Terahertz Achromatic Quarter Wave Plate: Design, Fabrication, and Characterization

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Abstract

The terahertz achromatic quarter wave plate is realized for the first time in the frequency band of 1.3 ~ 1.8 THz via design, fabrication, and characterization. The wave plate is made from 9 pieces of birefringent crystalline quartz plates with different thickness, which are precisely stacked together with different rotation angles. The designed quarter wave plate shows phase retardance of 90°±3° with a stable optical axis. Experimental results show that the linearly polarized terahertz wave converts into circularly polarized when transmitting through the fabricated quarter wave plate placed at 45°. The quarter wave plate is also used to measure the known input state of polarization. The optimization method introduced in this paper is applicable to design the wave plates in the other frequency bands as well.

Keywords: Terahertz, polarization, wave plate, spectroscopy

Introduction

Terahertz (THz) frequency range, which locates between microwave and infrared electromagnetic waves, is defined as the electromagnetic wave with the frequency range of 0.1 to 10 THz (1 THz=10^{12} Hz). THz technique presents a new frontier of numerous technical applications, such as cancer early detection, homeland security, and quality assurance in food industry.[1, 2] As a result of the wide applications, much interest has been focused on controlling and manipulating the THz beams: divergence, spatial or frequency modes, and profiles. [3-5] Especially, polarization state characterization in terahertz system can yield more information of the objects under measurement. In our previous studies, the measurement of Stokes parameters [6], Muller matrix [7], and polarization dependent loss (PDL) [8] etc have been realized in THz band. A THz polarizer with high extinction ratio has also been fabricated.[10] THz wave plate, which is one of the most important polarization devices, can be used in THz imaging and detection.[9, 10] Especially, an achromatic THz wave plate, which can work over a broad terahertz frequency range, is largely demanded [11, 12]. In Ref. 11, an ultra-broad terahertz wave plate from 0.2 to 1.8 THz has been successfully demonstrated. But only the phase delay was taken into consideration. In wave plate design, especially for practical application, besides phase delay, the other two parameters, output linearity when the incident angle is 0° and stable optical axis over a broadband, are also the key factors to ensure the good performance of a wave plate.

The standard wave plate made from birefringent materials can only be used at a single frequency, as the retardance depends strongly on the frequency. Achromatic wave plate has been well developed in the visible and infrared frequency regimes [13], while there are few reports in THz regime. Another key challenge in THz achromatic wave plate is that the naturally occurring materials showing birefringence is not widely available.

In this study, an achromatic wave plate, which works in the frequency range of 1.3 ~ 1.8 THz, is successfully designed, fabricated, and characterized. The achromatic wave plate is made of 9 pieces of crystalline quartz plates with different thickness and different rotation angles using the partial cancellation of the change of retardance with respect to the frequency by combining multiple pieces of birefringent materials. [14-16] The fabricated quarter wave plate exhibits good agreement with the design, which successfully convert linearly polarized THz wave into circularly polarized when the incident polarization is 45 º. The research focus is the design and realization of quarter wave plate, while method demonstrated in this paper can be generalized to half wave plate or the wave plate with arbitrary retardance.

Design

The combination of quartz plates is illustrated in Fig. 1 (a). The quarter wave plate consists of 9 pieces of crystalline quartz plates. Crystalline quartz is a suitable material to make terahertz achromatic wave plate with the absorption below 0.05 cm^{-1} and an absorption anisotropy α_o-α_e below 0.02 cm^{-1} at 1 THz, which has a negligible influence on retardation. [17] Meanwhile, it also presents strong birefringence with the difference of ordinary and extraordinary refractive indices of 0.046 in the lower THz band. The refractive indices n_o and n_e at particular frequency f in the range of 1.0 to 2.0 THz as follows:

$$\begin{aligned} n_o &= 2.1068 + 10^{-3} f + 3\times 10^{-3} f^2 \\ n_e &= 2.1576 + 5\times 10^{-3} f + 4\times 10^{-3} f^2 \end{aligned}, \quad (1)$$

which is retrieved from Ref. 17 and f is the frequency in THz.

To achieve a high-performance quarter wave plate, its phase delay must be around 90 ° with nearly consistent optical axis and linear output polarization state over a broad frequency range, so that there are 3 parameters to be taken into consideration, (1) phase delay, (2) optical axis, and (3) output linearity when incident polarization is 0 ° angle (In the following explanation, 'output linearity' will be used for short.), which are illustrated as follows.

Each quartz plate can be described by its corresponding Jones matrix J_i (i=1~9):

$$J_i(\delta_i,\theta_i)=\begin{bmatrix} \cos\frac{\delta_i}{2}+i\cos 2\theta_i\sin\frac{\delta_i}{2} & i\sin 2\theta_i\sin\frac{\delta_i}{2} \\ i\sin 2\theta_i\sin\frac{\delta_i}{2} & \cos\frac{d_i}{2}-i\cos 2\theta_i\sin\frac{\delta_i}{2} \end{bmatrix}, \quad (2)$$

which depends on two parameters: the dephasing $\delta_i=\frac{1}{f}\frac{d_i(n_e-n_o)}{c}$ and the rotation angle θ_i of each piece of quartz plate. The dephasing δ_i is wavelength dependent. Therefore, d_i and θ_i are the key parameters in the design.

When 9 pieces of crystalline quartz are stacked together to form the quarter wave plate, it can be represented by the following equation:

$$J=\prod_{i=1}^{9}J_i(\delta_i,\theta_i)=\begin{pmatrix} A & B \\ -B^* & A^* \end{pmatrix}. \quad (3)$$

As discussed before, there are 3 parameters to be taken into consideration, phase delay, optical axis, and output linearity. In terms of Jones matrix, these 3 parameters can be expressed as follows.

The eigenvalues D and the eigenvectors V of matrix J can be expressed in the matrix format:

$$D = \begin{pmatrix} D_{1,1} & 0 \\ 0 & D_{2,2} \end{pmatrix}, \tag{4}$$

$$V = \begin{pmatrix} V_{1,1} & V_{1,2} \\ V_{2,1} & V_{2,2} \end{pmatrix}, \tag{5}$$

where $D_{1,1}$ and $D_{1,2}$ are the eigenvalues of matrix *J*.

The phase difference (PD) between the two eigenvalues is the phase delay of the terahertz achromatic wave plate consisting of several crystalline quartz plates:

$$PD = AngleD_{1,1} - AngleD_{2,2}\ , \tag{6}$$

where $AngleD_{1,1}$ and $AngleD_{2,2}$ are the phase of elements $D_{1,1}$ and $D_{2,2}$ in matrix D.

The optical axis *AX* and output linearity *LI* are related to the eigenvectors *V*, which are given below. The output linearity LI means the output should be linearly polarized when the angle between the optical axis of the quarter wave plate and the incident polarization is 0 °:

$$AX = \tan\left(\frac{V_{2,1}}{V_{1,1}}\right), \tag{7}$$

$$LI = AngleV_{1,1} - AngleV_{2,1}\ , \tag{8}$$

where $AngleV_{1,1}$ and $AngleV_{2,1}$ represent the phase of the elements $V_{1,1}$ and $V_{2,1}$ in the matrix *V*.

The phase difference *PD*, the optical axis *AX* and linearity *LI* are retrieved from the above eigenvalues *D* and the eigenvectors *V* of matrix J from Eq. (3), so that all these 3 variables are wavelength dependent. To achieve an achromatic terahertz wave plate, the loss function *F* consisting of phase delay *PD*, optical axis *AX*, and output linearity

LI with specific weights w_{PD}, w_{AX}, and w_{LI} respectively. The value of loss function will be reduced by optimization, so that the values of 3 variables are also reduced.

$$F = \sum_j \left(w_{PD}(PD_j - 90) + w_{AX}(AX_j - AX_1) + w_{LI}(LI_j - 0) \right). \tag{9}$$

The weights are used for optimization as the loss function F consists of 3 parameters which cannot be optimized at the same time. Usually the optimization takes a few rounds to make sure all the 3 parameters are within the required ranges, so that the weights are tuned during the optimization process. To increase the weights can squeeze the corresponding parameters into the preset ranges.

Simulated annealing

9 pieces of crystalline quartz plates are used in the design to achieve the achromatic terahertz wave plate. Each quartz plate has two parameters, the thickness d_i and the rotation angle θ_i. Therefore, there are 18 variables involved in the design process. In order to deal with many variables, simulated annealing is used for optimization, which was developed by S. Kirkpatrick, et al [18]. Simulated annealing is a generic probabilistic algorithm method of finding a good approximation of a system to the global optimal results of a given function in a large search space with many variables. The given function, which is also called as 'cost function', represents a quantitative measurement of the goodness of the complex system, which depends on the detailed configuration of the many parts of the system. In this study, cost function is equation (9). Our target is to find the suitable composition of the several crystalline quartz plates to minimize the value of cost function which meets the requirements as shown below.

$$\begin{array}{l} \text{Phase delay}: \ 90° \pm 3° \\ \text{Variation of axis}: \ -3° \sim 3° \ . \\ \text{Output linearity}: \ -3° \sim 3° \end{array} \tag{10}$$

Simulated annealing is a type of local search algorithm instead of global search starts with an initial solution chosen at random. A neighbor of this initial solution is then generated and the change in cost function is calculated. If a reduction in cost function is found, the initial solution is replaced by the generated next neighboring solution; otherwise the initial solution is retained. This process is repeated until the cost function corresponding to the generated solution is within the predefined criteria.

Results and discussion

Via simulated annealing method, the parameters of the 9 pieces of quartz plates have been optimized to minimize the cost function shown in equation (9) over the frequency range of 1.3 ~ 1.8 THz. As expected, the bandwidth is extended with the number of quartz plates. The thickness and angle of each piece are summarized in Table 1.

The phase retardance shows significant stability over the targeted frequency range of 1.3 ~ 1.8 THz, which is around 90° with a variation of ±3°, as shown in Fig. 2 (a). In the ideal case, the optical axis should keep unchanged and the output state should keep linearly polarized when the incident THz wave polarization is in the same direction of the optical axis. However, in practice, the optical axis will change a little bit and the output state is changed into elliptically polarized. The purpose of optimization process is to minimize the change. Especially, the output linearity optimization is aimed to reduce the amount of the short axis in the elliptical polarization state when the incident polarization is 0°. Figure 2 (b) and (c) show the optical axis and output linearity of the terahertz wave plate. It can be observed that the optical axis is fixed around 38° with a variation of 2° and the output state keep almost linearly polarized.

The achromatic wave plate is fabricated according to the simulation results. The thickness of each quartz plate is with high precision with a roughness smaller than 10 μm. The total thickness of the quarter wave plate is around 5 cm. The achromatic terahertz wave plate is shown in Fig. 1 (b). The quartz plates are stacked up tightly in an optical tube so as to minimize the transmission loss induced by Fabry-Perot reflection among the plates. The throughput is measured to be ~50%.

The characterization of the achromatic quarter wave plate is carried out by a terahertz frequency domain spectroscopy system. The emitter is a coherent tunable THz wave from the efficient parameteric scattering in nonlinear crystal $MgO{:}LiNbO_3$ pumped by a Q-switch YAG laser. The pulse width, repetition rate, and maximum output energy are 27 ns, 200 Hz, and ~25 mJ/pulse, respectively. The detector is a high sensitivity bolometer. The detailed experimental setup can be found in Ref. 19. The intensity of the transmitted terahertz wave is measured at a step of 0.05 THz over the frequency range of 1.3 ~ 1.8 THz. The experimental setup is illustrated in Fig. 3 (a). The achromatic wave plate is placed between two rotatable polarizers. The polarizers are the key devices in the measurement which will be explained as follows. The state of polarization is supposed to be changed after passing through the wave plate and polarizers.

The achromatic wave plate is placed with a rotation angle of 45°, keeping the tube axis parallel to the terahertz wave. The terahertz wave is linearly polarized in the vertical direction when it passes through the first polarizer P_1 which is set as 0 °. Then the polarization changes into circularly polarized when it transmits through the achromatic wave plate as the retardance of the terahertz wave passing through the quarter-wave plate is 90 °. The second polarizer P_2 is rotated during the characterization and it changes the circular polarization into linear one with the same amplitude in any rotation angle. Since the detector measures the intensity of the transmitted terahertz wave, the value should keep as a constant when the second polarizer is rotated into any angle. Figure 3 (b) presents the intensities measured in different rotation angles at different frequencies. The intensity of the terahertz wave at single frequency keeps similar over a broad frequency band from 1.3 to 1.8 THz.

Quarter wave plate can be used to characterize the state of polarization (SOP) of an incident THz wave. [20] The SOP is the normalized three dimensional Stokes vector. In order to further prove the performance of the designed achromatic wave plate, the measurement of a known SOP in the incident THz wave is carried out. Two polarizers are used in this experiment, one before the wave plate and another one after the wave plate. The first polarizer P_1 is rotated to 45 ° to generate the known SOP of $(0, 1, 0)^T$. As P_1 is used to generate the THz wave with certain SOP, the THz wave after P_1 is defined as input. The second polarizer P_2 is fixed at an angle of 0 °. At a particular frequency, the quarter wave plate is rotated against the tube axis to four angles θ_i, i=1, 2, 3, 4 and the corresponding intensity is measured. The relation between the intensity and the Stokes vectors is shown below. [20]

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & \cos^2\theta_1 & \cos 2\theta_1 \sin 2\theta_1 & -\sin 2\theta_1 \\ 1 & \cos^2\theta_2 & \cos 2\theta_2 \sin 2\theta_2 & -\sin 2\theta_2 \\ 1 & \cos^2\theta_3 & \cos 2\theta_3 \sin 2\theta_3 & -\sin 2\theta_3 \\ 1 & \cos^2\theta_4 & \cos 2\theta_4 \sin 2\theta_4 & -\sin 2\theta_4 \end{pmatrix} \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix}. \quad (11)$$

In this study, the angle θ_i is chosen as (-45°, 30°, 0°, 60°). Therefore, eq. (13) is converted into

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0.25 & 0.433 & -0.866 \\ 1 & 0.25 & -0.433 & -0.866 \end{pmatrix} \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix}. \tag{12}$$

The characterization result is shown in Fig. 4. The Stokes parameters of the input terahertz wave is $(0, 1, 0)^T$, which meets the expectation very well.

Conclusions

In conclusions, an achromatic quarter terahertz wave plate in the frequency range of 1.3 ~ 1.8 THz has been successfully designed, fabricated and characterized. Simulated annealing is used in the simulation process to optimize three critical parameters, phase retardance, optical axis, and output linearity when the incident polarization is 0°. The wave plate, which is fabricated by the combination of 9 pieces of crystalline quartz plates, shows superior performance in terms of constant phase retardance, stable optical axis, and nearly linear polarization (when the incident polarization is 0°). The method of wave plate design developed in this paper can be extended to the other terahertz frequency band as well. This high-performance achromatic terahertz wave plate can be applied in terahertz sensing, detection, and imaging applications.

Acknowledgements

This work is supported by the international joint project between A-STAR of Singapore and JST of Japan with A-STAR/SERC/SICP grant No. 1021630069.

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Figure captions

Fig.1. (a) Illustration of the achromatic wave plate made from crystalline quartz plates, (b) image of the achromatic wave plate.

Fig. 2. Simulation results by simulated annealing of (a) phase delay, (b) optical axis, and (c) output linearity.

Fig. 3. (a) Characterization setup to test the achromatic wave plate and (b) characterization result of circularly polarized output THz wave when the incident polarization is 45°.

Fig. 4. The characterization results of the known state of polarization (SOP).

Fig. 1

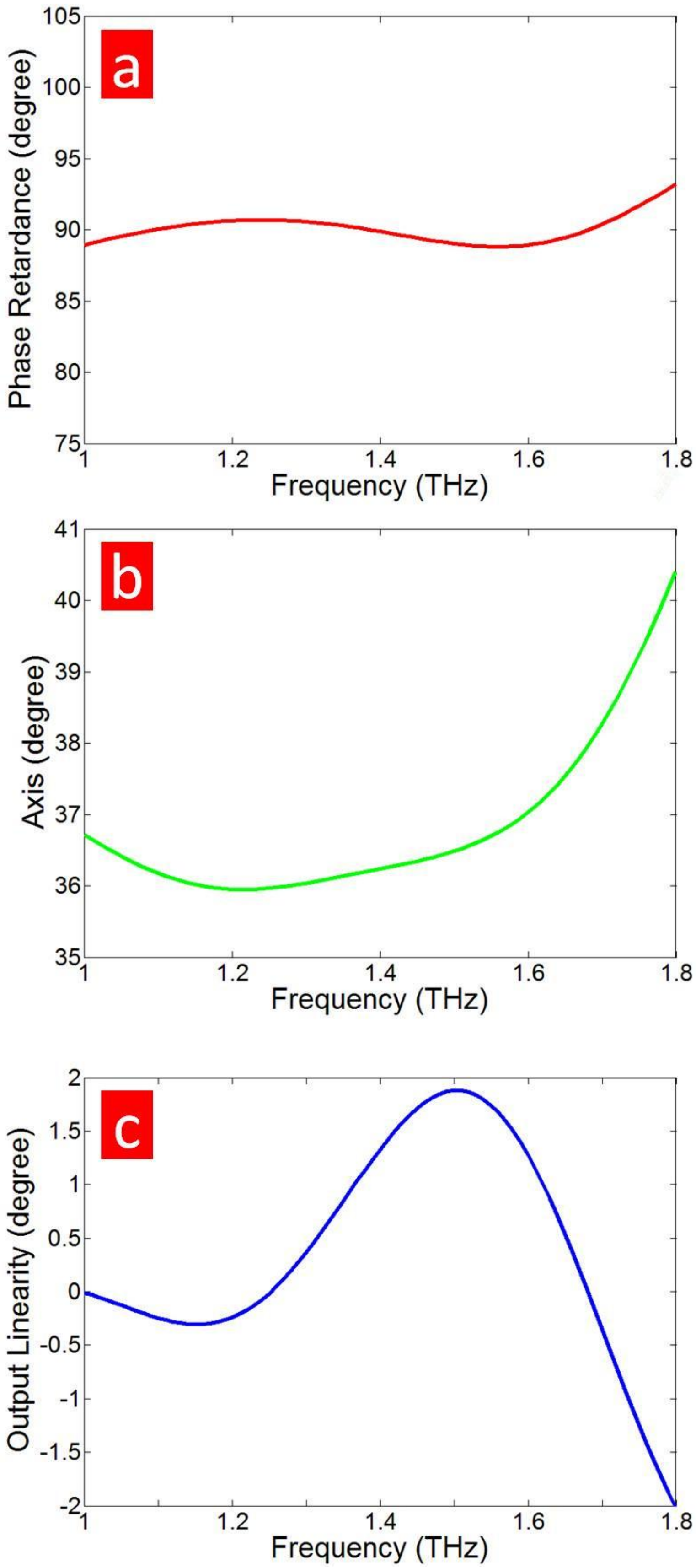
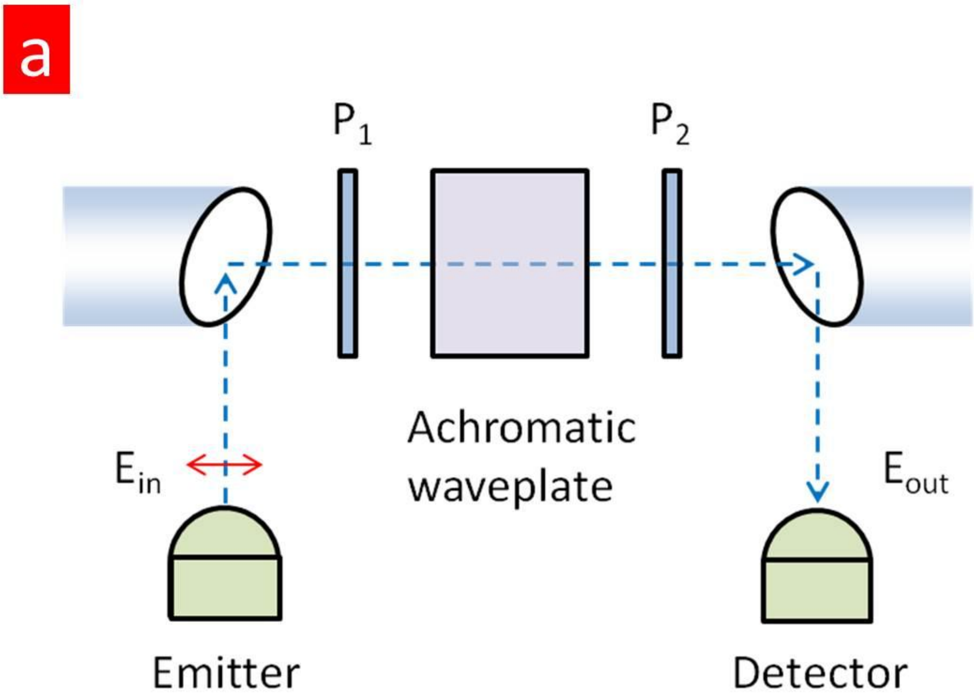

a
Phase Retardance (degree)
105
100
95
90
85
80
75
1
1.2
1.4
1.6
1.8
Frequency (THz)
b
Axis (degree)
41
40
39
38
37
36
35
1
1.2
1.4
1.6
1.8
Frequency (THz)
c
Output Linearity (degree)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
1
1.2
1.4
1.6
1.8
Frequency (THz)

Fig. 2


a
P1
P2
Achromatic
waveplate
Ein
Eout
Emitter
Detector

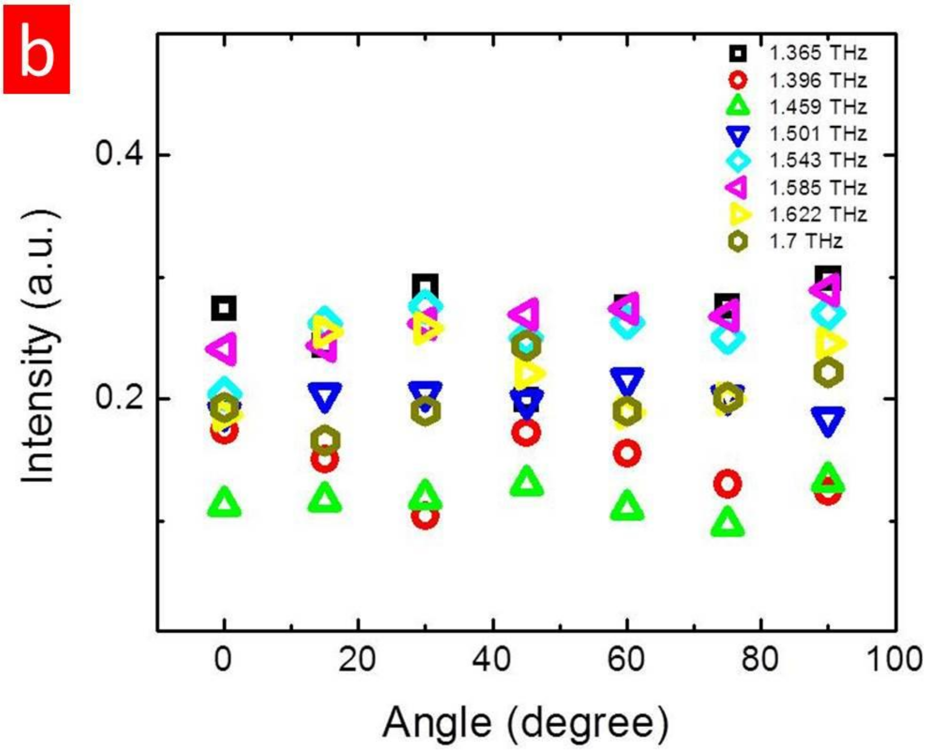

b
1.365 THz
1.396 THz
1.459 THz
1.501 THz
1.543 THz
1.585 THz
1.622 THz
1.7 THz
0.4
0.2
Intensity (a.u.)
0
20
40
60
80
100
Angle (degree)

Fig. 3

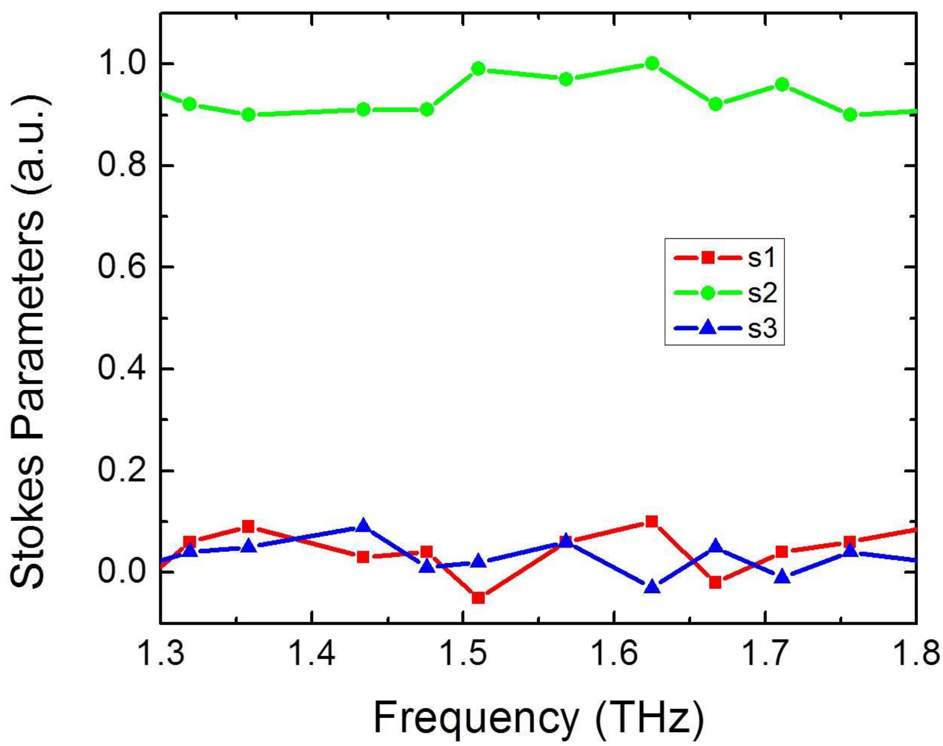

Stokes Parameters (a.u.)
1.0
0.8
0.6
0.4
0.2
0.0
1.3
1.4
1.5
1.6
1.7
1.8
Frequency (THz)
s1
s2
s3

Fig. 4

Table 1. Simulation results of the thicknesses and angles of quartz plates

Quartz plates	1	2	3	4	5	6	7	8	9	Total thickness
Thickness (mm)	7.38	3.17	9.91	1.64	3.32	2.38	8.49	11.86	3.57	51.72
Rotation angle (°)	16.0	15.0	104.0	68.0	358.0	79.0	0.0	90.0	5.0	NA