Quantifying the Benefits to Consumers for Demand Response with a Statistical Elasticity Model

Wenxian Yang, Rongshan Yu, Milashini Nambiar
Signal Processing Department, Institute for Infocomm Research, A*STAR, Singapore
Email: {wyang, ryu, nambiarm}@i2r.a-star.edu.sg

Abstract

This paper considers the benefits of Real-Time Pricing (RTP), both to individual customers and to the society as a whole, in a deregulated hybrid electricity retail market where both RTP and traditional flat pricing programs coexist. The framework is based on a statistical elasticity model, where the optimal RTP is calculated to maximize social welfare. The customers are characterized by their input load profile shape and their activeness or extent of response in participating the RTP program by rescheduling their load. With increasing penetration of RTP scheme in the hybrid market, the economic impact of the dual price scheme to all parties involved is analyzed. Indices compared include the actual load, Peak-to-Average Ratio (PAR) of the actual load, utility, generation cost and social welfare. Actual load and unit price are compared among customers with different characteristics. Simulation on typical power systems demonstrates that base on the statistical elasticity model, RTP, as a price incentive, effectively rewards customers who provide flexibility in energy consumption.

1 Introduction

Conceptually, it has been commonly agreed that Demand Response (DR) can bring benefits to all parties involved. Such benefits include direct economic benefits to power suppliers and customers, contribution to social welfare, reliability benefits and other indirect environmental benefits [1]. DR can be implemented by providing incentives to customers such that they can reschedule their electricity consumption by load shedding and load shifting with an objective that eventually, the entire power consumption would have a reduced peak or more flattened shape. One of the widely accepted incentive is RTP which incentivize customers to reschedule electricity consumption in return of financial benefit. The aggregate impact of load rescheduling will result in a flatter load curve which is much more economically efficient for power generation. The eventual reduction in generation cost can then be used to reward customers who had participated in the RTP scheme and contributed to this cost reduction by means of a reduction in electricity bills.

Traditionally, DR has been implemented by reliability-driven Direct Load Control (DLC) with large Commercial and Industrial (C&I) customers only. It is nontrivial to deploy DR in a retail market, where the traditional flat tariff, which is based on average cost of service, is offered [2]. To enable DR in a retail market, the hardware and infrastructure for smart meters need to be developed. Secure and real-time information exchange between generators and customers needs to be available. Without such infrastructure, it would just be tough to initiate DR at local energy marketplace. While this is so, grid operators remain hesitant and adopt a wait-and-see attitude towards the rolling out of the costly infrastructure as situation is still hazy. The economic benefits of DR to both grid operators and end customers are dependent on many factors and the figure of savings for them remain uncertain and unknown. Policy makers and market participants need assurance that DR programs produce net benefits and are interested in the distribution of the benefits to market participants. Fairness is another major concern of end customers as there may exist cross subsidy between different types of customers in a deregulated retail market, especially when multiple price schemes coexist. From the society's point of view, long-run economical and environmental benefits to the society as a whole are to be assured as it is possible that DR may stimulate electricity consumption due to reduced price. A more detailed discussion on the difficulties to implement DR can be found in [3].

This paper addresses the issue of quantifying the economic benefits of DR to consumers in retail markets. One of the key inputs to quantify DR benefits is *demand elasticity*, i.e., the extent of the response of customers to the incentive given. The elasticity of energy consumers is dependent on many variables and is very complex to model accurately. It differs among different types of customers, e.g., industrial, commercial and residential customers. It may vary with demand level, the nominal level of prices and incomes, weather conditions and geographic locations, etc. For example, it has been reported that electricity demand elasticity under Time-of-Use (TOU) pricing schemes can range from -0.07 to -0.33 for residential consumers, while under dynamic price, elasticity can reach high values as -0.7 [2]. Recently, it has been recognized that another important factor to understand the benefits of DR and to efficiently engage DR programs in smart grid is *customer segmentation* [4]. In [5] and [6], it is pointed out that the price response varies with many different factors, including the heterogeneity in customer characteristics. By taking customer segments into account, utilities can enhance the performance of smart grid promoting efforts, products, programs and engagement strategies. Some existing works segment customers based on demographic profiles, e.g., the square footage of home, credit history and average monthly bill amount. Attitudinal segmentation which clusters customers into groups based on differences and/or similarities in how they think, perceive, decide and act on energy, have been largely unused. In [7], US Federal Energy Regulatory Commissions (FERC) called for investigation of DR participation rate as a function of customer type (e.g., residential, C&I; gender and income level) and customer profile (e.g., age, geographic location).

1.1 Literature Review

There have been many initiatives to quantify the benefits of DR programs in a variety of market settings and conditions [1]. These can be categorized into empirical studies [8, 9, 10, 11, 12, 13] that are based on data analysis and modeling from pilot DR programs, and analytical studies [14, 15, 16, 17, 18, 19, 20] that adopt various DR models and reasonable assumptions on market and customer characteristics.

Many pilot DR programs have been launched mainly in U.S. and Europe [21]. The work in [8] analyzed the social welfare and short-run wealth transfer among market participants in PJM electricity market in U.S. under an economic DR program, where incentive payments are made to customers with load curtailment when demand is high. Gain in social welfare is analyzed, along with the aggregate benefits to consumers who do not curtail load (price-taking customers) and price-responsive consumers.

The study in [9] estimated a Constant Elasticity of Substitution (CES) model based on the consumption data from the smart energy pricing pilot in U.S. during the same time period in 2008 and 2009 for over 1000 customers. It is found that participants reduced their peak usages in the range of 18% to 33% consistently in the two summers. The daily price-elasticity, which is the daily average self-elasticity, is found to be -0.039. The substitution elasticity is found to be -0.096, which implies that a 1% change in the ratio of peak to off-peak prices leads to -0.096% change in the ratio of peak to off-peak consumption. With new enabling technologies, the substitution elasticity can reach -0.18.

The work in [10] studied a U.K. market (Duke Power) for large C&I customers where optional day-ahead hourly price derived from marginal cost is engaged. The effects of customer characteristics and temperature conditions are considered when estimating the self and cross price elasticities for industrial customers. The study also includes social welfare analysis of the potential gain if customers transfer from TOU to RTP and finds that the net benefit is approximately 4% of the average customer's bill, and much greater than metering costs.

The Energy-Smart Pricing Plan (ESPP) in Chicago launched in 2003, which claims to be the first program to expose residential consumers to hourly RTP, is studied in [11]. The ESPP is a self-selected program and therefore participating households are statistically significantly price elastic. The program's benefits to consumers are analyzed in terms of average annual surplus. However it is commented that the precise magnitude of the price elasticity and welfare results from this experiment may not generalize to other settings and time periods, due to the differences in participating customers, smart grid technologies and price structures, etc.

The work in [12] analyzed a 6-rate real-time tariff program in French, which divides a year into 3 types of days and each day into peak and off-peak periods. Both empirical data and analytic models based on demand theory are used to explain the data collected from the DR program. Individual characteristics, including the type of heating system, ownership of different appliances and different energy management systems are considered. Results show positive improvement of welfare to the majority of the participants. However, the authors mentioned that their results cannot be generalized because the sample built during this experiment has a small size and is quite homogeneous which, in part, prevents them from measuring the influence of individual characteristics or other environmental variables on electricity consumption.

For more discussions on empirical studies, an overview of the value of dynamic pricing in mass market can be found in [2]. A summary of 10 typical empirical studies on the benefits of DR can be found in [1]. A more recent review can be found in [13] where 5 studies assessing potential costs and benefits of DR in the UK are summarized and 8 types of economic benefits are discussed.

Despite all these efforts, many studies found that it is a difficult task to quantify the economic benefits of DR programs to different participating individual customers or households in the retail market. First of all, it is both cost-expensive and time-consuming to conduct pilot DR programs in the retail market. Secondly, DR programs may not cover sufficient participating customers. Thus, studies based on pilot programs can hardly be generalized due to the limitation in participating samples, region and weather conditions, price structures etc. Thirdly, in many cases the consumption patterns and participation levels of individuals are unknown to researchers. Therefore, most existing empirical studies are more focusing on aggregated or average production cost-savings rather than benefits to individual customers.

To overcome these limitations, efforts have been made to theoretically predict the potential quantitative benefits that DR programs may bring in hypothetic market structure and conditions. Based on an RTP simulation model [22], the work in [14] analyzed the long-run efficiency gains from adopting RTP in competitive electricity markets. A hybrid market structure is assumed where some customers see real-time prices and others see time-invariant prices. The benefits when the portion of consumers enrolling with RTP increases are analyzed. Results are reported for different levels of customer response with various elasticity values, which revealed that the magnitude of efficiency gains from RTP can be significant even with small elasticity. The study in [14] also demonstrates that the incremental benefit of increasing the shares of RTP is likely to decline. While a different demand-price model is adopted in this paper, these issues are also discussed here.

The work in [15] studied the retail mass market which includes residential and small C&I customers, and quantified the benefits to customers and utilities of dynamic pricing programs with specific examples using a suite of models called the Pricing Impact Simulation Model (PRISM) Suite. The PRISM is built based on the data collected in the 2003-2005 California Statewide Pricing Pilot (SPP). The PRISM Impacts model simulates the response of customers to dynamic pricing and the PRISM Benefits model estimates financial benefits to customers and utilities. A range of utility benefits resulting from dynamic pricing is analyzed, including capacity benefits, energy savings benefits, Transmission & Distribution (T&D) benefits and price mitigation benefits. The individual customer bill impacts are simulated for different load shapes and customers with/without central air conditionings.

In [16], the impact of increasing participation of demand side in electricity markets on various categories of market participants is evaluated for a wholesale market with a day-ahead market-clearing mechanism. The presumed market allows price-sensitive demand-side bids and takes into account the load shifting behavior of consumers. The elasticity of demand is measured by the Load Participation Factor (LPF) which is the ratio of price-responsive demand to total possible demand. This work analyzed the benefits to aggregate customers that are price-responsive or price-taking, together with total social welfare. The authors showed that market-clearing price reduces with increasing level of demand shifting, and thus benefits all bidders even if they do not participate in load shifting.

In [17], the potential cost savings for residential customers are estimated by analyzing the DR capabilities of different electrical appliances that a household possesses. A model for residential DR is developed to minimize the electricity bill that a household needs to pay, using the historical spot market clearing price as a reference. Results indicate that residential DR can be cost-effective on the long term and demonstrate that increased price volatility will result in more energy cost savings per household.

An empirical demand model is presented in [18] to analyze the effectiveness of DR with TOU pricing in retail market. The rescheduling of different types of loads is pre-determined and it is assumed that the level of customer participation in DR is a linear function of the increase in TOU price as compared to the base rate. The seasons (summer and winter), different Plug-in Hybrid Electric Vehicle (PHEV) penetration levels and charging strategies are considered to determine the different levels of response. The reduction in peak demand is analyzed.

The main objective of [19, 20] is to propose optimal RTP algorithms for DR, while the economic benefits of the optimal RTP are also discussed to prove its advantages. In [19], quadratic utility functions are adopted to compute the optimal RTP that maximizes the aggregate utility of all users and minimize total generation cost, while observing the generating capacity constraints. Ten randomized customers were tested and results show that all of them can gain much higher welfare under RTP than under fixed pricing. The optimal day-ahead Time-Dependent Pricing (TDP) proposed in [20] can help flatten electricity usage and therefore reduce generation cost by 27%. The price-responsive behaviors were simulated with two sample customers of different patience indices. Results show that the patient customer could save 26% and the impatient customer could save 16% from load shifting.

The above-reviewed papers are compared with this paper in terms of methodologies, customer segmentation and economic impact analysis, as summarized in Table 1.

1.2 Our Work

This paper focuses on the quantitative analysis of economic benefits of DR to all participants, including welfare to the society as a whole, production cost for electricity generators, and economic impacts on different individual customers. Note that estimates of net benefits brought by DR vary significantly with a number of factors including the characteristics of the underlying customer base, the underlying cost curves for electricity supply, and the behavioral patterns of consumers. To conduct a valid and convincing benefit analysis, the following issues are considered in this paper.

Firstly, it is foreseeable that in a deregulated retail market, various pricing programs will arise for end customers to subscribe with DR aggregators. Therefore a hybrid market structure is assumed where multiple pricing schemes coexist. In such a hybrid scheme, the market share of RTP scheme becomes an important market index and fairness on cross subsidies becomes a major concern of the consumers.

The second concern regards to the level of customer acceptance, participation and response to price incentives. These factors determine the elasticity which is critical to the performance of RTP programs. It is essential to model properly how consumers might respond to time-dependent prices for electrical energy. This paper is based on a recently proposed statistic model of the price elasticity for electricity demand [23]. The model captures the change in demand, including both load shedding and load shifting, with regard to the dynamic price signal using explicit utility function with a set of parameters that are random variable of predefined statistical distributions. Although it is a simplified model without considering the uncertainty in demand and supply, T&D costs etc., it provides a tool to quantify the economic benefits for RTP program to all involving parties.

Thirdly, while RTP is deemed to be beneficial to customers as a whole, it is not clear if the benefit to an individual is proportionate to the value that he contributes by load rescheduling [24]. The impact to individual customers depends on his own profile and proper assessment of smart grid potentials requires better understanding of customers' behavior in power markets. Therefore customer modeling is considered in this paper. A preliminary version of the proposed customer modeling can be found in [25].

More specifically, a deregulated hybrid retail market is considered, where both RTP and traditional flat pricing schemes coexist, and the benefits that RTP in retail market brings to all parties involved are evaluated. The proposed framework calculates optimal RTP to retail customers to induce their consuming behavior in the retail market. At the same time, to achieve maximal welfare, requested demand is rescheduled according to the RTP signal and the statistical elasticity model. The percentage of customers enrolled in the RTP scheme is referred to as the market share of RTP, denoted by κ ($0 \leq \kappa \leq 1$). With the advancement in smart grid, including the maturity of relevant policy, promotion of smart meters and smart home appliances, education of smart grid and DR to customers, the market share of RTP is expected to increase. The economic benefits of RTP to end customers are then analyzed with different market shares of RTP during DR deployment. The actual load and unit price for end customers, and generation cost for generators as well as aggregate social welfare for the society as a whole, are used to quantitatively measure

Table 1: Comparison of related literature with our study.

	Paper	Market	DR options	Customer segmentation		Economic impact		
				Willingness	Capability	Society	GenCo.	Individuals
empirical studies	[8]	PJM competitive wholesale market	economic DR where incentive is payed for load curtailment when demand is high	price-taking and price-responsive loads	N	Y	Y	Calculate the maximum annual incentive DR payment that could be earned by a DR participant in PJM with a sensitivity analysis of the expected revenues
	[9]	retail market	2 types of dynamic pricing: Critical Peak Pricing (CPP) & Peak Time Rebate (PTR)	N	N	N	N	N
	[10]	UK market for large C&I customers	RTP (day-ahead hourly price) which is marginal cost	N	considers aspects of customer production processes that facilitate response to hourly prices	N	N	social welfare analysis of the potential gain if customers can transfer from TOU to RTP
	[11]	retail market	RTP based on day-ahead wholesale prices	optional residential RTP	N	N	N	Annual average household welfare gain of approximately $10, (1-2% of the average household's electricity expenditure
	[12]	retail market, residential customers	6-rate real-time tariff (3 types of days and each day has peak and off-peak periods)	N	consider individual characteristics on the type of heating system, on the ownership of different appliances and on the energy management system	N	N	analyze results on sample customers, results for individuals according to type of heating system
analytical studies	[14]	competitive wholesale and retail market	RTP & flat rate	simulated with different self-elasticities	simulated with different load profile peakiness	Y	Y	qualitative remarks on the efficiency of RTP with heterogeneous customers
	[15]	mass market	CPP, PTR, TOU	N	different load shape peakiness, customers with/without central air condition	N	Y	calculate individual customer bill impacts
	[16]	wholesale market	RTP	price-taking and price-responsive customers	N	Y	Y	2 sample customers with given parameters
	[17]	hypothetic retail market	day-ahead RTP (historical day-ahead spot-market clearing price)	N	DR capability for different appliances	N	N	calculated average cost reduction per household per day
	[18]	retail market	TOU	N	consider 2 seasons (summer and winter), 2 PHEV penetration levels (low and high) and 2 PHEV charging strategies (normal and quick)	N	analyze peak load reduction	N
	[19]	hypothetic retail market	RTP	randomized sample customers	N	Y	N	10 sample customers with given (random) parameters
	[20]	hypothetic retail market	day-ahead TDP	patient and impatient customers	N	Y	Y	2 sample customers with given parameters
	Our study	hypothetic retail market	both day-ahead RTP & flat rate	different LoP	different load profile peakiness	Y	Y	all customers with continuous parameters

* In the table, Y stands for discussed with quantitative analysis and N stands for not discussed.

the economic benefits. To address the fairness concern, the direct economic benefits received are compared among customers subscribing to different pricing schemes, and among different types of customers that subscribe to RTP scheme. Although this work is preliminary, the results do provide important insights for policy makers and related participants of electricity markets.

The remaining of this paper is structured as follows. Section 2 briefly introduces the system model, including the hybrid market structure and the algorithms for load scheduling and optimal RTP. Simulation results on example power systems and analysis are presented in Section 3, followed by conclusions drawn in Section 4.

2 System Model

Assume a day-ahead RTP scheme such that consumers have sufficient knowledge to reschedule their load. The system model is composed of two parts. Firstly, with a given price vector, which is either fixed or dynamic, and a request load profile, the Energy Management Controller (EMC) [26] embedded in each customer's smart meter will schedule the operation of appliances in response to the price vector to maximize individual's welfare. Secondly, in the retail market the retail company predicts the price vector that maximizes the society's total welfare.

Let the period of study be divided into T time slots, where $T \triangleq |\Gamma|$ and Γ is the set of all time slots under consideration. Let $\mathbf{p} = \{p_t\}$, $t \in \Gamma$ denote the RTP vector from a retail company and p_0 denote the fixed rate under the flat pricing scheme. Assume that there are in total M individual tasks $\{m : m = 1, \cdots, M\}$ that are to be initialized by all customers within Γ, and each task will consume x_m MWh electricity. To simplify the problem, it is assumed that each task can be completed within one time slot. The nomenclature used in this paper is summarized in Table 2.

2.1 Load Rescheduling by EMC

Given the price vector $\mathbf{p}$, EMC will determine, for each individual task m, the best execution time slot such that the welfare brought by task m to the consumer is maximized. Let $u_m(t)$ denote the marginal utility function, i.e., utility per unit power consumption, and let

$$w_m(t) = u_m(t) - p_t \tag{1}$$

denote the marginal welfare. The optimal time slot for task m would be t_m where

$$w_m(t_m) > 0, \quad w_m(t_m) > w_m(\tau), \quad \forall \tau \in \Gamma. \tag{2}$$

If no such t exists, i.e., $w_m(t) \leq 0$, $\forall t \in \Gamma$, then the task will be shed.

The marginal utility function of loads is then modeled as realizations of the following parametric stochastic process,

$$u(t) = \begin{cases} \beta - \delta(t - \alpha), & \alpha \leq \mathrm{t} \leq \alpha+\gamma; \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Here the parameters $\alpha, \beta, \gamma, \delta$ are random variables that characterize different aspects of the utility function. α is the time when a task is initially requested, β is the marginal utility if the task starts at requested time, $\gamma \in [1, \gamma_{max}]$ is the maximum tolerable delay of executing the task and $\delta \geq 0$ is the utility decay rate. Here, it is assumed that the marginal utility of each task decays

Table 2: Table of nomenclature.

κ	Market share of RTP.
T	Number of time slots.
Γ	Set of all time slots.
$\mathbf{p}$	Real-time price vector.
p_t	Real-time price at time slot t.
p_0	Flat price at all time slots.
M	Number of individual tasks.
x_m	Power consumption of task m (MWh).
$U_m(t)$	Utility function of task m.
$W_m(t)$	Welfare function of task m.
$u_m(t)$	Marginal utility function of task m.
$w_m(t)$	Marginal welfare function of task m.
t_m	Scheduled time slot for task m.
$u(t)$	Parametric marginal utility function.
α, β, γ, δ	Parameters in $u(t)$. α: request time; β: initial marginal utility; γ: maximum tolerable delay; δ: decay rate of marginal utility.
$f(\alpha, \beta, \gamma, \delta)$	Probability distribution of α, β, γ and δ.
$d_m(\mathbf{p}, t)$	Demand curve of task m.
$D_t(\mathbf{p})$	Aggregated demand curve.
ϵ_{tt}, $\epsilon_{t\tau}$	Self and cross-time elasticity factors.
$\hat{D}_t(\mathbf{p})$	Estimated aggregated demand curve.
$\hat{\epsilon}_{tt}$, $\hat{\epsilon}_{t\tau}$	Estimated self and cross-time elasticity factors.
X_0	Sum of energy consumption of all tasks.
$P(t)$	Probability of a task being scheduled to time slot t.
$\hat{U}_t(\mathbf{p})$	Expected utility of time slot t.
$W(\mathbf{p})$	Social welfare function.
I	Number of generator buses.
J	Number of load buses.
C	Energy generation cost.
a_j, b_j, c_j	Generator cost parameters of generator j.
ρ_t^j	Spot price of node j at time slot t.
η	Load profile peakiness factor.

linearly with time, which represents the cost of inconvenience caused by the delay. Under this model, the scheduling of each individual task is now a random event whose probability distribution is controlled by the stochastic process $u(t)$, and

$$P(t) \triangleq \Pr\{w(t) > 0, w(t) \geq w(\tau); \forall \tau \in \Gamma\} \tag{4}$$

is the probability that a task will be scheduled at time slot t, which can be calculated from the probability distribution $f(\alpha, \beta, \gamma, \delta)$ as follows:

$$P(t) = \sum_{\gamma=1}^{\gamma_{\max}} \sum_{\alpha=t-\gamma+1}^{t} \int_{\delta_t^l}^{\delta_t^h} \int_{\beta_t^l}^{\infty} f(\alpha, \beta, \gamma, \delta)\, d\beta d\delta. \tag{5}$$

The upper and lower bounds of the integrals are given by:

$$\begin{aligned} \beta_t^l &= p_t + \delta(t-\alpha), & (6)\\ \delta_t^h &= \min\left(\delta_{max}, \min_{\tau:\tau\in[\alpha,t-1]}\left(\frac{p_t - p_\tau}{\tau - t}\right)\right), & (7)\\ \delta_t^l &= \max\left(\delta_{min}, \max_{\tau:\tau\in[t+1,\alpha+\gamma]}\left(\frac{p_t - p_\tau}{\tau - t}\right)\right). & (8) \end{aligned}$$

2.2 Welfare and Optimal RTP

In conventional supply-demand economics, the notion of consumer utility is defined as the value expressed in monetary units that a consumer derives from the consumption of a good. The consumer's benefit, or welfare, is then defined as utility minus cost. Given the aggregated demand as a function of electricity price, it is now possible to calculate the utility from electricity consumption as well as the associated social welfare, i.e., the aggregated welfare from all demands for all customers [27].

Following [23], the expected utility $\hat{U}_t$ of time slot t, is calculated as:

$$\hat{U}_t(\mathbf{p}) = p_t \hat{D}_t(\mathbf{p}) + \int_0^{\infty} \hat{D}_t(\mathbf{p} + \sigma \mathbf{I})\, d\sigma, \tag{9}$$

where $\mathbf{I} = (1, \ldots, 1)^{\mathrm{T}}$ is a vector of all ones, and the aggregated demand is estimated through expectation with respect to the distribution of $u(t)$, which is given as follows:

$$\begin{aligned} \hat{D}_t(\mathbf{p}) &= E\left\{\sum_{m=1}^{M} d_m(\mathbf{p}, t)\right\} = \sum_{m=1}^{M} E\{d_m(\mathbf{p}, t)\} \\ &= \sum_{m=1}^{M} x_m \Pr\{w(t) > 0, w(t) \geq w(t'); \forall t' \in \Gamma\} \\ &= X_0 \cdot P(t). \end{aligned} \tag{10}$$

$E\{\cdot\}$ is stochastic expectation operation, $X_0 \triangleq \sum_{m=1}^{M} x_m$ is the total energy consumption to be scheduled in the target operation cycle.

It can be seen that utility is a function of the electricity price vector $\mathbf{p}$, and has the following partial derivatives:

$$\frac{\partial \hat{U}_t(\mathbf{p})}{\partial p_t} = p_t \hat{\epsilon}_{tt} + \hat{D}_t(\mathbf{p}) + \int_0^\infty \frac{\partial \hat{D}_t\left(\mathbf{p} + \sigma \mathbf{I}\right)}{\partial p_t} d\sigma, \tag{11}$$

$$\frac{\partial \hat{U}_\tau(\mathbf{p})}{\partial p_t} = p_\tau \hat{\epsilon}_{\tau t} + \int_0^\infty \frac{\partial \hat{D}_\tau\left(\mathbf{p} + \sigma \mathbf{I}\right)}{\partial p_t} d\sigma, \quad t \neq \tau, \tag{12}$$

where $\hat{\epsilon}_{tt}$ and $\hat{\epsilon}_{t\tau}$ are, respectively, the self and cross-time elasticity from the aggregated demand curve $\hat{D}(\mathbf{p})$. Furthermore, it can be shown that the partial derivatives of the global utility, which is defined as the total utility over the whole time range, with respect to price are given by:

$$\frac{\partial}{\partial p_t}\left[\sum_{\tau=1}^{T} \hat{U}_\tau(\mathbf{p})\right] = \sum_{\tau=1}^{T} p_\tau \hat{\epsilon}_{\tau t}, \quad \forall t \in \Gamma. \tag{13}$$

From the above results, it is now possible for a retail company to derive the optimal RTP that maximizes social welfare, which is given by the solution to the following optimization problem

$$\max_{\mathbf{p}, G_t^i} \quad W(\mathbf{p}) \triangleq \sum_{t=1}^{T} [\sum_{j=1}^{J} \hat{U}_t^j\left(\mathbf{p}\right) - \sum_{i=1}^{I} C_t^i\left(G_t^i\right)], \tag{14}$$

$$s.t. \qquad \mathbf{L}_t(\mathbf{G}_t, \hat{\mathbf{D}}_t(\mathbf{p})) = 0; \quad \forall t \in \Gamma; \tag{15}$$

$$\underline{\mathbf{G}}_t \leq \mathbf{G}_t \leq \overline{\mathbf{G}}_t; \quad \forall t \in \Gamma; \tag{16}$$

$$\mathbf{Z}_t(\mathbf{G}_t, \hat{\mathbf{D}}_t(\mathbf{p})) \leq 0; \quad \forall t \in \Gamma. \tag{17}$$

Here I and J are numbers of generator buses and load buses, respectively. $\hat{\mathbf{D}}_t(\mathbf{p})$ is the column vector of aggregated demands from different load buses, $\mathbf{G}_t$ is the column vector of individual productions. U_t^j is the utility of customers from different locations, and C_t^i is the energy production cost at time slot t for generator i. $\mathbf{L}_t$ is a set of M equality constraints that describes the power balancing conditions of power system operation, $\underline{\mathbf{G}}_t$ and $\overline{\mathbf{G}}_t$ are the lower and upper capacities of individual generators, respectively. $\mathbf{Z}_t$ is a set of K inequalities that represents system operating constraints such as voltage limits and transmission line/equipment overloads.

Assume that the same RTP tariff is applied to all customers from different load buses. Let $\{\rho_\tau^j\}_{j=1}^J, \forall \tau \in \Gamma$ denote the nodal prices that reflect the marginal energy costs at each individual node. The optimal RTP can be calculated by solving a set of linear equations [23]:

$$\sum_{\tau=1}^{T}\sum_{j=1}^{J} \hat{\epsilon}_{\tau t}^j p_\tau = \sum_{\tau=1}^{T}\sum_{j=1}^{J} \hat{\epsilon}_{\tau t}^j \rho_\tau^j; \quad \forall t \in \Gamma. \tag{18}$$

In this optimal RTP algorithm, since the main objective is to maximize long-term social welfare, only the average behaviors but not the variance (i.e., uncertainties) of the aggregated load profile are considered in the optimization problem. Note that although the consumption patterns of individual tasks can be very stochastic, as guaranteed by the law of large number, their aggregated behavior would still be regular for system with a large number of independent tasks. Nevertheless, the assumption of large quality of independent loads may no longer hold for other problems, e.g., operational control in a small service area. In such a case, the uncertainties of the proposed statistical framework would certainly deserve further investigation before one could extend the proposed technique to those problems.

2.3 Hybrid Retail Market and Flat Pricing

As mentioned above, a hybrid retail market is considered here. Let κ denote the market share of RTP, i.e., κ users subscribe to RTP program and $(1-\kappa)$ users subscribe to flat pricing scheme. For simplicity, it is assumed that the percentage of users subscribe to RTP program equals for all load buses.

Using the proposed algorithm for calculating the optimal RTP, an optimal flat price can also be calculated for a given load request to maximize social welfare. However, it then becomes “daily” dynamic price rather than a fixed price and differs from RTP only in the price changing rate. Therefore the flat price is simply set to the market clearing price given the spot prices from the wholesale market, which is the price that would allow producers to break even if it were charged as flat price to all customers, defined as [14]

$$p_0 = \sum_{j=1}^{J} \rho_j \cdot D / \sum_{j=1}^{J} D, \tag{19}$$

where ρ is the nodal price and D is the request demand.

Note that this flat price is only fixed for the time period under consideration and varies with the nodal price and request demand. In application, different time periods can be used for flat pricing and RTP calculation respectively. The flat price should be calculated as a long term average.

Assume that customers subscribing to flat pricing program do not differ systematically from those under RTP program, i.e., they behave the same regarding rationality in consuming of electricity. Because the proposed statistical demand model has an upper limit for marginal utility, there will still be load shedding if all customers are under flat price when the the upper bound of the marginal utility does not justify the price. Thus the demand-price elasticity is non-zero even if only flat price is used. This is a significant difference from [14].

Based on this assumption, given the flat price, customers that subscribe to the flat pricing scheme will, similar to those under RTP, reschedule their loads. However, only load shedding will occur as flat pricing provides no incentive for load shifting.

2.4 Consumers Modeling

In the proposed modeling of customers, two indices are defined to characterize a customer including an objective index which is the peakiness of the customer’s load profile, and a subjective index which is the customer’s Level-of-Participation (LoP) in the RTP program.

2.4.1 Load Profiles

In general, electricity customers can be roughly classified based on the peakiness of their load profiles [28] which include peaky load profile, average load profile and flat load profile. Compared to customers with average load profiles, those with peaky load profiles consume less during off-peak hours and more during peak hours. A peaky load curve is thus steeper than an average load curve. On the contrary, customers with flat load profiles consume relatively more during off-peak hours and less during peak hours compared to customers with average load profiles, and have a flat load curve.

Under the flat pricing scheme where price is independent of time, bills of different customers only depend on the total amount of the electricity they consumed. It is well recognized that under such a pricing scheme, the customers with flat load profiles are subsidizing the customers with peaky load profiles as the marginal cost of electricity is much higher during peak period than during off-peak period, which leads to the fairness concern [28].

Although the fairness criterion under dynamic pricing schemes is still an arguable issue and has not been well understood, it is generally believed that the above "unfairness" will be alleviated by an RTP scheme that charges higher electricity prices which more truly reflect the electricity production cost during peak hours.

Different load profiles for individual customers are simulated by introducing a *peakiness* parameter η. Let $\mathbf{x}$ denote the normalized load profile for all aggregated loads, and let $\bar{x}$ denote the average of $\mathbf{x}$. Define an individual load profile to be $\mathbf{y}$ which is the normalized vector of $\mathbf{z}$, i.e., $\mathbf{y} = \mathbf{z}/\sum \mathbf{z}$, where $\mathbf{z}$ is defined by

$$z(i) = \begin{cases} \bar{x} + |x(i) - \bar{x}|^{\eta}, & \text{if} \quad x(i) > \bar{x}; \\ \bar{x} - |x(i) - \bar{x}|^{\eta}, & \text{otherwise.} \end{cases} \tag{20}$$

The peakiness parameter η is a positive number. When $\eta = 1$, the simulated individual load curve coincides with the aggregated load curve. The simulated load curve is more peaky than the aggregated load curve when $\eta < 1$ and is flatter when $\eta > 1$. Three example load curves with different peakiness $\eta = 0.9, 1, 1.1$, respectively, are shown in Fig. 1. Note that the smaller the η is, the more peaky the load curve will be, and vice versa.

2.4.2 Level-of-Participation

In [27], the customers are classified as long-range rational, short-range rational and real-world customers. Long-range rational customers take a long range view in consumption decision making, short-range rational customers set consumption from consideration pertaining to a very short time span, and real-world customers have a consideration period in-between the above two extreme cases. Customers may have different consideration periods if they are unaware of the electricity price outside this period. In an RTP scheme where price information is available to all customers one day ahead, their different price responsive behavior actually reflects their willingness to participate in the RTP program.

Define a customer's willingness to participate as his LoP, which can be captured by the statistical model of the proposed marginal utility function. The upper bound of the maximum tolerable delay time, γ_{max}, is an important parameter that reflects the customers' LoP in a DR program. It measures how much the customer would respond to the dynamic price change. When a customer is characterized by tasks with zero tolerable delay time, the customer is in effect not responsive to price differences in RTP and all the loads are expected to be realized at requested time as long as they can justify the electricity price at that time.

It is assumed that 20% of the energy consumption are from non-deferrable loads such as critical lighting, air-conditioning, TV, etc. The remaining loads are deferrable such as washing machine and charging of electric vehicles, etc. The distribution of γ is then defined as (21)

$$p_r(\gamma) = \begin{cases} 0.2, & \text{if} \quad \gamma = 0; \\ \frac{0.8}{\gamma_{max}}, & \text{if} \quad \gamma \in \{1, \cdots, \gamma_{max}\}; \\ 0, & \text{otherwise.} \end{cases} \tag{21}$$

The larger the value of γ_{max} is, the higher the LoP will be.

Table 3: Input data for generators

Bus	Generation limits		Generation cost		
	$\overline{G}$ (MW)	$\underline{G}$ (MW)	a	b	c
1	240	25	0.1599	11.669	213.1
2	240	18.75	0.2667	10.333	200
3	240	22.5	0.2223	10.833	240

3 Numerical Examples and Benefits Analysis

3.1 Solving the Optimization Problem

The optimization problem (14)-(17) has a non-linear objective function and non-linear constraints. The objective function and constraints are once continuously differentiable almost everywhere on their domains, but are not twice continuously differentiable. The IPOPT [30] algorithm uses the primal dual interior point method with a filter line search to determine the local minima of large scale non-linear optimization problems that are at least once continuously differentiable. Thus, it is called to solve the optimization problem starting from a feasible point, which was determined using the standard OPF algorithm as implemented in the MATPOWER package [31].

It is worth to point out that the IPOPT algorithm may not find the global optimal solution due to the non-convexity of the optimization problem. However, since the focus of this paper is to propose a framework for studying the benefits of price-based DR programs without specifying the actual price design methodology being used, it is adopted in our simulation to find the price vector due to its low complexity and the results show that RTP vector determined by IPOPT can well incentivize load rescheduling comparing to the flat pricing scheme. To find the global optimal RTP, global optimizers such as NOMAD [32], which employs the Mesh Adaptive Direct search algorithm to solve global non-linear programs, can be used. However, it involves many more iterations with expensive function evaluations and has much higher complexity comparing to IPOPT.

3.2 Simulation Setup

Simulation is done with the 6-bus example case from [29] and IEEE 57 bus test case, which are hereafter referred to as case1 and case2, respectively. The power system for case1 is as shown in Fig. 2 with three generators and three load buses. The impedances are per unit on a base of 100 MVA. Table 3 and Table 4 give the input data for the 3 generators and 11 branches.

The optimization is performed for a 24-hour interval. The distribution of α is pre-determined by a typical 24-hour load curve from a utility company whereby peak demands appear from 10 am to 7 pm, and demand valleys appear early in the morning from 12 am to 7 am. It is assumed that the total requested demand is equally distributed among the three load buses. For simplicity, it is assumed that β and δ are uniformly distributed. The ranges of β and δ are $[\beta_{min}, \beta_{max}] = [45, 70]$ and $[\delta_{min}, \delta_{max}] = [0.001, 3]$ for case1 and $[\beta_{min}, \beta_{max}] = [50, 120]$ and $[\delta_{min}, \delta_{max}] = [0.001, 4]$ for case2, respectively. Furthermore, in both test cases, it is assumed that 20% of energy consumption are from non-deferrable tasks while the remaining loads are deferrable. Given the maximum tolerable delay time γ_{max} for deferrable loads, assume that the discrete random variable γ is uniformly distributed within range $[1, \gamma_{max}]$. The probability function of γ is given by (21).

The effects of parameter settings for β and δ on the simulation can be explained as follows.

Table 4: Input data for branches

From bus	To bus	r (pu)	x (pu)	b (pu)	MVA limit
1	2	0.10	0.20	0.04	50
1	4	0.05	0.20	0.04	120
1	5	0.08	0.30	0.06	90
2	3	0.05	0.25	0.06	40
2	4	0.05	0.10	0.02	140
2	5	0.10	0.30	0.04	70
2	6	0.07	0.20	0.05	90
3	5	0.12	0.26	0.05	80
3	6	0.02	0.10	0.02	170
4	5	0.20	0.40	0.08	20
5	6	0.10	0.30	0.06	40

Under flat price scheme, the actual load can be simply calculated as

$$\text{Actual Load}(\%) = \Big(\frac{\beta_{max} - p_0}{\beta_{max} - \beta_{min}}\Big) \times 100\%, \tag{22}$$

where p_0 is the flat tariff. Under RTP scheme, β relates to load shedding. The value of β and price value p_t are compared to determine whether the load at time t can be realized or not at this time. δ relates to load shifting and the value of δ is in accordance with the average time-distance normalized price difference

$$p_{\text{diff}}(t, \tau) \triangleq \frac{p_t - p_\tau}{t - \tau}. \tag{23}$$

If $\delta > p_{\text{diff}}(t, \tau)$, then load shifting from t to τ is discouraged as the utility decreases faster than price. Otherwise, load shifting is encouraged.

In this paper, uniform distributions are assumed for parameters β and δ. The actual distributions of the parameters of the statistical elasticity model have to be established through individual's consumption behaviors from RTP programs. Unfortunately, such data is not widely accessible at this moment. In the future, new simulations can be readily done using the tools and the framework provided in this paper to examine the economic benefits of RTP programs once the actual data on consumer behaviors becomes available.

3.3 Benefits to Generators and Society

First, we demonstrate the societal benefits brought by RTP in the hybrid retail market. The flat price is calculated as in (19). The optimal RTP vector is calculated using the proposed algorithm base on aggregate customers. Assume that aggregate customers exhibit average behaviors with input load profile peakiness $\eta = 1$ and maximum tolerable delay $\gamma_{max} = 12$ hours. We test the cases where κ increases from 0 (i.e., all customers subscribe to flat-pricing scheme) to 1 (i.e., all customers subscribe to RTP) with a step size of 0.1.

As shown in Table 5, with more customers subscribing to RTP scheme, a slight increase (1.84% for case1 and 3.43% for case2) of the total actual load is observed. This coincides with [14] that RTP will not necessarily reduce energy consumption. It was pointed out in [14] that, in theory, total quantity consumed could either increase or decrease. As the market share of RTP increases,

Table 5: Benefits of a hybrid dual-price scheme to society with different RTP market shares κ.

(a) case1

κ	Actual Load (%)	PAR	Utility from flat-pricing	Utility from RTP	Total Utility	Generation Cost	Social Welfare
0	0.9093	1.5157	335900.66	0	335900.66	181541.78	154358.87
0.1	0.9091	1.4078	302310.59	32964.62	335275.21	179438.56	155836.65
0.2	0.9093	1.3325	268720.52	66283.72	335004.24	178110.60	156893.65
0.3	0.9101	1.2921	235130.46	99898.87	335029.33	177328.52	157700.81
0.4	0.9115	1.2580	201540.39	133764.97	335305.36	176967.93	158337.42
0.5	0.9134	1.2292	167950.33	167825.78	335776.11	176916.80	158859.31
0.6	0.9156	1.2051	134360.26	202022.39	336382.65	177085.34	159297.31
0.7	0.9180	1.1848	100770.20	236306.40	337076.60	177409.54	159667.06
0.8	0.9206	1.1675	67180.13	270675.77	337855.90	177869.26	159986.64
0.9	0.9232	1.1530	33590.07	305054.47	338644.54	178383.16	160261.38
1	0.9260	1.1406	0	339477.65	339477.65	179094.44	160383.21

(b) case2

κ	Actual Load (%)	PAR	Utility from flat-pricing	Utility from RTP	Total Utility	Generation Cost	Social Welfare
0	0.8897	1.5157	498060.79	0	498060.79	206080.40	291980.39
0.1	0.8918	1.3992	448254.71	49603.03	497857.74	204484.59	293373.16
0.2	0.8943	1.3350	398448.63	99809.56	498258.19	203893.65	294364.53
0.3	0.8973	1.2978	348642.55	150385.70	499028.25	203897.64	295130.61
0.4	0.9006	1.2671	298836.47	201187.36	500023.84	204275.27	295748.57
0.5	0.9040	1.2415	249030.39	252114.44	501144.83	204885.03	296259.80
0.6	0.9074	1.2201	199224.32	303096.99	502321.30	205630.67	296690.63
0.7	0.9108	1.2020	149418.24	354080.62	503498.86	206440.18	297058.68
0.8	0.9141	1.1865	99612.16	405056.59	504668.75	207291.91	297376.84
0.9	0.9172	1.1732	49806.08	455989.07	505795.15	208140.67	297654.48
1	0.9202	1.1615	0	506882.65	506882.65	208983.77	297898.88

the PAR of the aggregate actual load curve decreased significantly with a maximum value of 24.75% for case1 and 23.37% for case2. This is because more and more RTP participants reschedule their load to off-peak hours. As for utility, on one hand, the utility received by customers subscribing to flat-price scheme decreases linearly as the percentage of flat-price users decreases. On the other hand, the utility received by customers subscribing to RTP scheme increases nonlinearly. As more customers subscribe to RTP, these customers will receive more unit utility. However, generation cost does not necessarily decrease along the direction of RTP deployment. The total generation cost decreases first, due to lower PAR. As the actual load increases, generation cost increases again when more customers subscribe to RTP although PAR keeps decreasing. It should be noted that in this work only the cost for electricity production is considered, which is calculated as

$$C = \sum_{t=1}^{T} \sum_{j=1}^{J} (a_j \cdot D_{j,t}^2 + b_j \cdot D_{j,t} + c_j), \tag{24}$$

where (a_j, b_j, c_j) are parameters for generator bus j (values for case1 given in Table 3). The total cost should decrease much more significantly if unit commitment is taken into considerations. This is because the PAR of the result load curve is much smaller. To cater for such flatter load curves, the peak units that provide electricity only for the peak hours are no longer needed, therefore reducing total cost in the long run. Social welfare, which is the total utility minus generation cost, increases when more customers subscribe to RTP instead of flat-pricing scheme. Comparing the two extreme cases, increment of 3.9% for case1 and 2% for case2 in social welfare is observed when the entire market shifts from flat-pricing to RTP.

3.4 Benefits to Customers

In this subsection, the impact of RTP on different types of individual customers is analyzed. First, we sample nine customers, with peaky ($\eta = 0.9$), average ($\eta = 1$) and flat ($\eta = 1.1$) load profiles, and with different LoP ($\gamma_{max} = 4, 12, 23$, respectively). The *impacts* considered here include the actual load realized, in percentage w.r.t. initial requested load, and the unit price that a customer bears. The unit price is defined as one's total bill divided by the total quantity he consumes, i.e.,

$$p_{\text{unit}} = \frac{\sum_{t=1}^{T} \hat{\mathbf{L}}(t) \cdot p_t}{\sum_{t=1}^{T} \hat{\mathbf{L}}(t)}, \tag{25}$$

where $\hat{\mathbf{L}}(t), (t = 1, 2, \cdots, T)$ is his actual load over the considered time period. Given a fixed market structure (i.e., fixed κ), a customer can either subscribe to a flat-pricing scheme or to an RTP scheme. If he subscribes to the former, then regardless of his load profile, the actual load and unit price will not differ. On the contrary, subscribing to the latter would cause the actual load and unit price to depend on the respective input load profile and LoP.

The following observations can be made from Tables 6 and 7. First of all, customers with flat load profiles receive more benefits from RTP than those with peaky load profiles. When the market share of flat pricing and RTP is fixed, i.e., given κ, customers with flat load profiles can have more actual load comparing to those with peaky load profiles if their LoP equals. The unit price that they pay is also less than those with peaky load profiles. Secondly, active participation will bring benefits to customers who subscribe to RTP scheme. If customer actively participates and reschedule his loads with longer maximum delay time γ_{max}, the actual load will increase, and the unit price will decrease. Thirdly, when more customers subscribe to RTP scheme, the benefits to existing subscribers will decrease. The actual load for an existing customer may either increase or decrease, but the unit price that he needs to pay increases. Last but not least, RTP will always be more beneficial to consumers than flat-pricing. In any case, when a customer changes his power consumption plan from flat-pricing to RTP, the unit price that he pays will decrease. The benefit in lower unit price will no doubt attract more customers to join the RTP scheme. This also provide incentives for them to reshape their demand curves to be more flat, and reschedule their load more actively according to the price change.

The curves of actual load and unit price of consumers with different load profiles and different LoP values are also shown in Fig. 3 and Fig. 4. In [28] it is noted that all users should benefit from dynamic pricing, regardless of their response. This viewpoint is supported by Fig. 3 (d) and Fig. 4 (d), where the unit price that RTP users bear is always smaller than that of users under flat pricing scheme, unless their load profile is impractically peaky (Fig.3 (b) and Fig. 4 (b)).

Results demonstrate that comparing to the flat pricing, RTP is more fair as the unit price a customer pays is linked with the shape of his load profile, and the extent that he reschedules his load according to the dynamic price. The cross-subsidy problem of the flat pricing scheme, if not eliminated, is much mitigated. It should be pointed out that strict elimination of cross-subsidy is not possible. For example, it is hard to justifiably separate the generation cost incurred by customers under flat pricing and that incurred by customers under RTP.

4 Conclusions

A framework for quantitative analysis of economic benefits that DR programs can bring to all parties involved in a dual-price retail market has been presented in this paper. The main contribution of this work is that it provides a tool for theoretical analysis of economic benefits that RTP brings to various market participants. This paper also discusses customer modeling and the impact of RTP on individual customers, depending on their different levels of subjective willingness and objective capabilities. The analysis demonstrates that RTP, as a price incentive, effectively rewards customers who provide flexibility in energy consumption. Results suggest that efforts should be made to encourage retailers to design proper RTP schemes and publicize to end users. It should be noted that the assumptions about customer acceptance and participation rates could significantly affect the estimated DR benefits. Although reasonable values are chosen for the model parameters, we point out that the accuracy of this estimate depends on the accuracy that the statistical elasticity model captures the actual circumstances, while modeling customer behaviors in electricity market is worth further investigating in future research for theoretical studies.

References

[1] US DOE. Benefits of demand response in electricity markets and recommendations for achieving them. US. Department of Energy; 2006.

[2] Faruqui A, George SS. The value of dynamic pricing in mass markets. The Electricity Journal. 2002 July;15:45–55.

[3] US FERC. A National Assessment of Demand Response Potential. US. Federal Energy Regulatory Commission (FERC); 2009.

Table 6: Actual load for different customers in a hybrid retail market with dual-price scheme where RTP market share κ increases.

(a) case1

		Peakiness	$\eta = 0.9$ (Peaky Load Profile)			$\eta = 1$ (Average Load Profile)			$\eta = 1.1$ (Flat Load Profile)		
		LoP	4	12	23	4	12	23	4	12	23
Subscribe to RTP	κ	0.1	0.8818	0.8928	0.8974	0.8968	0.9064	0.9104	0.9066	0.9152	0.9188
		0.2	0.8869	0.8960	0.9001	0.9011	0.9091	0.9127	0.9104	0.9175	0.9208
		0.3	0.8917	0.8994	0.9031	0.9052	0.9119	0.9152	0.9139	0.9200	0.9230
		0.4	0.8962	0.9028	0.9061	0.9090	0.9147	0.9177	0.9173	0.9225	0.9251
		0.5	0.9002	0.9059	0.9089	0.9124	0.9174	0.9200	0.9203	0.9248	0.9272
		0.6	0.9037	0.9086	0.9114	0.9154	0.9197	0.9221	0.9229	0.9268	0.9290
		0.7	0.9066	0.9110	0.9135	0.9179	0.9217	0.9238	0.9251	0.9285	0.9305
		0.8	0.9092	0.9131	0.9153	0.9201	0.9234	0.9254	0.9270	0.9301	0.9319
		0.9	0.9113	0.9147	0.9168	0.9218	0.9248	0.9266	0.9285	0.9312	0.9329
		1	0.9132	0.9162	0.9181	0.9233	0.9260	0.9276	0.9298	0.9322	0.9338
Subscribe to Flat-pricing			0.9093								

(b) case2

		Peakiness	$\eta = 0.9$ (Peaky Load Profile)			$\eta = 1$ (Average Load Profile)			$\eta = 1.1$ (Flat Load Profile)		
		LoP	4	12	23	4	12	23	4	12	23
Subscribe to RTP	κ	0.1	0.8893	0.8989	0.9038	0.9023	0.9106	0.9150	0.9106	0.9182	0.9222
		0.2	0.8942	0.9018	0.9060	0.9063	0.9129	0.9166	0.9141	0.9201	0.9235
		0.3	0.8983	0.9045	0.9081	0.9096	0.9151	0.9183	0.9169	0.9219	0.9248
		0.4	0.9017	0.9068	0.9100	0.9124	0.9169	0.9197	0.9193	0.9234	0.9260
		0.5	0.9043	0.9086	0.9114	0.9145	0.9183	0.9208	0.9210	0.9246	0.9268
		0.6	0.9062	0.9100	0.9124	0.9160	0.9193	0.9214	0.9222	0.9253	0.9273
		0.7	0.9076	0.9109	0.9130	0.9169	0.9198	0.9218	0.9229	0.9256	0.9274
		0.8	0.9086	0.9115	0.9134	0.9176	0.9201	0.9219	0.9234	0.9257	0.9273
		0.9	0.9093	0.9119	0.9136	0.9180	0.9202	0.9218	0.9235	0.9256	0.9270
		1	0.9098	0.9120	0.9136	0.9181	0.9201	0.9215	0.9235	0.9253	0.9266
Subscribe to Flat-pricing			0.8897								

Table 7: Unit price paid by different customers in a hybrid retail market with dual-price scheme where RTP market share κ increases.

(a) case1

	Peakiness		$\eta = 0.9$ (Peaky Load Profile)			$\eta = 1$ (Average Load Profile)			$\eta = 1.1$ (Flat Load Profile)		
	LoP		4	12	23	4	12	23	4	12	23
Subscribe to RTP	κ	0.1	45.1923	44.3511	43.8160	44.3993	43.6649	43.1980	43.9009	43.2329	42.8083
		0.2	45.4765	44.7616	44.2856	44.7255	44.0984	43.6825	44.2528	43.6804	43.3018
		0.3	45.6803	45.0731	44.6554	44.9766	44.4419	44.0763	44.5329	44.0435	43.7105
		0.4	45.8401	45.3245	44.9610	45.1837	44.7283	44.4098	44.7693	44.3515	44.0611
		0.5	45.9713	45.5321	45.2155	45.3598	44.9707	44.693	44.9733	44.6155	44.3621
		0.6	46.0811	45.7031	45.4241	45.5111	45.1752	44.9300	45.1504	44.8408	44.6169
		0.7	46.1750	45.8485	45.6031	45.6434	45.3524	45.1366	45.3068	45.0381	44.8408
		0.8	46.2559	45.9728	45.7572	45.7597	45.5069	45.3170	45.4451	45.2114	45.0377
		0.9	46.3266	46.0801	45.8904	45.8625	45.6421	45.4750	45.5682	45.3642	45.2112
		1	46.3887	46.1732	46.0058	45.9541	45.7610	45.6134	45.6782	45.4993	45.3642
Subscribe to Flat-pricing			47.2665								

(b) case2

	Peakiness		$\eta = 0.9$ (Peaky Load Profile)			$\eta = 1$ (Average Load Profile)			$\eta = 1.1$ (Flat Load Profile)		
	LoP		4	12	23	4	12	23	4	12	23
Subscribe to RTP	κ	0.1	55.0691	53.5096	52.5221	53.8133	52.4447	51.5805	53.0208	51.7718	50.9848
		0.2	55.3392	54.1125	53.3033	54.1873	53.1062	52.3974	53.4591	52.4695	51.8234
		0.3	55.4809	54.4982	53.8334	54.4285	53.5598	52.9767	53.7623	52.9653	52.4335
		0.4	55.5731	54.7648	54.2041	54.6084	53.8916	53.3993	53.9970	53.3380	52.8884
		0.5	55.6445	54.9705	54.4952	54.7570	54.1580	53.7402	54.1941	53.6424	53.2607
		0.6	55.7074	55.1387	54.7329	54.8874	54.3810	54.0241	54.3668	53.8999	53.5736
		0.7	55.7658	55.2803	54.9313	55.0046	54.5718	54.2645	54.5211	54.1217	53.8406
		0.8	55.8217	55.4035	55.1011	55.1124	54.7390	54.4727	54.6615	54.3167	54.0730
		0.9	55.8748	55.5110	55.2471	55.2110	54.8860	54.6535	54.7889	54.4886	54.2757
		1	55.9256	55.6066	55.3747	55.3022	55.0170	54.8126	54.9056	54.6420	54.4548
Subscribe to Flat-pricing			57.7233								

[4] Smart Grid Consumer Collaborative. 2012 State of the Consumer Report; 2012.

[5] Reiss PC, White MW. HoDehold electricity demand, Revisited. Review of Economic Studies. 2005;72(3):853–883.

[6] Faruqui A, Gerge S. Quantifying customer response to dynamic pricing. The Electricity Journal. 2005;18(4):53–63.

[7] US FERC. National Action Plan on Demand Response. US. Federal Energy Regulatory Commission (FERC); 2010.

[8] Walawalkar R, Blumsack S, Apt J, Fernands S. An Economic Welfare Analysis of Demand Response in the PJM Electricity Market. Energy Policy. 2008;36(10):3692–3702.

[9] Faruqui A, Sergici S. Dynamic Pricing of Electricity in the Mid-Atlantic Region: Econometric Results from the Baltimore Gas and Electric Company Experiment. Journal of Regulatory Economics. 2011;40:82–109.

[10] Taylor TN, Schwarz PM, Cochell JE. 24/7 hourly response to electricity real-time pricing with up to eight summers of experience. Journal of Regulatory Economics. 2005;27(3):235–262.

[11] Allcott H. Rethinking real-time electricity pricing. Resource and Energy Economics. 2011;33(4):820–842.

[12] Aubin C, Fougere D, Husson E, Ivaldi M. Real time pricing of electricity for residential customers: Econometric analysis of an experiment. Journal of Applied Econometrics. 1995;10(S1):S171–S191.

[13] Bradley P, Leach M, Torriti J. A Review of the Costs and Benefits of Demand Response for Electricity in the UK. Energy Policy. 2012;52:312–327.

[14] Borenstein S. The long-run efficiency of real-time electricity pricing. The Energy Journal. 2005;26(3):93–116.

[15] Faruqui A, Wood L. Quantifying the benefits of dynamic pricing in the mass market. The Brattle Group; 2008.

[16] Su CL, Kirschen DS. Quantifying the effect of demand response on electricity markets. IEEE Transactions on Power Systems. 2009 August;24(3):1199–1207.

[17] Abdisalaam A, Lampropoulos I, Frunt J, Verbong GPJ, Kling WL. Assessing the economic benefits of flexible residential load participation in the Dutch day-ahead auction and balancing market. In: International Conference on the European Energy Market (EEM); 2012.

[18] Shao S, Zhang T, Pipattanasomporn M, Rahman S. Impact of TOU rates on distribution load shapes in a smart grid with PHEV penetration. In: Transmission and Distribution Conference and Exposition, IEEE PES. IEEE; 2010. p. 1–6.

[19] Samadi P, Mohsenian-Rad AH, Schober R, Wong VWS, Jatskevich J. Optimal Real-Time Pricing Algorithm Based on Utility Maximization for Smart Grid. In: IEEE International Conference on Smart Grid Communications (SmartGridComm); 2010. p. 415–420.

[20] Joe-Wong C, Sen S, Ha S, Chiang M. Optimized Day-Ahead Pricing for Smart Grids with Device-Specific Scheduling Flexibility. IEEE Journal on Selected Areas in Communications. 2012;30:1075–1085.

[21] Simoes MG, Roche R, Kyriakides E, Suryanarayanan S, Blunier B, McBee KD, et al. A comparison of smart grid technologies and progresses in Europe and the U.S. IEEE Transactions on Industry Applications. 2012;48(4):1154–1162.

[22] Borenstein S, Holland SP. Investment efficiency in competitive electricity markets with and without time-varying retail prices. Center for the Study of Energy Markets Working Paper, University of California Energy Institute; 2003. 106. Available from: `http://www.ucei.org/PDF/csemwp106.pdf`.

[23] Yu R, Yang W, Rahardja S. A Statistical Demand-Price Model with Its Application in Optimal Real-Time Price. IEEE Transactions on Smart Grid. 2012;3(4):1734–1742.

[24] Kirschen DS, Strbac G, Cumperayot P, de Paiva Mendes D. Factoring the elasticity of demand in electricity prices. IEEE Trans on Power Systems. 2000 May;15(2):612–617.

[25] Yang W, Yu R, Rahardja S. Benefits to Consumers under RTP with a Statistical Demand Model. In: IEEE PES Innovative Smart Grid Technologies - Asia (ISGT Asia); 2012.

[26] Xiong G, Chen C, Kishore S, Yener A. Smart (in-home) power scheduling for demand response on the smart grid. In: IEEE PES Innovative Smart Grid Technologies (ISGT); 2011.

[27] David AK, Li YZ. Consumer rationality assumptions in the real time pricing of electricity. Generation, Transmission and Distribution, IEE Proceedings. 1992 July;139:315–322.

[28] Faruqui A. The Ethics of Dynamic Pricing. The Electricity Journal. 2010 July;23(6):13–27.

[29] Wood AJ, Wollenberg BF. Power Generation, Operation, and Control, 2nd Edition. John Wiley & Sons; 1996.

[30] Wachter A, Biegler LT. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. Mathematical Programming. 2006;106:25–57.

[31] Zimmerman RD, Murillo-Sánchez CE, Thomas RJ. MATPOWER: Steady-State Operations, Planning and Analysis Tools for Power Systems Research and Education. IEEE Transactions on Power Systems. 2011 Feb;26(1):12–19.

[32] S. Le Digabel, Algorithm 909: NOMAD: Nonlinear Optimization with the MADS algorithm, ACM Transactions on Mathematical Software, 2011; 37(4):1–15.

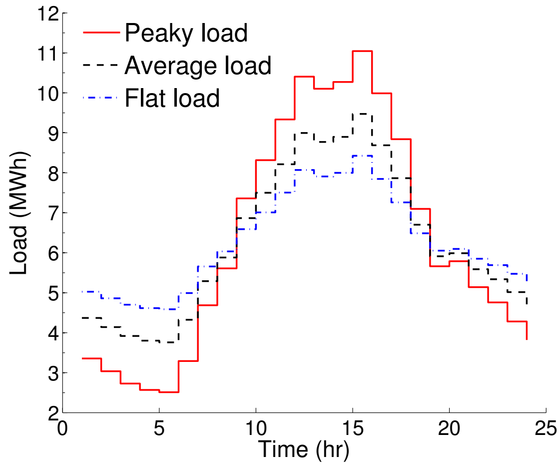


Figure 1: Three example load curves with different peakiness, $\eta = 0.9, 1, 1.1$ for peaky load, average load and flat load, respectively.

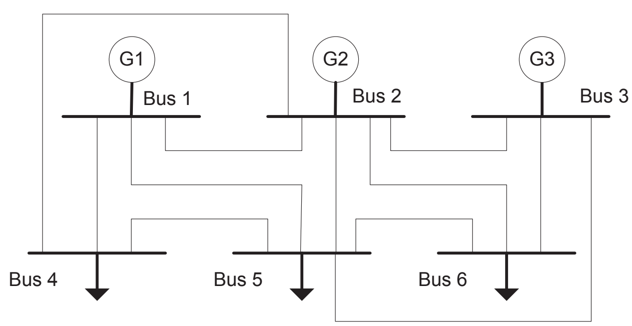
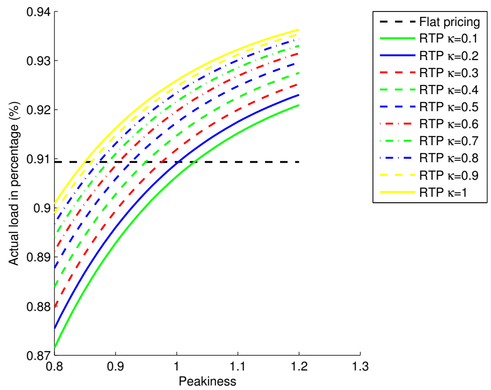
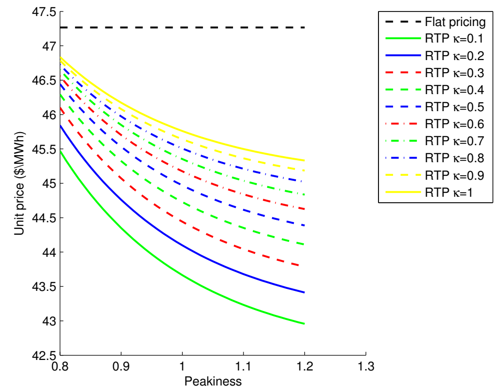


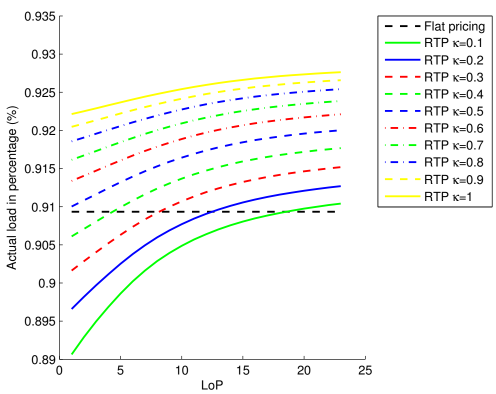
Figure 2: Example power system with three generators and three load buses.



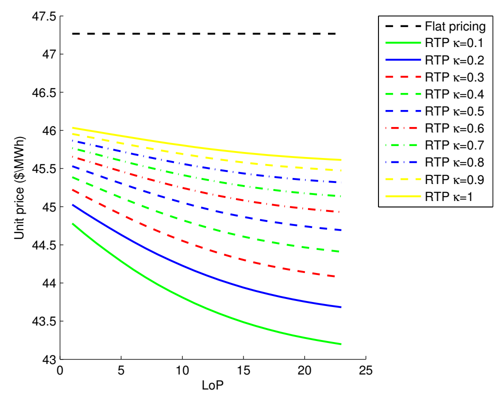
(a) Actual load w.r.t. peakiness



(b) Unit price w.r.t. peakiness

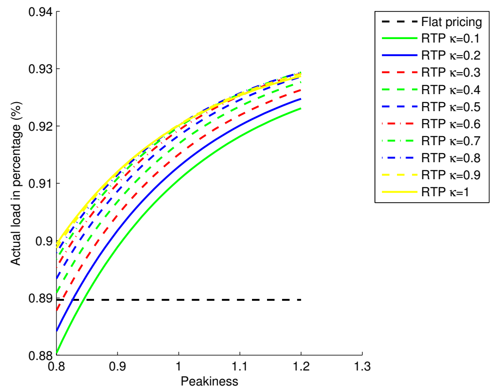


(c) Actual load w.r.t. level-of-participation

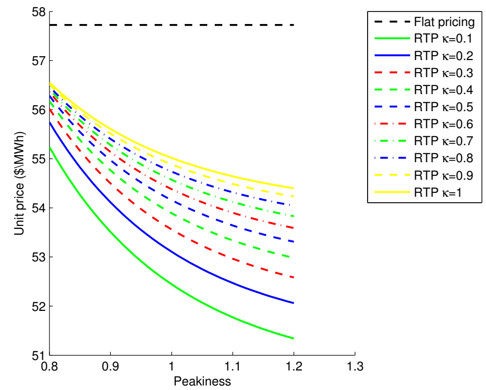


(d) Unit price w.r.t. level-of-participation

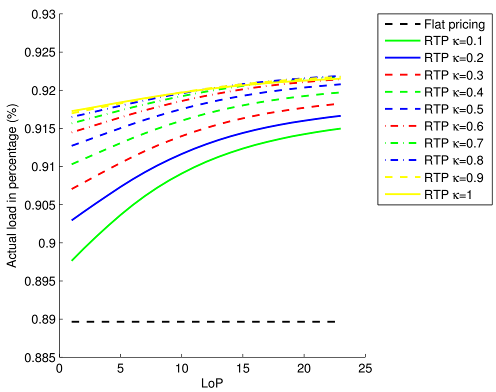
Figure 3: Results for case1, actual load and unit price for individual customers with different load profile peakiness and LoP.



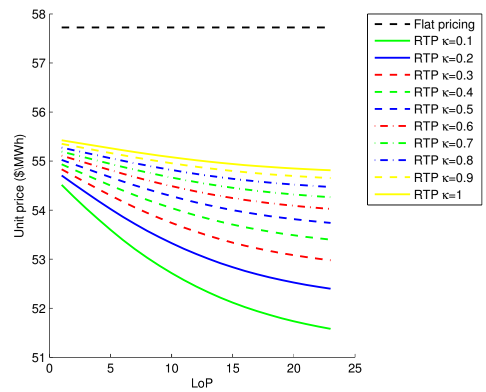
(a) Actual load w.r.t. peakiness



(b) Unit price w.r.t. peakiness



(c) Actual load w.r.t. level-of-participation



(d) Unit price w.r.t. level-of-participation

Figure 4: Results for case2, actual load and unit price for individual customers with different load profile peakiness and LoP.