A Modified Slip Model for Gas Lubrication at Nanoscale Head Disk Interface

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ABSTRACT

In this paper, we derive a modified slip model for gas lubrication based on the effective viscosity concept and a kinetic-theory based slip boundary condition. A more accurate expression for the effective mean free path is proposed, which can overcome the deficiency of other slip models in predicting effective mean free path. In comparison with the existing slip models, such as the first, second and 1.5 order slip models, the current slip model agrees well with F-K model over the entire flow regimes and therefore is expected to be more accurate for ultra-thin gas film lubrication at head-disk interface.

Keywords: effective viscosity, effective mean free path, slip flow, gas lubrication, head-disk interface, hard disk drive

1 INTRODUCTION

In the modern magnetic storage devices, air bearing sliders are normally used to position the magnetic head at the target track over the surface of the magnetic media. Between the surfaces of magnetic head and disk is a thin layer of air film to form high stiffness air-bearing cushion and support the head float over the disk, which constitutes a successful application of gas lubrication. The minimum thickness of air film, or called the slider-disk mechanical spacing or flying height, is only about 1~2 nm in the current hard disk drives. At such a small gap, the air flow at the head-disk interface is highly rarefied, which leads to the corresponding Knudsen number, which is defined as the ratio of mean free path to the spacing of the slider/disk interface, well beyond the continuum region of $Kn<0.01$. Normally, the rarefied flow can be classified into four flow regimes: continuum flow ($Kn<0.01$), slip flow ($0.01\leq Kn<0.1$), transition flow ($0.1\leq Kn<10$) and free molecule flow ($Kn\geq 10$) [1] according to its Knudsen number. In today's hard disk drive, as the head-disk spacing continues to decrease for a higher recording areal density, the corresponding Knudsen number is also driven to a level well above the unity. Therefore, the traditional macroscopic Reynolds equation based on the continuum assumption with non-slip boundary conditions is no longer valid. Some modifications with the consideration of the slip flow boundary conditions are required.

Previously, there are several established slip models, including the first order slip model of Burgdorfer [2], the second order slip model of Hsai and Domoto [3], the 1.5 order slip model of Mitsuya [4], and the new first and second order slip models of Wu and Bogy [5]. Although these models are claimed to be applicable in the slip or even transition regime, the physical justification of their applicability to the high Knudsen number regions is not clear as the continuum description is valid only as long as the Knudsen number is sufficiently small. To solve this problem, Fukui and Kaneko [6] derived a generalized Reynolds-type lubrication equation based on the linearized Boltzmann equation and proved its validity for arbitrary Knudsen number. Their results showed that the Burgdorfer's first order slip model underestimates the flow rate while the classic second order slip model overestimates it in the transition and free molecular regimes. Since then, F-K model has been widely used in the air bearing simulations for the head-disk interface in hard disk drives. It is also believed to give the most accurate results for arbitrary Knudsen number among the existing compressible lubrication equations [7].

However, compared with molecular-based F-K model, the conventional slip models are simpler and more efficient to be implemented into the air bearing code because they don't require interpolating the database [8], or using complex power series approximations for Poiseuille flow rate coefficient calculation, which are usually difficult to get when the surface accommodation coefficient is below 1.0. Therefore, the efficiency of air bearing simulation code can be greatly improved. By recognizing the merits of conventional slip models, Shen *et al.* [9] derived a new slip model from the first order solution of the Boltzmann equation and showed that this new slip model predicts results much closer to the F-K model. They also claimed that their slip model has the advantages in that it is intrinsically first order in nature, is consistently derived from the kinetic theory, and is more accurate than existing models over the entire Knudsen number range [9]. Although their efforts to derive a new slip model based on Boltzmann equation are commendable, some unclear area in the paper still needs to be clarified. For example, as mentioned by Chen and Bogy [7], the application of the Matthiessen rule in calculating effective mean free path may not be suitable for rarefied gas dynamics. In this paper, we will address this issue as well and propose a new formula for the effective mean free path based on the effective viscosity concept. Then we will start from Shen's kinetic-theory based slip boundary condition to derive a modified slip model which overcomes the deficiency of Shen's model in using the Matthiessen rule to predict effective mean free path. Finally, we will compare this model with other established slip models, especially Peng's model [10], which is also based on modification of the mean free path.

2 EFFECTIVE VISCOSITY AND EFFECTIVE MEAN FREE PATH

The concept of effective viscosity is first originated from first order slip model proposed by Burgdorfer [2]. Considering a Couette flow between two parallel plates with the lower plate moving at a velocity of U and the upper plate kept stationary, the shear stress at one of the plate surfaces is

$$\tau = \frac{\mu}{h + 2\lambda} U \tag{1}$$

where h is the spacing between two plates and μ is viscosity coefficient. The contribution of mean free path λ can be included in the effective viscosity as

$$\mu_{eff} = \frac{\mu}{1 + 2Kn} \tag{2}$$

where $Kn = \lambda / h$, Knudsen number is a parameter to measure the rarefaction effect.

In order to find a more accurate model for the effective viscosity, the linearized Boltzmann equation should be applied instead of the first order slip model. Based on the pioneer work of Cercignani and Pagani [11], Loyalka *et al.* [12] and Sone *et al.* [13], Veijola and Turowski [14] developed an approximate model for the effective viscosity by curve fitting the exact values from Boltzmann equation. They found that the maximum relative error of this approximation, at any value of *Kn*, is less than ±0.6%. The expression for their model can be written as

$$\mu_{eff} = f(Kn) \cdot \mu = \frac{\mu}{1 + 2Kn + 0.2 \cdot Kn^{0.788} \cdot e^{-Kn/10}} \tag{3}$$

The effective mean free path is related to the effective viscosity by the following relation [15]

$$\lambda_{eff} = f(Kn) \cdot \lambda = \frac{\lambda}{1 + 2Kn + \beta \cdot Kn^{0.788} \cdot e^{-Kn/10}} \tag{4}$$

where β is a curve-fitting parameter, which will be used to best fit the data from F-K model. Its physical meaning may be related to the molecular model used in the study. For hard sphere model, β equals to 0.2. However, for more realistic variable hard sphere (VHS) or variable soft sphere (VSS) model, β should be larger than 0.2 due to the shorter mean free path obtained from VHS and VSS models [16].

Previously, Peng *et al.* [10] and Shen *et al.* [9] used probabilistic mean method and the Matthiessen rule, respectively, to derive the effective mean free path. In their expressions, the effective mean free path can be written as

$$\text{Shen's equation: } \frac{1}{\lambda_{eff}} = \frac{1}{\lambda/\sqrt{3}} + \frac{1}{h/2} \tag{5}$$

$$\text{Peng's equation: } \lambda_{eff} = \begin{cases} \lambda\left(1 - \dfrac{\lambda}{4h}\right), & h \geq \lambda \\ \lambda\left(\dfrac{3h}{4\lambda} - \dfrac{h}{2\lambda} In\left(\dfrac{h}{\lambda}\right)\right), & h < \lambda \end{cases} \tag{6}$$

The differences among equations (4-6) can be seen more clearly by using Fig. 1, which shows the variation of normalized effective mean free path λ_{eff} / λ with the inverse Knudsen number, $D = \sqrt{\pi}/(2Kn)$. We can see that the effective mean free path based on the effective viscosity concept (equation (4) with $\beta = 0.2$) agrees well that in Shen's slip model using the Matthiessen rule (equation (5)) when D<1. However, as D value increases further for a continuum flow regime, the λ_{eff} / λ predicted by Shen's equation approaches to $1/\sqrt{3}$ instead of 1.0, which means that the Matthiessen rule may not be suitable for rarefied gas dynamics. As for Peng's equation (6), it predicts a much longer λ_{eff} at each D value. This is probably because they neglected the randomness of molecular collisions in the derivations. From Fig. 1, we can also see that the λ_{eff} / λ will be reduced as β value increases. A bigger β value means more molecular collisions and shorter λ_{eff} at each D value. We will discuss more about β effects in the next sections.

3 DERIVATION OF A MODIFIED SLIP MODEL USING THE NEW λ_{eff}

Our derivation is mostly based on the new slip boundary condition proposed by Shen *et al.* [9]. For air bearing problem as shown in Fig. 2, the Navier-Stokes equation can be simplified as

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2} \tag{7}$$

$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2} \tag{8}$$

$$\frac{\partial p}{\partial z} = 0 \tag{9}$$

$$\frac{\partial}{\partial x}\left(\rho\int_0^h udz\right)+\frac{\partial}{\partial y}\left(\rho\int_0^h vdz\right)=0 \tag{10}$$

where p is pressure, ρ is gas density, u and v are air velocity at x and y directions.

By using Shen's slip boundary condition [9], which is derived based on linearized Boltzmann equation, and considering accommodation coefficient effect on the air bearing surfaces, we have

$$u(z=0)=\frac{2-a}{a}\tau\left(\frac{\pi kT}{2m}\right)^{1/2}\frac{\partial u}{\partial z}|_{z=0}-\frac{\lambda_{eff}}{amn}\left(\frac{2\pi m}{kT}\right)^{1/2}\frac{\partial p}{\partial x}+U \tag{11}$$

$$u(z=h)=-\frac{2-a}{a}\tau\left(\frac{\pi kT}{2m}\right)^{1/2}\frac{\partial u}{\partial z}|_{z=h}-\frac{\lambda_{eff}}{amn}\left(\frac{2\pi m}{kT}\right)^{1/2}\frac{\partial p}{\partial x} \tag{12}$$

where $\lambda=\tau(\pi kT/2m)^{1/2}$, a is the surface accommodation coefficient, τ is the relaxation time, m is molecular mass, n is number density, T is the gas temperature and k is Boltzmann constant.

The first terms on the right-hand sides of above equations are slip velocity caused by the velocity gradient in the z-direction. The second terms are due to the pressure gradient along the flow direction in the Knudsen layer. These two terms are both first order, which is consistent with the Boltzmann equation [9]. From Shen's derivation, we can also see that the effective mean free path λ_{eff} should be considered in the second term only instead of the first term. This is different from Peng's model, which considers λ_{eff} in both first and second terms.

Integrating equations (7) and (8) with z and using the above boundary conditions, we can have the following expressions for velocity field in the air bearing

$$u=-\frac{1}{2\mu}\frac{\partial p}{\partial x}(\frac{2-a}{a}\lambda h+hz-z^2)+U\left(1-\frac{z+(2-a)\lambda/a}{2(2-a)\lambda/a+h}\right)-\frac{\lambda_{eff}}{amn}\left(\frac{2\pi m}{kT}\right)^{1/2}\frac{\partial p}{\partial x} \tag{13}$$

$$v = -\frac{1}{2\mu}\frac{\partial p}{\partial y}(\frac{2-a}{a}\lambda h + hz - z^2) - \frac{\lambda_{eff}}{amn}\left(\frac{2\pi m}{kT}\right)^{1/2}\frac{\partial p}{\partial x} \tag{14}$$

Substituting equations (13) and (14) into equation (10), we have non-dimensional Reynolds equation as

$$\frac{\partial}{\partial X}\left(\widetilde{Q}_p PH^3\frac{\partial P}{\partial X} - \Lambda PH\right) + \frac{\partial}{\partial Y}\left(\widetilde{Q}_p PH^3\frac{\partial P}{\partial Y}\right) = 0 \tag{15}$$

where $P = p/p_a$, $H = h/h_0$, $X = x/l$, $Y = y/l$ are non-dimensional pressure, bearing clearance, x and y coordinate, respectively; p_a, h_0 and l are ambient pressure, slider's minimum clearance and slider's length, respectively; $\Lambda = 6\mu Ul/(p_a h_0^2)$ is the bearing number; $\widetilde{Q}_p$ is the flow factor, which is defined by

$$\widetilde{Q}_p = \frac{Q_p(D)}{Q_{con}(D)} \tag{16}$$

where Q_{con} is the non-dimensional continuum flow rate coefficient and equal to *D/6*; Q_p is the non-dimensional Poiseuille flow rate coefficient and it is defined as

$$Q_p = -\frac{q_p\sqrt{2kT/m}}{h^2\partial p/\partial x} \tag{17}$$

where q_p is the Poiseuille flow rate between two parallel plates with a spacing of *h*, which can be written as

$$q_p = \int_0^h \rho u dz = -\frac{\rho}{2\mu}\frac{\partial p}{\partial x}(\frac{2-a}{a}\lambda h^2 + \frac{h^3}{6}) - \frac{\rho\lambda_{eff}h}{amn}\left(\frac{2\pi m}{kT}\right)^{1/2}\frac{\partial p}{\partial x} \tag{18}$$

Substituting equation (18) into equation (7) and using the relations $\rho = mn$ and $\rho/\mu = (\pi m/2kT)^{1/2}/\lambda$ based on kinetic theory, we have

$$Q_p = \frac{h\sqrt{\pi}}{12\lambda} + \left(\frac{2}{a} - 1\right)\frac{\sqrt{\pi}}{2} + \frac{2\sqrt{\pi}\lambda_{eff}}{ah} = \frac{D}{6} + \left(\frac{2}{a} - 1\right)\frac{\sqrt{\pi}}{2} + \frac{\pi}{aD_{eff}} \tag{19}$$

where $D_{eff} = \sqrt{\pi}/(2Kn_{eff})$ and Kn_{eff} is the effective Knudsen number, which can be written as

$$Kn_{eff} = \lambda_{eff} / h = \frac{Kn}{1 + 2Kn + \beta \cdot Kn^{0.788} \cdot e^{-Kn/10}} \tag{20}$$

Equation (19) displays a modified slip model we derived based on Shen's slip boundary condition and effective viscosity concept. The non-dimensional Poiseuille flow rates from various classical slip models are plotted as a function of the inverse Knudsen number in Fig. 3. We can see that the first order slip model underestimates the flow rate and the 1.5 order and second order slip models overestimate it under small D flow regimes. The flow rate predicted by the present model is in a good agreement with that from F-K model. When β value is set as 4, it will be best fit to data from F-K model over the entire flow regimes.

We also compare the present model with those based on the modification of mean free path, such as Shen's model [9], Peng's model [10] and Zhang's model [17]. In general, these models can be summarized as follows:

$$Q_p = \begin{cases} \dfrac{D}{6} + \dfrac{\sqrt{\pi}}{2} + \dfrac{\pi}{\pi + \sqrt{3}D}, & \text{Shen's model} \\ \dfrac{D}{6N_P} + \dfrac{\sqrt{\pi}}{2} + \dfrac{\pi N_P}{4D}, & \text{Peng's second order model} \\ \dfrac{D_{eff}}{6} + \dfrac{\sqrt{\pi}}{2}, & \text{Zhang's model} \end{cases} \tag{21}$$

where $N_P = \lambda_{eff} / \lambda$ is the nanoscale effect function and λ_{eff} is given in equation (6) for Peng's model.

Figure 4 shows the comparison of non-dimensional Poiseuille flow rates among various slip models based on the modification of mean free path. We can see that the present model with β=4 provides better prediction of flow rate than other models at arbitrary Knudsen number. When β value is 0.2, the flow rates predicted by the present model will be very close to those from Shen's model, even though the methods for calculating the effective mean free path are totally different. As for Zhang's model, it also used effective viscosity concept, but its results are not consistent with F-K model, especially in small *D* value. This is because this model doesn't consider the slip velocity caused by the pressure gradient along the flow direction in the Knudsen layer.

Another major advantage of our slip model is that it can be used for arbitrary surface accommodation coefficient. This is very important for an accurate simulation as the surface accommodation coefficient at the head-disk interface could be as low as 0.6 in some extreme conditions [18]. Fig. 5 compares the non-dimensional Poiseuille flow rate for various slip models when the surface accommodation coefficient is 0.8. Again, the present model with β=4 shows good agreement with F-K model over entire flow regimes. Compared with F-K model, the present model doesn't require huge computer resources to generate database for Poiseuille flow rate at arbitrary surface accommodation coefficient. Therefore, the efficiency of air bearing code can be improved greatly.

4 CHARACTERISTICS OF GAS LUBRICATION

The characteristics of gas lubrication can be numerically calculated by expressing Reynolds equation (equation (15)) as

$$\frac{\partial}{\partial X}\left(\tilde{Q}_p(D_0 PH)PH^3\frac{\partial P}{\partial X}-\Lambda PH\right)+\frac{\partial}{\partial Y}\left(\tilde{Q}_p PH^3\frac{\partial P}{\partial Y}\right)=0 \qquad (22)$$

where D_0 is a characteristic inverse Knudsen number, $D_0 = \sqrt{\pi} \cdot h_0 /(2\lambda_0)$, while λ_0 is ambient mean free path.

The load carrying capacity w is obtained by integrating the pressure distribution over the bearing length, which can be expressed as

$$w = \int_0^l (p - p_a) b \mathrm{d}x \tag{23}$$

$$W = w/(p_a l b) \tag{24}$$

where b is the width of the slider bearing, and W is the normalized load capacity.

Figure 6 compares normalized load capacity among various slip models for the case of $h_1/h_0 = 2$ and bearing number $\Lambda = 10$. This case has also been used by Shen *et al.* [9] and Peng *et al.* [10] to validate their models. It is found that the first order slip model overestimates the load while the 1.5 order and second order slip models underestimate the load when D_0 is less than 1.0. Compared with other slip models, the loads predicted by the current slip model agree well with those from F-K model over the entire flow regimes and β=4 gives the best fit to data from F-K model.

We also compare pressure distributions on the slider surface for various slip models, as shown in Fig. 7. Again, the current model with β=4 predicts a pressure profile very close to the F-K model. The first order slip model overestimates the pressure while the second order slip models underestimate it. It is also found that as β value increases, the pressure on the slider surface becomes larger. This is caused by more molecular collisions on the boundary at a bigger β value.

5 CONCLUSION

A modified slip model is derived based on Shen's kinetic-theory based slip boundary condition. A new formula is proposed for calculating effective mean free path using effective viscosity concept. In this formula, a parameter β, which is related to the molecular model based on kinetic theory, is used to curve fit the data from F-K model. Our simulation studies show that the current slip model with $\beta=4$ can predict flow rate, load and pressure profile very close to the F-K model in the whole Knudsen number range. In comparison with the existing slip models, such as the first, second and 1.5 order slip models, the current slip model is expected to be more accurate for ultra-thin gas film lubrication.

REFERENCES

1 **Zhou, W. D., Liu, B., Yu, S. K.,** and **Hua, W.** Rarefied-gas heat transfer in micro- and nanoscale Couette flow. *Phys. Rev. E*, 2010, **81**, 011204.

2 **Burgdorder, A.** The influence of the molecular mean free path on the performance of hydrodynamic gas lubrication bearings. *ASME J. Basic Eng.*, 1959, **81**, 94-100.

3 **Hsia, Y. T.** and **Domoto, G. A.** An experimental investigation of molecular rarefaction effects in gas lubricated bearings at ultra-low clearance. *Trans. ASME J. Tribol.*, 1983, **105**, 120-130.

4 **Mitsuya, Y.** Modified Reynolds equation for ultra-thin film gas lubrication using 1.5-order slip-flow model and considering surface accommodation coefficient. *Trans. ASME J. Tribol.*, 1993, **115**, 289-294.

5 **Wu, L.** and **Bogy, D. B.** New first and second order slip models for the compressible Reynolds equation. *Trans. ASME J. Tribol.*, 2003, **125**, 558-561.

6 **Fukui, S.** and **Kaneko, R.** Analysis of ultra-thin gas film lubrication based on linearized Boltzmann equation: first report-derivation of a generalized lubrication equation including thermal creep flow. *Trans. ASME J. Tribol.*, 1988, **110**: 335-341.

7 **Chen, D.** and **Bogy, D. B.** Comparison of slip-corrected Reynolds lubricant equations for the air bearing film in the head-disk interface of hard disk drives. *Tribol. Lett.*, 2010, **37**, 191-201.

8 **Fukui, S.** and **Kaneko, R.** A database for interpolation of Poiseuille flow rates for high Knudsen number lubrication problems. *Trans. ASME J. Tribol.*, **112**, 1990, 78-83.

9 **Shen, S.** and **Chen, G.** A kinetic-theory based first order slip boundary condition for gas flow. *Phys. Fluids*, **19**, 2007, 086101.

10 **Peng, Y., Lu, X.,** and **Luo, J.** Nanoscale effect on ultrathin gas film lubrication in hard disk drives. *Trans. ASME J. Tribol.*, **126**, 2004, 347-352.

11 **Cercignani, C.** and **Pagani, C. D.** Variational approach to boundary-value problems in kinetic theory. *Phys. Fluids*, **9**, 1966, 1167-1173.

12 **Loyalka, S. K., Petrellis, N.,** and **Storvick, T. S.** Some exact numerical results for the BGK model: Couette, Poiseuille and thermal creep flow between parallel plates. *J. Appl. Math. Phys.*, **30**, 1979, 514-521.

13 **Sone, Y., Takata, S.,** and **Ohwada, T.** Numerical analysis of the plane Couette flow of a rarefied gas on the basis of the linearized Boltzmann equation for hard-sphere molecules. *Eur. J. Mech. B*, **9**, 1990, 273-288.

14 **Veijola, T.** and **Turowski, M.** Compact damping models for laterally moving microstructures with gas rarefaction effects. *J. Microelectromech. Syst.*, **10**, 2001, 263-272.

15 **Chan, W. K.** and **Sun, Y.** Analytical modeling of ultra-thin-film bearings. *J. Micromech. Microeng.*, **13**, 2003, 463-473.

16 **Zhou, W. D., Liu, B., Yu, S. K., Hua, W.,** and **Wong, C. H.** A generalized heat transfer model for thin film bearings at head-disk interface. *Appl. Phys. Lett.*, **92**, 2008, 043109.

17 **Zhang, H., Zhu, C.,** and **Yang, Q.** Characteristics of micro gas journal bearings based on effective viscosity. *Trans. ASME J. Tribol.*, **131**, 2009, 041707.

18 **Zhou, W. D., Liu, B., Yu, S. K.,** and **Hua, W.** Numerical simulations of accommodation coefficient effects at the head-disk interface. *Japn. J. Appl. Phys.*, **49**, 2010, 095206.

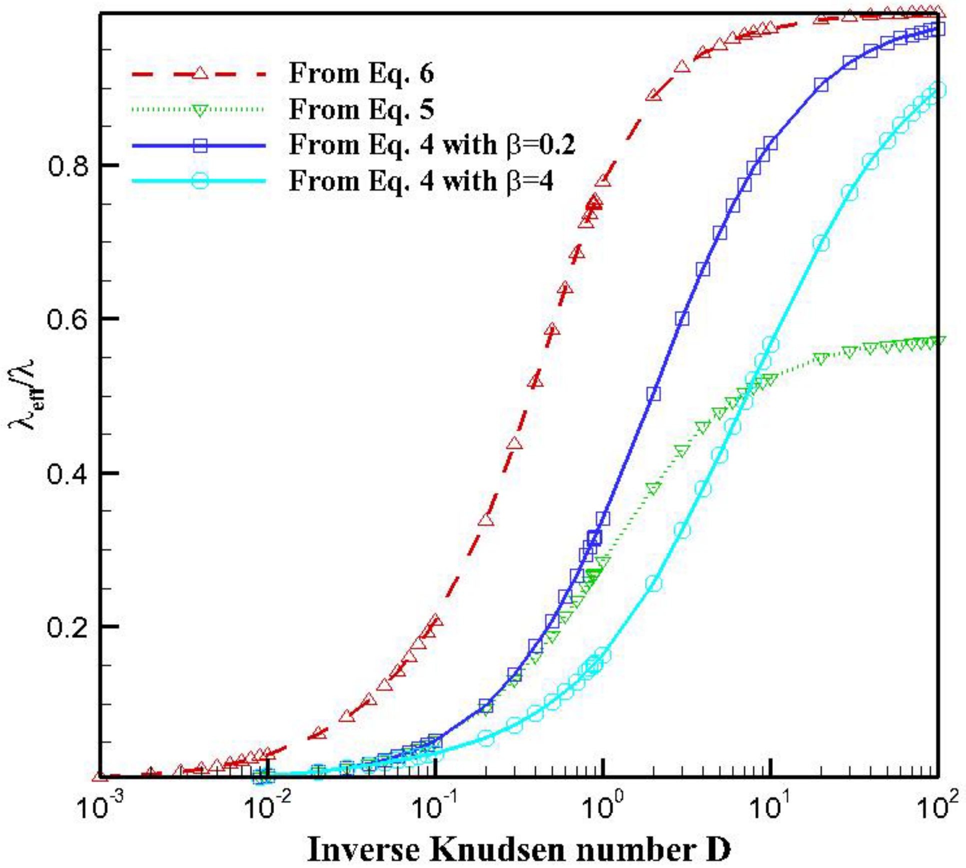

Fig. 1 The normalized effective mean free path λ_{eff} / λ as a function of inverse Knudsen number D

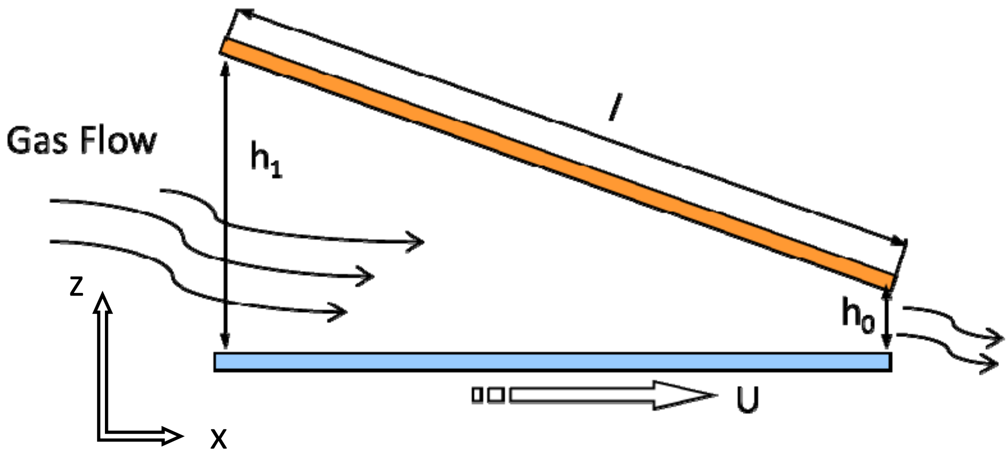

Fig. 2 The inclined plane slider bearing

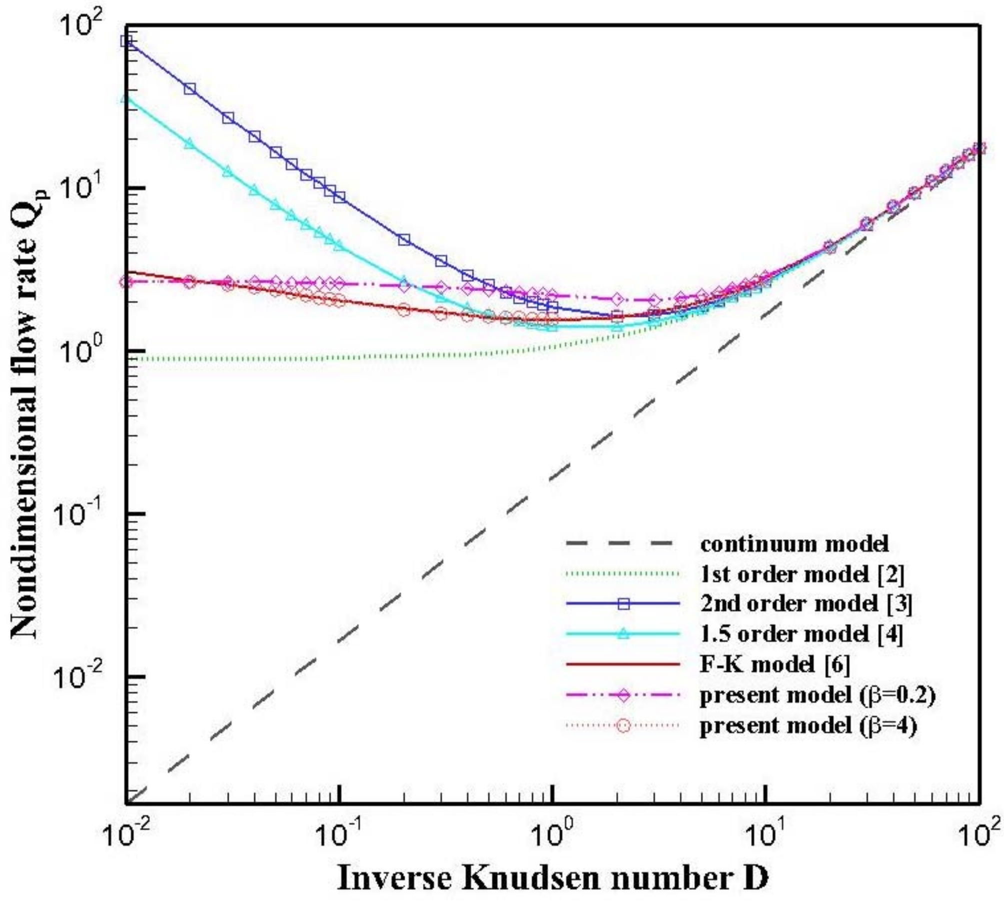

Fig. 3 The non-dimensional Poiseuille flow rates for various classical slip models

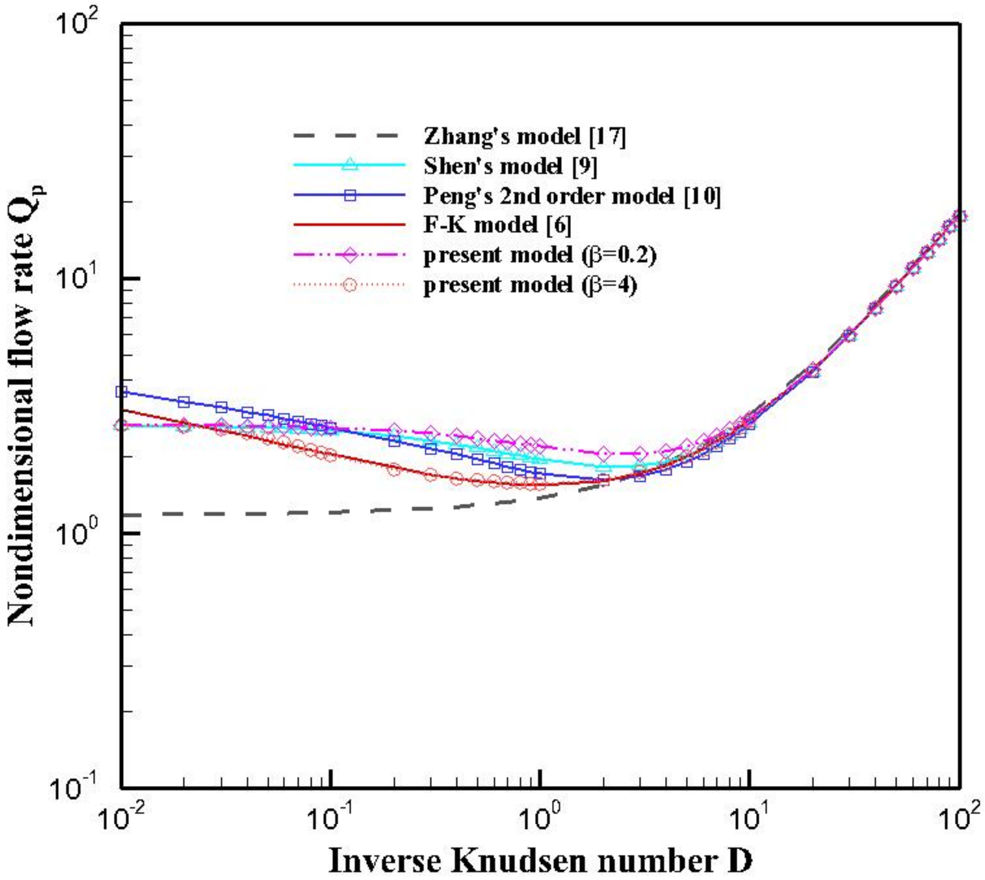

Fig. 4 Comparison of non-dimensional flow rate among various slip models based on the modification of mean free path

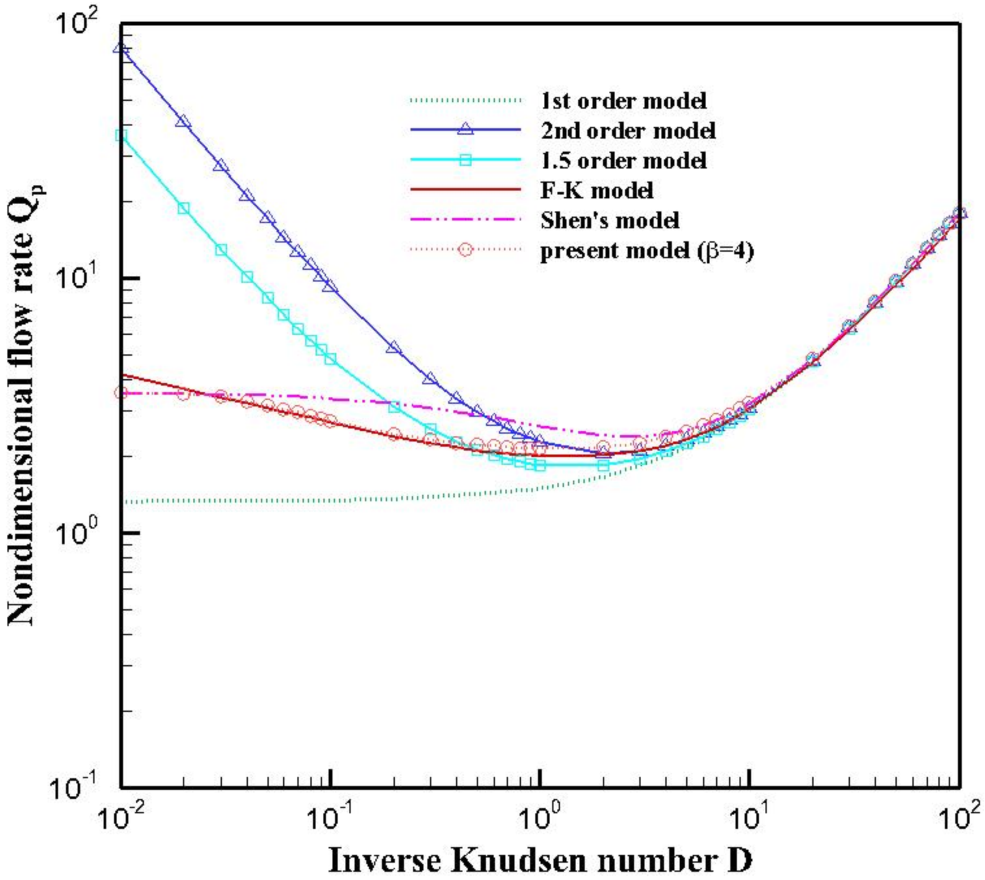

Fig. 5 The non-dimensional Poiseuille flow rates for various slip models when surface accommodation coefficient a is 0.8

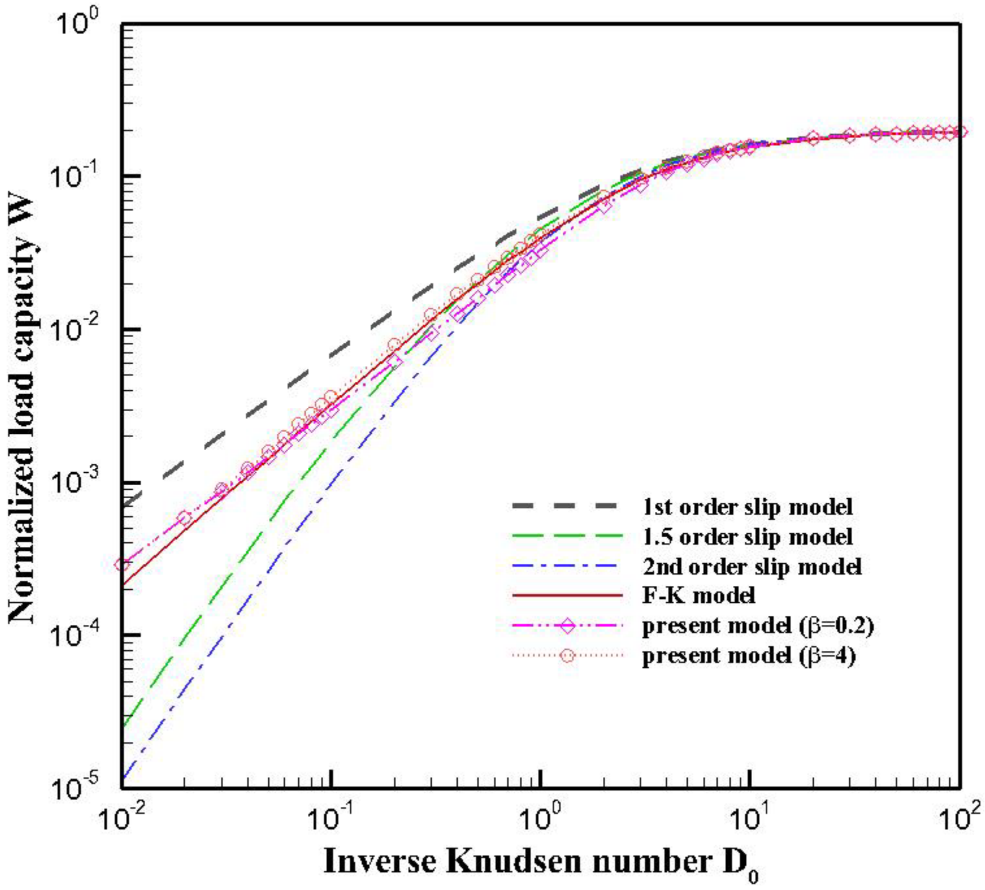

Fig. 6 Comparison of normalized load capacity among various slip models. The parameters used are: h_0 = 50 nm, h_1/h_0 = 2 and Λ = 10

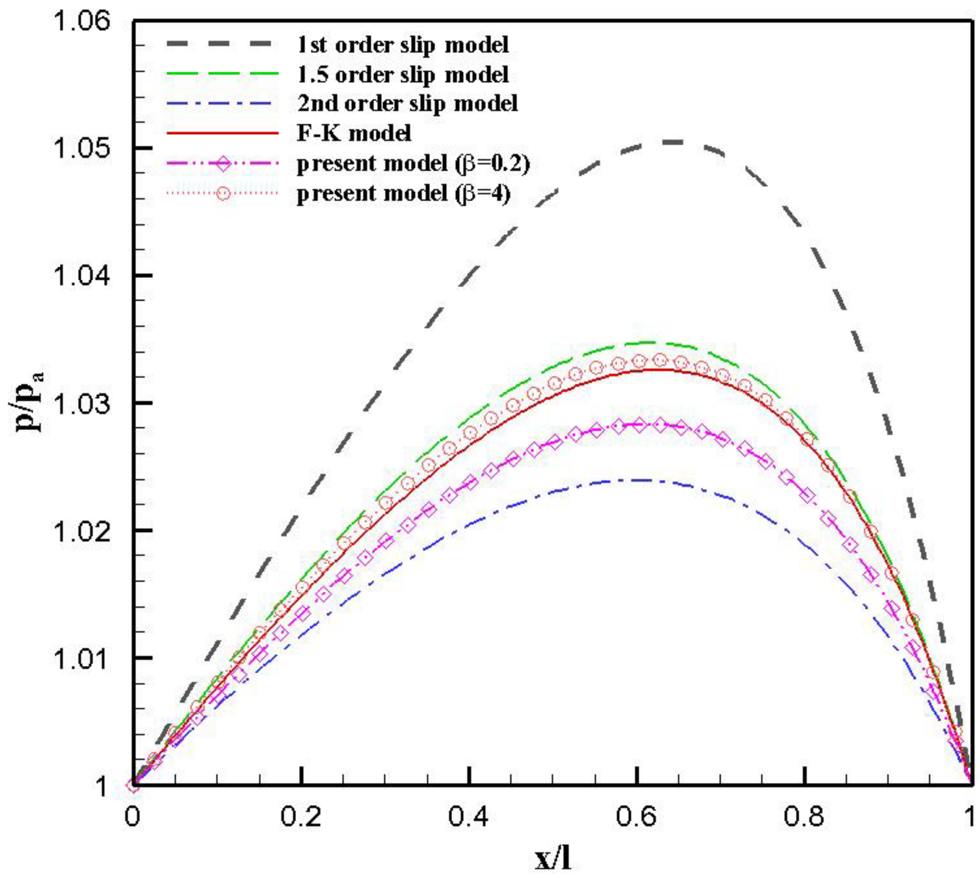

Fig. 7 Pressure distributions on the slider surface for various slip models. The parameters used are: h_0 = 50 nm, h_1/h_0 = 2, Λ = 10 and D_0=0.5