, \dots, N-M\}$ are robot $[i]$'s two neighboring robots. Define the corresponding angle errors as

$$e_{[i]1}(t) = \alpha_{[j_1][i][j_2]}(t) - \alpha_{[j_1][i][j_2]}^* \quad (69)$$

$$e_{[i]2}(t) = \alpha_{[j_1][j_2][i]}(t) - \alpha_{[j_1][j_2][i]}^* \quad (70)$$

where $\alpha_{[j_1][i][j_2]}^* \in (0, 2\pi)$, $\alpha_{[j_1][j_2][i]}^* \in (0, \pi)$. Then, one has the results for the N -robot case.

Theorem 6. For the N -UAV and $(N-M)$ -robot formation under the control laws (7)-(9), (32), (43)-(44) and (68), if the initial angle errors $e_{[i]1}(0)$ and $e_{[i]2}(0)$ are small, then all the errors $e_{[d]1}, e_{[d]2}, 4 \leq d \leq N-M$ exponentially converge to zero and the desired N -robot formation is achieved.

The proof of this theorem can be obtained by following Theorem 4 and Theorem 5. If the robot [3] initially localizes at a wrong side with respect to the robots [1], [2], it can also use the control law $u_{[3]} = b_{[3][1]} + b_{[3][2]}$ to reach the right side with respect to the robots [1], [2], and then execute the control command (57). Note that the control law (57) designed for the follower robots is different from the previous work [37].

V. EXPERIMENTS AND RESULTS

In this section, we validate proposed aerial-ground collaborative formation algorithms using a heterogeneous UAV and ground robot swarm. Firstly, we validate Theorem 4 and Theorem 5 with 2 fix-placed leader quadrotor UAVs (holding mode) and 3 movable ground robots. Afterward, Theorem 2 and Theorem 6 are verified with four quadrotor UAVs and three ground robots in the translational motion experiment. Lastly, the application experiment on obstacle avoidance is conducted to evaluate the performance of the proposed method over complex formation maneuverings. More details can be found in the experiments video at <https://youtu.be/KyGeF6sNGDA>.

A. Experiment Setup

The experimental setup consists of DJI Tello Edu UAVs¹ controlled by the Tello SDK 2.0², GoPiGo3 unicycle ground robots³ driven by Raspberry Pi 3 computers with Robot Operating System (ROS) Kinetic, a PC running Ubuntu 18.04 OS and ROS Melodic as a computing center, a TP-Link wireless router for data communication and an OptiTrack motion capture system to track the positions of the UAVs and robots. Note that at this stage, the angle measurements in (14) are simulated from the position data streamed by the motion capture system in real time.

The experiments are conducted in a $6\text{m} \times 8\text{m} \times 3\text{m}$ (width $\times$ length $\times$ height) indoor flying enclosure which is covered by the motion capture system. All the UAVs, ground robots, the center computer and the motion capture system are connected to the wireless router with assigned static IP addresses within 192.168.0.100 – 192.168.0.125. The master-slaves mode is established in the ROS communication between the computing center and each ground robot; hence, all the

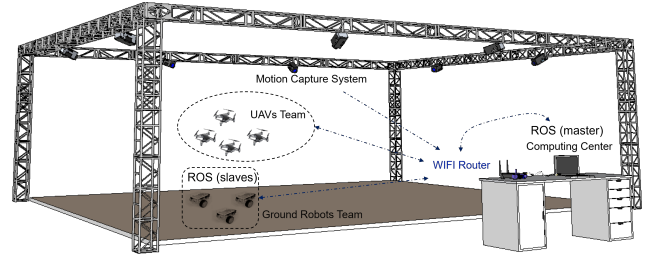


Fig. 9: Illustration of experiment setup and network topology for the air-ground collaborative formation. All agents are connected and communicated with a WIFI router. The control laws are computed at the computing center side as a master ROS node. The relative position measurements and angle measurements are simulated from the position data streamed by the motion capture system.

UAVs and ground robots can be controlled by the computing center via multiple ROS nodes. Note that the ground robots only subscribe to the corresponding command velocity topics with their own onboard computers while UAVs are directly driven by Tello SDK 2.0 via WIFI. The environment setup and the communication topology are illustrated in Fig. 9.

During the collaborative formation maneuvering, all ground robot nodes are launched first via remote Secure Shell (SSH) protocol from the computing center. UAVs are launched to connect with the computing center with Tello ROS drivers. Then the motion capture system streams the UAVs' position and ground robots' position in the system's world frame to the computing center at a frequency of 240Hz. Thereafter, the computing center calculates the velocity commands based on the aforementioned control laws with simulated angle measurements and desired angle constraints and publishes the velocity commands to corresponding agent's ROS node at a frequency of 500Hz. Finally, the UAVs and ground robots are driven to perform collaborative formation maneuvering with real-time velocity commands. All the control gains are set to $k_p = 1.0$ and $k_v = 1.0$ for the following experiments.

B. Holding Mode Experiment

In the holding mode experiment, two leaders, UAV 1 and UAV 2, are fixed at the positions $[1.03, 0.15, 1.27]^T \text{m}$ and $[1.09, -0.53, 1.27]^T \text{m}$ in the preset world frame, respectively. The desired angle constraints for the ground robot 1 are $\alpha_{r1}^* = 80^\circ$ and $\alpha_{r2}^* = 60^\circ$. The desired position of ground robot 2 w.r.t. ground robot 1 is set to $\delta_{[1][2]} = [0, -0.6, 0]^T \text{m}$. The desired angle constraints for the ground robot 3 are $\alpha_{[3][1][2]}^* = 60^\circ$ and $\alpha_{[1][3][2]}^* = 60^\circ$. The evolution of formation and the angle errors, namely $e_{r1}, e_{r2}, e_{[3]1}$ and $e_{[3]2}$, of the 3 ground robots are illustrated in Fig. 10 and Fig. 11, respectively. From Fig. 11, we can observe that the magnitudes of the angle errors become increasingly smaller, which implies the aerial-ground swarm converges to the desired formation with proposed control laws (Theorem 4 and Theorem 5). Moreover, we validated the stability of the proposed control law by placing these three ground robots at different initial positions. All these ground robots can reach desired positions

¹<https://www.ryzerobotics.com/tello>

²<https://dl-cdn.ryzerobotics.com/downloads/Tello/Tello%20SDK>

³<https://www.dexterindustries.com/gopigo3/>

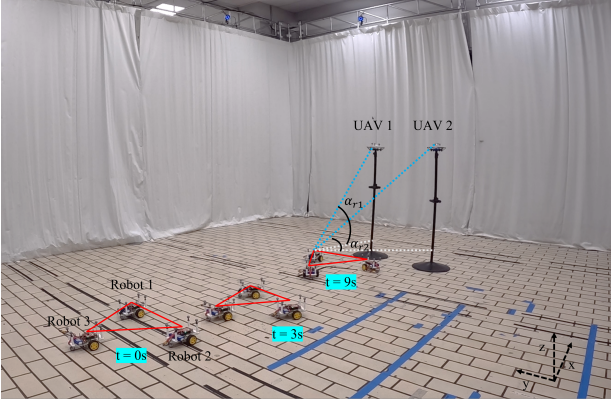


Fig. 10: The trajectories of ground robots in the holding mode. The two leaders, UAV 1 and UAV 2 are fixed at the stand holders, while 3 ground robots move to generate a desired formation with the UAVs. Robot 1 managed to reach the position at time $t = 9s$ driven by angle constraints $\alpha_{r1}^* = 80^\circ$ and $\alpha_{r2}^* = 60^\circ$ while other two robots maintained the triangular formation with constraints $\delta_{[1][2]} = [0, -0.6, 0]^\top m$, $\alpha_{[3][1][2]}^* = 60^\circ$ and $\alpha_{[1][3][2]}^* = 60^\circ$.

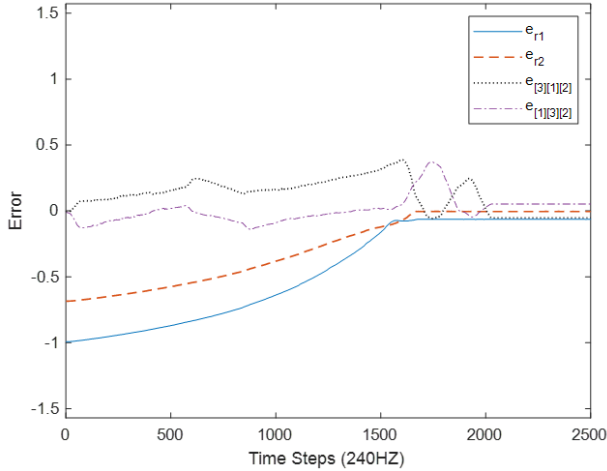


Fig. 11: The evolution of angle errors of ground robots in the holding mode experiment. Errors converge to nearly zero from large initial errors. The steady-state errors come from the friction of the ground surface and the inefficient power of robots' motors.

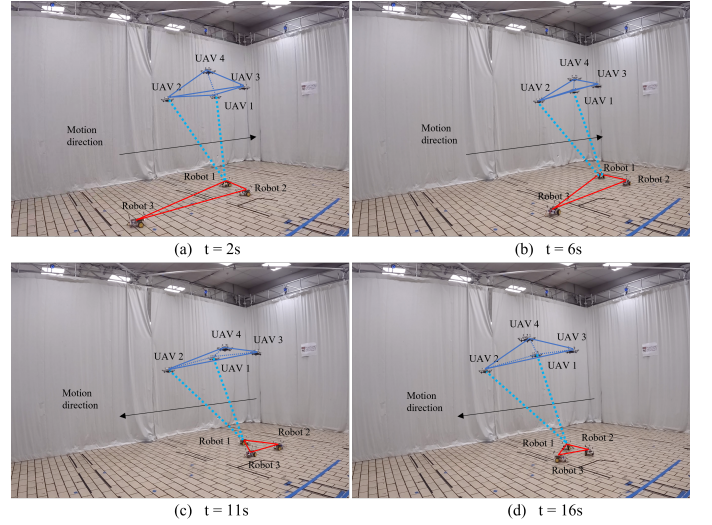


Fig. 12: Snapshots of the translational maneuvering performed by aerial-ground collaborative formation. Ground robot 3 was launched **two seconds** later after other ground robots were launched to verify the stabilization of the proposed method. The aerial-ground formation was maintained after $t = 9s$.

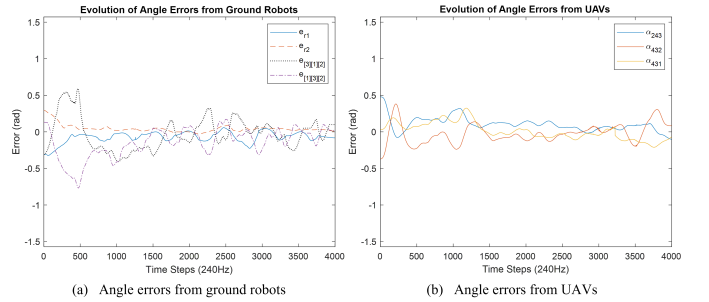


Fig. 13: The evolution of angle errors of ground robots (a) and UAVs (b) in the translational maneuvering experiment.

angle constraints for UAV 4 are $\alpha_{243}^* = 60^\circ$, $\alpha_{432}^* = 60^\circ$ and $\alpha_{431}^* = 60^\circ$. The constraints for the ground robots are the same as those in the holding mode, i.e., $\delta_{[1][2]} = [0, -0.6, 0]^\top m$, $\alpha_{[3][1][2]}^* = 60^\circ$ and $\alpha_{[1][3][2]}^* = 60^\circ$. To verify the stabilization capability of our method, we launched ground robot 3 with two seconds later after the other ground robots were launched.

Fig. 12 shows the snapshots of the translational maneuvering that the aerial-ground swarm performs at different time. Meanwhile, UAVs under the hovering mode are validated as shown in Fig. 12 (d). The angle errors from ground robots and UAVs during the maneuvering are demonstrated in Fig. 13. Even though robot 3 started to move far behind the other agents, it still managed to catch up and maintain the desired formation after 9s. All the angle errors converge to zero during the translational motion, which validated the results from Theorem 2 and Theorem 6. Note that the noise of angle errors is caused by the delay of actuator executions and the friction between the ground robots and the ground surface.

D. Obstacle Avoidance Experiment

Obstacle avoidance is important for robots to perform tasks in cluttered environments. The air-ground collaborative for-

with formations constrained by the designed angles. Note that, the ground robot is also constrained by its unicycle model and the rough surface of the ground, resulting in some arc-shaped routes in the experiments.

C. Translational Motion Experiment

In translational motion, a heterogeneous aerial-ground collaborative formation is achieved with four UAVs and three ground robots. The UAV 1 is guided to track a straight-line trajectory forward and backward for 3m. The desired relative positions for UAV 2 and UAV 3 w.r.t UAV 1 are $\delta_{12} = [-0.6, -0.6, 0.0]^\top m$ and $\delta_{13} = [0, -0.6, 0]^\top m$. The desired

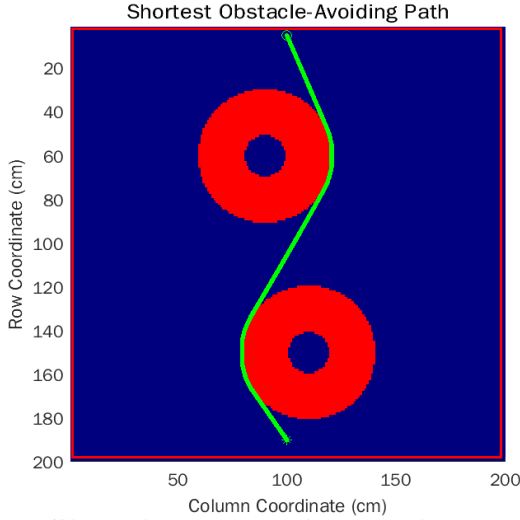


Fig. 14: Offline trajectory generation to avoid two obstacles of radius 30cm based on the shortest path solver [38].

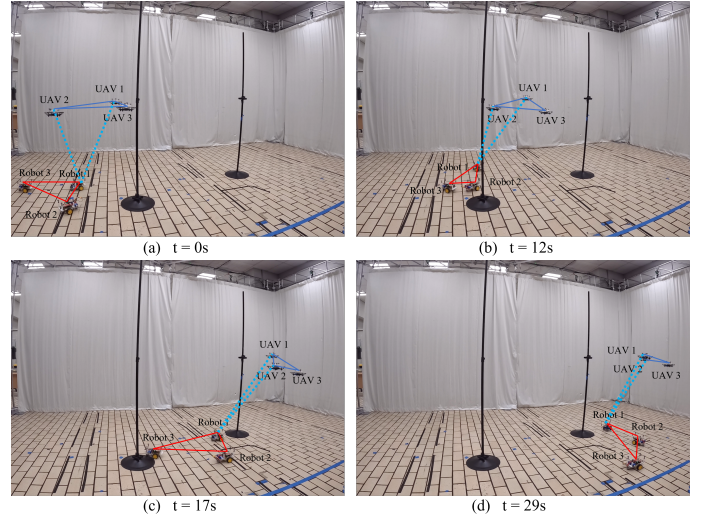


Fig. 15: Obstacle avoidance performed by aerial-ground collaborative formation maneuvering. The leader UAV 1 was guided to avoid the obstacles (two black stands) while other agents kept a desired formation with relative position constraints and angle constraints.

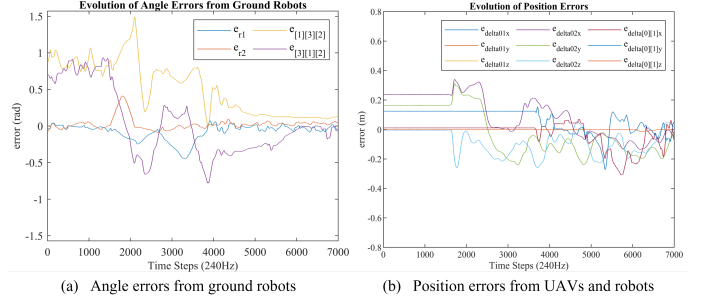


Fig. 16: The evolution of errors in the obstacle avoidance experiment.

mode are validated as shown in Fig. 15 (c) and (d). The angle errors and position errors are illustrated in Fig. 16.

With large initial state errors, the UAVs and ground robots managed to form and maintain the desired formation and avoid these two obstacles as shown in Fig. 15. As observed in Fig. 16, even though angle errors and position errors started from large initial values and fluctuated during the formation maneuvering due to the friction between the ground surface and ground robots and the delay of actuator execution (unmodeled uncertainties), the state errors converged to nearly zero. It verified the results from Theorem 6, which shows the robustness and stability of proposed heterogeneous formation control laws. Note that, compared to the Mecanum wheels, the unicycle wheels used in our ground robots pose more challenges for the formation. However, our control laws still showcase the capability to stabilize the desired heterogeneous formation under different scenarios.

E. Comparison and Discussion

We further highlight the advantages of proposed angle-specified heterogeneous formation approach by comparing

1 mation provides more comprehensive perception and decision
2 capabilities to avoid obstacles. To demonstrate the obstacle
3 avoidance capability of the proposed formation method, the
4 leader UAV 1 tracks an offline-generated trajectory.

5 1) *Offline Trajectory Generation*: Offline trajectory gener-
6 ation is widely adopted in obstacle avoidance, relying on a
7 global map and localization information. In this paper, the
8 trajectory is generated based on the shortest path with the
9 obstacle avoidance solver [38]. We first define a 2D 200cm $\times$
10 200cm grid map and denote the pixel values of obstacles and
11 boundaries as 1 while those within the open space as 0. The
12 start and desired positions are provided at the beginning of the
13 computation, i.e., [5cm, 100cm] and [190cm, 100cm] in our
14 experiments, respectively. The solver at first rapidly propagates
15 all possible paths to provide the shortest route and then
16 contracts this shortest route to follow closely around obstacles
17 based on the computed distance gradient. Quantization noise
18 is eliminated via linear interpolation. Two obstacles are placed
19 at the positions [60cm, 90cm] and [150cm, 110cm] with an
20 avoidance radius of 30cm allowing for formation maneuvering
21 with a certain formation size. The generated offline trajectory
22 is shown in Fig. 14.

23 2) *Collaborative Formation for Obstacle Avoidance*: In
24 the complex obstacle avoidance experiment, to avoid the
25 down-wash effect brought by UAV 4, three leader UAVs and
26 three ground robots were used to demonstrate the air-ground
27 collaborative formation capability of the proposed method in
28 an indoor environment. The relative position constraints for
29 the UAV 2 and UAV 3 are $\delta_{12} = [-0.6, -0.6, 0.0]^T$ m and
30 $\delta_{13} = [0.0, -0.6, 0.0]^T$ m, while the relative position con-
31 straint and the angle constraints are $\delta_{[1][2]} = [0, -0.6, 0]^T$ m,
32 $\alpha_{r1}^* = 80^\circ$, $\alpha_{r2}^* = 60^\circ$, $\alpha_{[3][1][2]}^* = 45^\circ$ and $\alpha_{[1][3][2]}^* = 45^\circ$
33 for ground robots. UAV 1 was guided to track the generated
34 offline trajectory shown in Fig. 14 to avoid the obstacles,
35 and other UAVs and robots were driven by desired relative
36 position constraints and angle constraints to maintain the
37 desired formation. Flight snapshots of obstacle avoidance are
38 demonstrated in Fig. 15. Meanwhile, UAVs under the hovering

TABLE I: Comparison of different formation control approaches

Methods	Position-based [5]	Displacement-based [7], [39]	Distance-based [40]	Bearing-based [11]	Angle-based (ours)
Shape description	Absolute positions	Relative positions	Distances	Bearings	Angles
Example	$p_1^* = [4; 5; 1]$	$(p_1 - p_2)^* = [3; 4; 0]$	$\ p_1 - p_2\ ^* = 5$	$(\frac{p_1 - p_2}{\ p_1 - p_2\ })^* = [\frac{3}{5}; \frac{4}{5}; 0]$	$\alpha_{[3][1][2]}^* = 60^\circ$
Measurement	Absolute position	Relative position	Relative position	Bearing	Bearing
Sensors (possible implementation)	GPS receiver	IMU+compass+radar+camera IMU+compass+lidar+camera	IMU+radar+camera IMU+stereo camera	IMU+compass+camera	IMU+camera
Estimated cost (USD/unit)	\$150-\$500	\$350-\$1200	\$300-\$1000	\$100-\$400	\$80-\$380
Estimated power (W/unit)	0.5-2	1-4	1-3	0.5-1.5	0.4-1.4

with existing solutions regarding the cost and energy efficiency of sensor measurements. We first compare the direction-based method with other traditional methods such as displacement-based methods [39] using cameras and 3D lidar sensors. The comparison is summarized in Table I. Note that the estimated cost and power are retrieved from the information of popular sensor products such as Intel RealSense cameras (<https://www.intelrealsense.com>) and Holybro GPS systems (<https://holybro.com/collections/h-rtk-gps>). As illustrated in Table I, our direction-based approach can leverage IMU+Zigbee+monocular camera sensors, which are inherently more energy-efficient and cost-effective than traditional GPS receivers, lidar, IMU+radar+camera, or other sensor combinations required for position, displacement, or distance-based methods. These traditional methods necessitate the use of absolute or relative position measurements, which often involve more power-intensive and expensive hardware, especially for large-scale missions such as light shows.

Within the direction-based method, we then compare our angle-specified method with bearing-constrained methods [11]. Different from bearing vectors, whose descriptions always depend on coordinate frames, angle constraints are scalars, which are independent of agents' coordinate frames [32]. Consequently, the scalar angle α_{ijk}^* remains consistent even when the agents' coordinate frames have different orientations. This intrinsic property of angle constraints inherently bestows more robustness to our formation control approach against the misalignment of agents' coordinate frames, a common challenge in dynamic environments. Such robustness is particularly advantageous in scenarios where precise coordination is critical, and it enhances the reliability of heterogeneous formations under varying conditions and uncertainties.

VI. CONCLUSION

This paper has developed an angle-specified heterogeneous leader-follower formation control approach that enables UAVs to fly in a 3D space and ground robots to move in a 2D plane. The UAVs maintain a desired 3D formation, constructed by a set of angle constraints originating from a triangular shape. For the ground robots, two robots track the UAV group, while a follower robot tracks two leader robots using only local bearing measurements. Unlike traditional methods that often rely on absolute positioning systems or relative position measurements, our approach utilizes local direction measurements, aligning with the industrial trend towards more practical, low-cost, and lightweight sensor technologies for various missions. By extending angle rigidity theory to 2D-3D heterogeneous formation control, our work overcomes the

limitations associated with the need for aligned coordinate frames among agents in existing bearing rigidity strategies. Several experiments have been conducted to demonstrate the effectiveness and robustness of proposed control algorithms.

Currently, we are only able to prove local convergence for the closed-loop nonlinear formation dynamics, and global convergence remains unknown due to the absence of a suitable Lyapunov function [41]. To address this, we can conduct numerical Monte Carlo simulations, which can determine the attraction region of the closed-loop dynamics, enhancing the method's practical applicability and safety. Future work will address measurement noise and global convergence issues.

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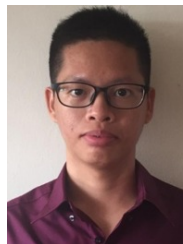
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