Energy Efficient Multiuser MIMO Systems with Distributed Transmitters

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***Abstract*—In this paper, we consider a distributed transmitter (D-TX) system, in which each TX has a dissimilar power amplifier with different maximum output power, and different number of transmit antennas. To improve energy efficiency (EE) of the D-TX system, we design a multiuser multiple-input multiple-output (MU-MIMO) precoding matrix, a transmit antenna selection (AS) matrix, and a power control (PC) matrix. A conventional zero-forcing based MU-MIMO precoding is shown to be EE optimal for given AS and PC. Optimal and heuristic PC methods are proposed for given AS and MU-MIMO precoding. For the AS, we also propose heuristic algorithms. Average transmit power, outage probability, and EE performance are evaluated to compare three AS algorithms, and to observe the performance gap between the optimal and heuristic PC methods. From the numerical results, we discuss a tradeoff between AS complexity and EE performance and provide a useful guide for energy efficient D-TX system design.**

***Index Terms*—Energy efficiency, multiuser MIMO, distributed transmitters, antenna selection, power control.**

I. Introduction

High spectral efficiency (SE) has been widely studied and it becomes more tangible with distributed systems that use, for example, coordination and/or cooperation among the distributed antennas and transmitters. On the other hand, energy efficiency (EE) has been recently emphasized in various wireless communication systems [1]–[10]. Hence, it is natural and timely to consider EE in the distributed systems, for example, a distributed transmitter (D-TX), each TX of which has dissimilar power amplifiers (PAs) with different maximum output power, and different number of transmit antennas. In [11], multiuser multiple-input multiple-output (MU-MIMO) precoding, antenna selection (AS), and power control (PC) are considered for energy efficient D-TX. To solve an EE maximization problem including coupled *nonlinear* constraints on *instantaneous* transmit power of each PA, a *heuristic* PC method has been proposed.

In this paper, we extend the design framework in [6] to be more systematic. To design MU-MIMO precoding, transmit AS, and PC matrices, we formulate an EE maximization problem. The original problem includes a non-convex objective function and integer optimization variables, and thus it is difficult to be solved directly. We decompose the original EE maximization problem into three subproblems, namely, i) an MU-MIMO precoding design subproblem, ii) an AS subproblem, and iii) a PC subproblem. In each subproblem, we focus on EE maximization and consider a per-antenna *average* transmit power constraint and a per-user data rate constraint. An outage is defined for the case when there is no feasible solution, which means at least one user equipment (UE) can not be supported. We show *analytically* that the conventional zero-forcing (ZF) MU-MIMO precoding is optimal with respect to EE for the given AS and PC. Furthermore, we propose three effective *AS algorithms* based on greedy-search, and *optimal PC method* for given antenna set and MU-MIMO precoding. The optimal PC and heuristic PC with three proposed AS algorithms are compared in computer simulation. From the results, we provide insight of a tradeoff between AS complexity and EE performance of the D-TX systems.

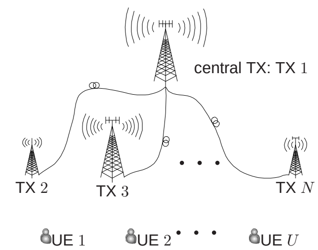


Fig. 1. Distributed transmitter (D-TX) system with N TXs and U UEs.

Notation: For any vector or matrix, the superscript $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^+$ denote transposition, Hermitian transposition, and pseudo-inverse, respectively; $\mathrm{tr}(\cdot)$ represents the trace of matrix; E stands for expectation of a random variable; for any scalar, column vector, and matrix, the notations $|\cdot|$, $\|\cdot\|_2$, and $\|\cdot\|_F$ denote the absolute value, 2-norm, and Frobenius-norm, respectively; $\boldsymbol{I}_a$ is an a-dimensional identity matrix; and the subscript $(\cdot)_{mn}$ or $[\cdot]_{mn}$ represents the (m,n)th element of a matrix. Throughout this paper, we use the superscript $(\cdot)'$ and $(\cdot)^*$ for a given constant value and an optimized variable, respectively.

II. System Model

We consider a D-TX system with N TXs and U UEs as illustrated in Fig. 1. Each UE has a single receive antenna, while TX n has M_n co-located transmit antennas. One central TX (e.g., a baseband unit and a signal processing center) performs any required central processing for communications, such as power control, resource allocation, and scheduling. $(N-1)$ extended transmitters are connected to the central TX through a noise-free wired backhaul (e.g., optical fiber) for coordinated and cooperative MU-MIMO communications. Specifically, the central TX performs i) computation of an MU-MIMO precoding matrix according to the estimated channels;

ii) selection of a transmit antenna set to be used for the MU-MIMO preprocessing; and iii) allocation of transmit power for each user's data stream to satisfy its the target rate, and at the same time to fulfill the maximum average transmit power of each selected transmit antenna. In this work, we assume that $M = \sum_{n=1}^{N} M_n$ antennas are available for the ZF-MU-MIMO preprocessing[1]. In other words, any channel matrix of the selected antennas from $\mathcal{M} = \{1, \ldots, M\}$ is assumed to be full rank. At least rank U MIMO channel is required to support U UEs with a ZF-MU-MIMO. Thus, with enough large number of TXs or their antennas, we assume $M \geq U$ to perfectly eliminate inter user interferences (IUIs) throughout the paper.

III. EE Maximization Problem Formulation

Denoting a received signal at UE $u \in \mathcal{U} = \{1, \ldots, U\}$ by y_u, its vector form $\boldsymbol{y} = [y_1 \cdots y_U]^T$ is written as

$$\boldsymbol{y} = \boldsymbol{HSW}\sqrt{\boldsymbol{P}}\boldsymbol{x} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{H}$ is a U-by-M MU-MIMO channel matrix; $\boldsymbol{S}$ is an M-dimensional transmit AS matrix which is a binary diagonal matrix, s.t. $s_{mm'} = 0$ if $m \neq m' \in \mathcal{M}$; $s_{mm} = 1$ if the mth antenna is selected, and $s_{mm} = 0$ otherwise; $\boldsymbol{W}$ is an M-by-U ZF-MU-MIMO preprocessing matrix; $\boldsymbol{P}$ is a U-dimensional diagonal matrix whose uth diagonal element p_{uu} determines a power portion assigned to UE u; $\boldsymbol{x} = [x_1 \cdots x_U]^T$ is a transmit signal vector where x_u is a transmit symbol to UE u with $\mathrm{E}\,|x_u|^2 = 1$; and $\boldsymbol{n} = [n_1 \cdots n_U]^T$ is an additive white Gaussian noise (AWGN) vector whose uth element n_u is an AWGN at UE u, and $n_u \sim \mathcal{CN}(0, \sigma^2)$. The (u, m)th element of $\boldsymbol{H}$ represents a channel gain $\sqrt{A_{um}}h_{um}$ consisting of the path loss $\sqrt{A_{um}}$ and the small scale fading h_{um} between a transmit antenna $m \in \mathcal{M}$ and UE u. The channels $\{h_{um}\}$ are assumed to be independent and identically distributed (i.i.d.). The mth transmit antenna is located at $\mathsf{TX}_{\pi(m)}$ where $\pi(m)$ is an one-to-one mapping function from antenna index m to TX index n, i.e., $\pi(m) = n$. To eliminate the IUIs perfectly, an M-by-U precoding matrix $\boldsymbol{W}$ satisfies a ZF property as

$$\boldsymbol{HSW} = \boldsymbol{I}_U;$$

therefore, ZF-MU-MIMO preprocessing yields the received SNR of stream u in (1) as $\mathsf{SNR}_u = p_{uu}/\sigma^2$.

With sufficient input backoff, we assume that a PA input signal is linearly amplified and the PA output signal has a Gaussian distribution [10], and we can further assume that UE u can correctly decode $\log_2(1 + \mathsf{SNR}_u)$-bit information per unit frequency and time ($\mathrm{bits/sec/Hz}$). Accordingly, we write the achievable throughput of UE u over bandwidth Ω Hz as $\Omega \log_2\left(1 + p_{uu}/\sigma^2\right),\ \forall u \in \mathcal{U}$, and obtain the system throughput per unit time ($\mathrm{bits/sec}$) as follows:

$$\mathsf{R}(\boldsymbol{P}) = \Omega \sum_{u \in \mathcal{U}} \log_2\left(1 + p_{uu}/\sigma^2\right). \tag{2}$$

[1]Since a ZF-MU-MIMO precoding is near optimal with respect to the SE if the signal-to-noise ratio (SNR) is high enough [12], [13], it is employed to the D-TX systems [14], [15].

From the power consumption model of MIMO system (refer to [16]), we now model the power consumption (watt) corresponding to $\mathsf{R}(\boldsymbol{P})$ as follows:

$$\mathsf{C}(\boldsymbol{P}, \boldsymbol{S}, \boldsymbol{W}) = f(\boldsymbol{W}, \boldsymbol{S}, \boldsymbol{P}) + g(\boldsymbol{S}), \tag{3}$$

where $f(\cdot)$ and $g(\cdot)$ are transmit power-dependent (TPD) and independent (TPI) terms, respectively. The average transmit power of transmit antenna m of $\mathsf{TX}_{\pi(m)}$ is derived as

$$\begin{aligned}\mathrm{E}\left|s_{mm}\boldsymbol{w}_m^T\sqrt{\boldsymbol{P}}\boldsymbol{x}\right|^2 &= \mathrm{E}\left(s_{mm}\boldsymbol{w}_m^T\sqrt{\boldsymbol{P}}\boldsymbol{x}\boldsymbol{x}^H\sqrt{\boldsymbol{P}}\boldsymbol{w}_m^{TH}s_{mm}\right)\\ &= s_{mm}\boldsymbol{w}_m^T\boldsymbol{P}\boldsymbol{w}_m^{TH}s_{mm}\\ &= \left[\boldsymbol{SWPW}^H\boldsymbol{S}\right]_{mm};\end{aligned}$$

where $\boldsymbol{w}_m$ is a precoding vector for the mth antenna and $\boldsymbol{W}^T = [\boldsymbol{w}_1 \cdots \boldsymbol{w}_M]$; thus, the TPD power consumption is defined as $f(\boldsymbol{W}, \boldsymbol{S}, \boldsymbol{P}) = c\sum_{m \in \mathcal{M}} \eta_m^{-1}[\boldsymbol{SWPW}^H\boldsymbol{S}]_{mm}$, where c is a system dependent power loss coefficient ($c > 1$) which can be empirically measured, and η_m is the efficiency of PA at the mth antenna ($0 < \eta_m < 1$). The TPI power consumption is modeled as $g(\boldsymbol{S}) = \sum_{m \in \mathcal{M}} P_{\mathrm{cc},\pi(m)}[\boldsymbol{S}]_{mm} + \mathrm{tr}(\boldsymbol{S})\Omega P_{\mathrm{sp1}} + \Omega P_{\mathrm{sp2}} + P_{\mathrm{fix}}$; $P_{\mathrm{cc},\pi(m)}$ is radio frequency (RF) circuit power consumption which is proportional to the number of RF chains and depends on the type of transmitter $\pi(m)$; P_{sp1} and P_{sp2} are signal processing related power consumption per unit frequency, where P_{sp1} is proportional to the number of active antennas, i.e., $\mathrm{tr}(\boldsymbol{S})$, while P_{sp2} is independent of $\mathrm{tr}(\boldsymbol{S})$; and P_{fix} is a fixed power consumption which is independent of $\mathrm{tr}(\boldsymbol{S})$ and Ω, for example, a part of power consumption at power supply and cooling systems.

A. Optimization Problem

From (2) and (3), we express a system EE ($\mathrm{bits/joule}$) as a function of $\boldsymbol{P}$, $\boldsymbol{S}$, and $\boldsymbol{W}$ as

$$\mathsf{EE}(\boldsymbol{W}, \boldsymbol{S}, \boldsymbol{P}) \triangleq \mathsf{R}(\boldsymbol{P})/\mathsf{C}(\boldsymbol{W}, \boldsymbol{S}, \boldsymbol{P}), \tag{4}$$

and formulate an EE maximization problem as follows:

$$\begin{aligned}\max_{\{\boldsymbol{W},\boldsymbol{S},\boldsymbol{P}\}}\quad & \mathsf{EE}(\boldsymbol{W}, \boldsymbol{S}, \boldsymbol{P}) && \text{(5a)}\\ \text{s.t.}\quad & \boldsymbol{HSW} = \boldsymbol{I}_U, && \text{(5b)}\\ & \left[\boldsymbol{SWPW}^H\boldsymbol{S}\right]_{mm} \leq P_m,\ \forall m \in \mathcal{M}, && \text{(5c)}\\ & \Omega\log(1 + p_{uu}\sigma^{-2}) \geq R_u,\ \forall u \in \mathcal{U}, && \text{(5d)}\\ & p_{u_1u_2} = 0, \forall u_1 \neq u_2 \in \mathcal{U}, && \text{(5e)}\\ & s_{m_1m_2} = 0, \forall m_1 \neq m_2 \in \mathcal{M}, && \text{(5f)}\\ & s_{mm} \in \{0, 1\}, \forall m \in \mathcal{M}, && \text{(5g)}\end{aligned}$$

where (5a) is the objective function; constraint (5b) follows from the ZF property; (5c) is a *per-antenna power* constraint with maximum output power P_m of antenna m; the inequalities in (5d) are *per-user rate* constraints with a target rate R_u of UE u, i.e., quality-of-service (QoS) constraints; constraints (5e) and (5f) follow from the diagonal structure of $\boldsymbol{P}$ and $\boldsymbol{S}$; and (5g) is for AS. Specifically, the right hand side of (5c) follows from the fact that the maximum transmit power of TX_n is limited by P_n^{max} in regulation; and the transmit power of antenna m is then limited by $P_m = P^{max}_{\pi(m)}/M_{\pi(m)}$ to cover the case when all co-located $M_\pi(m)$ antennas are selected for the MU-MIMO communications.

B. Problem Decomposition

Directly solving (5) is difficult due to the non-convex objective function in (5a) and integer optimization variables $\{s_{mm}\}$ in (5f) and (5g). Instead, we propose a suboptimal decomposition approach based on solving three computationally tractable subproblems with respect to $\boldsymbol{W}$, $\boldsymbol{S}$, and $\boldsymbol{P}$. We first give some intuition for our decomposition approach and will provide the details in subsequent sections.

The original problem (5) is reduced to an optimization problem to find $\boldsymbol{W}$ for given feasible $\boldsymbol{P}$ and $\boldsymbol{S}$ that satisfy (5d)–(5g). We denote this subproblem as P1. On the other hand, for given feasible $\boldsymbol{W}$ and $\boldsymbol{P}$ that satisfy (5b)–(5e), the original problem (5) becomes an AS problem to optimize the matrix $\boldsymbol{S}$. For fixed $\boldsymbol{S}$, let $G(\boldsymbol{S})$ denote the optimal (minimum) objective value of P1. Ideally, we wish to choose $\boldsymbol{S}$ as $\boldsymbol{S}^* = \arg\min_{\boldsymbol{S}} G(\boldsymbol{S})$. However, this AS problem is combinatorial in nature and can be computationally intractable when the number of antennas and users are prohibitively large. We therefore present three simple AS algorithms in Section V to obtain a computationally tractable suboptimal solution to this problem, denoted by P2. At last, we note that the optimization problem is quasi-convex over $\boldsymbol{P}$ if $\boldsymbol{S}$ and $\boldsymbol{W}$ are fixed and it can be solved efficiently (more details are provided in Section VI). This PC subproblem is denoted by P3.

In summary, our decomposition approach is as follows:

Step 1: Solve P1 to obtain an optimal ZF-MU-MIMO precoding matrix, i.e., $\boldsymbol{W}^*$, corresponding to fixed $\boldsymbol{S}'$ and $\boldsymbol{P}'$ (Section IV).

Step 2: Solve P2 to find a suboptimal set of transmit antennas to use, i.e., $\boldsymbol{S}^*$, for given $\boldsymbol{W}^*$ in Step 1 and $\boldsymbol{P}'$ (Section V).

Step 3: Solve P3 to obtain $\boldsymbol{P}^*$ for fixed $\boldsymbol{S}^*$ and $\boldsymbol{W}^*$ (Section VI).

It is clear that Steps 1 to 3 can be iterated repeatedly to find better suboptimal solutions, but we will include iteration only between Steps 1 and 2 for simplicity. Note that the AS in Step 2 is critical to the overall performance. Since the AS involves minimizing the same objectives in P1, i.e., $G(\boldsymbol{S})$, we will propose simple suboptimal algorithms which perform the iteration between Steps 1 and 2 to provide $\boldsymbol{W}^*$ and $\boldsymbol{S}^*$ for the subsequent Step 3. Alteration of the optimization, i.e., solving P2 first and P1 subsequently, remains as further work.

IV. Design of ZF-MU-MIMO Precoding Matrix $\boldsymbol{W}$

For given $\boldsymbol{S}'$ and $\boldsymbol{P}'$, which are supposed to satisfy (5d)–(5g), the objective is a function of only $\boldsymbol{W}$ and it can be maximized by minimizing $\mathrm{tr}(\boldsymbol{S}'\boldsymbol{W}\boldsymbol{P}'\boldsymbol{W}^H\boldsymbol{S}')$. Therefore, the original optimization problem (5) can be reformulated for $\boldsymbol{W}$ as

$$\mathsf{P1}: \boldsymbol{W}^* = \min_{\boldsymbol{W}} \mathrm{tr}\left(\boldsymbol{S}'\boldsymbol{W}\boldsymbol{P}'\boldsymbol{W}^H\boldsymbol{S}'\right), \text{ s.t. (5b)}. \quad (6)$$

We note that in this formulation, we have dropped the constraint (5c) in the original optimization problem. Constraint (5c) involves the optimization variable $\boldsymbol{W}$ and it is supposed to be included in P1. However, for this paper, we adopt a simpler approach, and drop (5c) to obtain a closed form solution for $\boldsymbol{W}$ in (6). The power constraints (5c) can be fulfilled in the subsequent Steps.

Using the general expression of a ZF-MU-MIMO precoding matrix which satisfies (5b), i.e., $\boldsymbol{W} = (\boldsymbol{H}\boldsymbol{S}')^+ + \mathrm{null}(\boldsymbol{H}\boldsymbol{S}')\boldsymbol{A}$, and the property that $\mathrm{tr}(\boldsymbol{A}\boldsymbol{A}^H) = \|\boldsymbol{A}\|_F^2$, without loss of optimality, we rewrite (6) to an unconstrained optimization problem with respect to $\boldsymbol{A}$ as

$$\boldsymbol{A}^* = \min_{\boldsymbol{A}} \left\| \boldsymbol{S}'(\boldsymbol{H}\boldsymbol{S}')^+\sqrt{\boldsymbol{P}'} + \boldsymbol{S}'\mathrm{null}(\boldsymbol{H}\boldsymbol{S}')\boldsymbol{A}\sqrt{\boldsymbol{P}'} \right\|_F^2 \quad (7)$$

where $\mathrm{null}(\cdot)$ takes the span of nullspace of a matrix and $\boldsymbol{A}$ is a U-dimensional arbitrary matrix. Since the lower bound of the objective function in (7) is obtained when $\boldsymbol{A}^*$ is a zero matrix (see the Appendix), the EE-aware precoding matrix becomes a conventional ZF-MU-MIMO precoding matrix as

$$\boldsymbol{W}^* = (\boldsymbol{H}\boldsymbol{S}')^+. \quad (8)$$

Note that $\boldsymbol{W}^*$ is a function of only $\boldsymbol{S}'$; thus, Step 1 can be associated with Step 2.

V. Design of Antenna Selection (AS) Matrix $\boldsymbol{S}$

We now design $\boldsymbol{S}$ for given $\boldsymbol{W}^*$ obtained in P1 and fixed $\boldsymbol{P}'$. The fixed PC matrix is defined as

$$\boldsymbol{P}' = \alpha\overline{\boldsymbol{P}}, \quad (9)$$

where $\overline{\boldsymbol{P}}$ is a diagonal matrix with the diagonal element $\overline{p}_{11}\cdots\overline{p}_{UU}$; $\overline{p}_{uu}$ is the relative power portion of UE_u, such that $p_{uu} = \alpha\overline{p}_{uu}$ and $\sum_u \overline{p}_{uu} = 1$; and α is a common power scaling factor for power limit and target rate. The relative power portion factors are determined, *heuristically*, based on the minimum required power for target rate as follows [11]:

$$\overline{p}_{uu} = \widetilde{p}_{uu}\Big/\sum_{k\in\mathcal{U}}\widetilde{p}_{kk}, \ \forall u \in \mathcal{U}, \quad (10)$$

where $\widetilde{p}_{uu}$ is the minimum required power to satisfy (5d), which is derived as

$$\widetilde{p}_{uu} = \sigma^2\left(2^{\frac{R_u}{\Omega}} - 1\right).$$

Using (9) and (10) to the power constraint (5c), we can derive the upper bound of α, denoted by $\overline{\alpha}(\boldsymbol{S})$, as

$$\alpha \le \min_{m\in\mathcal{M}} \left(P_m\big/\left[\boldsymbol{S}\boldsymbol{W}^*\overline{\boldsymbol{P}}\boldsymbol{W}^{*H}\boldsymbol{S}\right]_{mm}\right) \triangleq \overline{\alpha}(\boldsymbol{S}). \quad (11)$$

On the other hand, using (9) and (10) to the QoS constraint (5d), we can derive the lower bound of α as follows:

$$\alpha \ge \sigma^2\left(2^{\frac{R_u}{\Omega}} - 1\right)\Big/\overline{p}_{uu} = \frac{p'_{uu}}{\overline{p}_{uu}} = \sum_{u\in\mathcal{U}} p'_{uu} \triangleq \alpha_{LB}. \quad (12)$$

Thus, if α' satisfies (11) and (12), i.e., $\alpha_{LB} \le \alpha' \le \overline{\alpha}(\boldsymbol{S})$, any $\alpha'\overline{\boldsymbol{P}}$ satisfies (5c) and (5d). If $\overline{\alpha}(\boldsymbol{S}) < \alpha_{LB}$ for all candidates of $\boldsymbol{S}$, an outage happens and the transmitters stay in an idle mode. Since $\boldsymbol{W}^*$ and $\alpha'\overline{\boldsymbol{P}}$ satisfy (5b), (5d), and (5e), we can reformulate (5) for given $\boldsymbol{W}^*$ and $\alpha'\overline{\boldsymbol{P}}$ as follows:

$$\mathsf{P2}: \ \boldsymbol{S}^* = \min_{\boldsymbol{S}} G(\boldsymbol{S}), \ \text{s.t. (5f) and (5g)}, \tag{13}$$

where

$$G(\boldsymbol{S}) = \mathsf{C}(\boldsymbol{W}^*, \boldsymbol{S}, \boldsymbol{P}') = \frac{c}{\eta}\alpha' \left\| \boldsymbol{S}\boldsymbol{W}^* \sqrt{\overline{\boldsymbol{P}}} \right\|_F^2 + g(\boldsymbol{S}). \tag{14}$$

Note that the final EE performance after Step 3 depends on the initial setup of feasible α' as the optimal antenna set in Step 2 depends on initial α'. Further discussion on α' will be provided in Section VII.

The problem $\mathsf{P2}$ is a combinatorial problem over s_{mm}. Since $M \geq U$ as we assumed in Section II, the number of candidates of transmit antenna sets becomes $\binom{M}{U} + \binom{M}{U+1} + \cdots + \binom{M}{M}$, where $\binom{a}{b}$ represents the number of b-combinations from a set with a elements. To reduce the computational complexity of the combinatorial problem, we propose three AS algorithms that can reduce the candidates antenna set effectively. The proposed AS algorithms are associated with $\boldsymbol{W}^*$ in (8) and based on *greedy search*.

A. Channel Norm Based (CNB) Greedy Algorithm

One typically used AS method is based on the channel norm [17]. The channel norm based (CNB) algorithm is focused on the system SE; therefore, it may not be optimal for EE. In each greedy step i, one channel column is decided to be discarded based on channel column norm, i.e., $\|\boldsymbol{h}_m\|_2^2$. Thus, the searching time complexity is $\mathcal{O}(M-U)$. Note that the channel norms are computed at once before the iteration and used in a whole iteration step without requiring additional computational complexity. The pseudo-code of CNB algorithm is shown in Algorithm 1.

Algorithm 1 : CNB-greedy algorithm

1. setup: $\mathcal{M}_0 = [1, 2, \ldots, M]$ and $\boldsymbol{S} = \boldsymbol{I}_M$
2. compute $q_m = \|\boldsymbol{h}_m\|_2^2, \ \forall m \in \mathcal{M}_0$
3. **for** $i = 1$ to $M - U + 1$ **do**
4. find $\boldsymbol{W}^*$ from (8)
5. compute $G(\boldsymbol{S}_i)$ from (14)
6. $q^* = \arg\min_{j \in \mathcal{M}_i} q_j$
7. update: $s_{q^*q^*} = 0$; $\mathcal{M}_i = \mathcal{M}_{i-1} \setminus \{q^*\}$; and $i \leftarrow i+1$
8. **end for**
9. construct $\boldsymbol{S}^*$ such that $s_{mm} = 1$ for all $m \in \mathcal{M}_\ell$ where $\ell = \left(\arg\min_i G(\boldsymbol{S}_i)\right) - 1$ and other elements are zeros.

B. Precoding Norm Based (PNB) Greedy Algorithm

The second algorithm modifies line 6 of Algorithm 1. Specifically, instead of $\|\boldsymbol{h}_j\|_2^2$, we employ a scaled precoding vector norm $\alpha' \left\| \boldsymbol{w}_j^{*T} \sqrt{\overline{\boldsymbol{P}}} \right\|_2^2$ for discarding the channel column vector, hence calling it as a precoding norm based (PNB)-greedy algorithm. As a consequence, the PNB-greedy algorithm may outperform the CNB-greedy algorithm because $\alpha' \|\boldsymbol{w}_j^{*T} \sqrt{\overline{\boldsymbol{P}}}\|_2^2$ is directly related to the power consumption $G(\boldsymbol{S})$. Moreover, since the precoding vector norms are obtained while $G(\boldsymbol{S}_i)$ is computed, the time complexity of PNB-greedy algorithm is the same as CNB-greedy algorithm. The pseudo-code of PNB-greedy algorithm is shown in Algorithm 2.

Algorithm 2 : PNB-greedy algorithm

1. setup: same as line 1 in Algorithm 1
2. **for** $i = 1$ to $M - U + 1$ **do**
3. find $\boldsymbol{W}^*$ from (8)
4. compute $G(\boldsymbol{S}_i)$ from (14) and
5. store $q_m = \alpha' \left\| \boldsymbol{w}_m^{*T} \sqrt{\overline{\boldsymbol{P}}} \right\|_2^2, \ \forall m \in \mathcal{M}_{i-1}$
6. $q^* = \arg\min_{j \in \mathcal{M}_i} q_j$
7. update: $s_{q^*q^*} = 0$; $\mathcal{M}_i = \mathcal{M}_{i-1} \setminus \{q^*\}$; and $i \leftarrow i+1$
8. **end for**
9. construct $\boldsymbol{S}^*$: same as line 9 in Algorithm 1.

C. Power Consumption Based (PCB) Greedy Algorithm

More complex, yet tractable compared to a naive exhaustive search, a power consumption based (PCB)-greedy algorithm is introduced. For the channel update, the PCB-greedy algorithm compares the power consumption $G(\boldsymbol{S}_j)$, i.e., $q_j = G(\boldsymbol{S}_j)$. Since $G(\boldsymbol{S}_j)$ is an actual objective value, we can expect further performance improvement. However, $G(\boldsymbol{S}_j)$ has to be recomputed in each iteration i. Hence, the time complexity of PCB-greedy algorithm increases to $\mathcal{O}(M^2 + M - U^2 - U)$ as j's are elements of $\mathcal{M}_i$ with $|\mathcal{M}_i| = M - i + 1$. The pseudo-code of PCB-greedy algorithm is shown in Algorithm 3.

Algorithm 3 : PCB-greedy algorithm

1. setup: same as line 1 in Algorithm 1
2. **for** $i = 1$ to $M - U + 1$ **do**
3. find $\boldsymbol{W}^*$ from (8)
4. compute $G(\boldsymbol{S}_i)$ from (14)
5. **for** $\forall j \in \mathcal{M}_i$ **do**
6. set $s_{jj} = 0$ for test
7. compute $q_j = G(\boldsymbol{S}_j)$
8. reset $s_{jj} = 1$ and update $j \leftarrow j+1$
9. **end for**
10. $q^* = \arg\min_j q_j$
11. update $s_{q^*q^*} = 0$; $\mathcal{M}_i = \mathcal{M}_{i-1} \setminus \{q^*\}$; and $i \leftarrow i+1$.
12. **end for**
13. construct $\boldsymbol{S}^*$: same as line 9 in Algorithm 1.

VI. Design of Power Control (PC) Matrix $\boldsymbol{P}$

If we fix a ZF-MU-MIMO matrix and an AS matrix as $\boldsymbol{W}^*$ and $\boldsymbol{S}^*$, which are obtained in Sections IV and V and satisfy (5b), (5f), and (5g), the original problem (5) is given by

$$\mathsf{P3}: \ \boldsymbol{P}^* = \max_{\boldsymbol{P}} \frac{\mathsf{R}(\boldsymbol{P})}{\mathsf{C}(\boldsymbol{W}^*, \boldsymbol{S}^*, \boldsymbol{P})} \quad \text{s.t. (5c), (5d), and (5e).} \tag{15}$$

The optimal and heuristic solutions of P3 are provided in subsequent Subsections.

A. Convex Optimization with Bisection Search

By introducing an additional variable ξ, we can rewrite the optimization problem in (15) as

$$\mathsf{P3}': \ \boldsymbol{P}^* = \max_{\boldsymbol{P}} \xi \text{s.t.(5c), (5d), (5e), and} \quad (16a)$$

$$\mathsf{R}(\boldsymbol{P}) - \xi G(\boldsymbol{S}^*) \geq 0. \quad (16b)$$

This rewriting of the optimization problem introduces an additional constraint (16b) to the problem. However, for fixed ξ, the (5c) and (5d) are convex constraints and (5e) is linear constraint; therefore, the feasibility of this optimization problem can be checked through solving a *convex feasibility problem* [18]. This optimization problem is therefore quasi-convex and the optimal ξ can then be found through bisection and sequentially solving the convex feasibility problem at each step of the bisection. We present the bisection search in Algorithm 4.

Algorithm 4 : Bisection search

1. setup: $\xi_{LB} = 0$, $\xi_{UB} \simeq \infty$, and a tolerance value, $\delta > 0$
2. **while** $\xi_{UB} - \xi_{LB} > \delta$ **do**
3. $\quad \xi \leftarrow (\xi_{UB} - \xi_{LB})/2$
4. $\quad$ Solve convex feasibility problem with constraints (5c), (5d), (5e) and (16b), and find (update) $\boldsymbol{P}^*$.
5. $\quad$ **if** infeasible **then** $\xi_{UB} \leftarrow \xi$
6. $\quad$ **else** $\xi_{LB} \leftarrow \xi$ **end if**
7. **end while**
8. $\boldsymbol{P}^*_{optimal} = \boldsymbol{P}^*$

B. Heuristic Method with Problem Modification

In this subsection, we revisit the heuristic PC method proposed in [11]. For simple closed form solution of (15), we maximize EE lower bound instead of EE directly. Using (9), P3 can be modified to maximize the EE lower bound as follows (for details see [11]):

$$\mathsf{P3}'': \ \alpha^* = \ \arg\max_{\alpha} \frac{\Omega U \log_2 (1 + c_1 \alpha)}{c_2 \alpha + c_3} \quad (17a)$$

$$\text{s.t.} \ \ \alpha_{LB} \leq \alpha \leq \alpha_{UB}, \quad (17b)$$

where $c_1 \triangleq \min_u \{\overline{p}_{uu}\} \sigma^{-2}$; $c_2 \triangleq c \sum_{m \in \mathcal{M}} \eta_m^{-1} [\boldsymbol{S}^* \boldsymbol{W}^* \overline{\boldsymbol{P}} \boldsymbol{W}^{*H} \boldsymbol{S}^*]_{mm}$; $c_3 = g(\boldsymbol{S}^*)$; and $\alpha_{UB} = \overline{\alpha}(\boldsymbol{S}^*)$ defined in (11). Note that all c_1, c_2, c_3, α_{LB}, and α_{UB} in (17) are constant values for given $\boldsymbol{W}^*$, $\boldsymbol{S}^*$, and $\overline{\boldsymbol{P}}$. Now, we can readily find the maximizer α_o which makes the first derivative of the objective function in (17a) to zero as

$$\alpha_o = \frac{1}{c_1} \left(\exp\left(1 + \mathsf{W}\left(-\frac{1}{\exp(1)} + \frac{c_1 c_3}{c_2 \exp(1)}\right)\right) - 1 \right),$$

where $\exp(\cdot)$ is an exponential function and $\mathsf{W}(\cdot)$ denotes the Lambert W function that satisfies $q = \mathsf{W}(q) e^{\mathsf{W}(q)}$. Considering the feasible region (17b), we get the optimal feasible solution of (17) as

$$\alpha^* = [\alpha_o]_{\alpha_{LB}}^{\alpha_{UB}}, \quad (18)$$

where $[x]_a^b$ takes x if it is between a and b, and the closest boundary a or b otherwise. Consequently, we get the heuristic PC matrix as

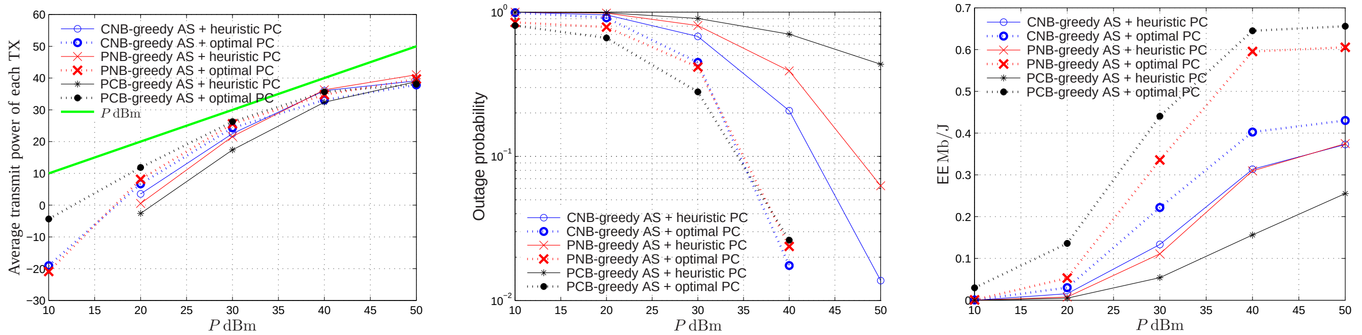
$$\boldsymbol{P}^*_{huristic} = \alpha^* \overline{\boldsymbol{P}}. \quad (19)$$

Since the solution in (19) is obtained from heuristic approach, namely EE lower bound maximization and fixed $\overline{P}$ in (10), it yields performance degradation compared to $\boldsymbol{P}^*_{optimal}$. However, the computational complexity for $\boldsymbol{P}^*_{huristic}$ is obviously lower than that for $\boldsymbol{P}^*_{optimal}$ as (19) is a tractable, closed form expression. Note that the α^* in (18) can be recursively used in Step 2 to update α' in (14) for additional EE improvement. This recursion will be considered in future work.

VII. Performance Evaluation and Discussion

In this section, we evaluate EEof D-TX systems with three AS algorithms and two PC methods. For comparison purpose, we show the system throughput and outage probability as well. For simple star topology network setup, we assume that i) two UEs, four TXs, and all TXs have two antennas, i.e., $U = 2$, $N = 4$, $M_n = 2, \forall n \in \{1, 2, 3, 4\}$; ii) three extended TXs are located at $0.5\,\text{km}$ from a central TX and they are equidistant from one another; and iii) UEs are distributed uniformly within $0.8\,\text{km}$ from the central TX. The path loss is modeled as $A_{um} = g - 128 + 10 \log_{10}(d_{um}^{-\nu})$ in dB scale, where g includes the transceiver feeder loss and antenna gains, $d_{um}^{-\nu}$ is the path loss for the distance d_{um} between TX antenna m and UE u, and ν is a path loss exponent. In our simulation, we set $g = 5\,\text{dB}$, $\nu = 3.76$, and $\sigma^2 = -174\,\text{dBm}/\text{Hz}$. The distance d_{um} depends on the UE location. The small scale fading is modeled as Rayleigh fading, i.e., i.i.d and zero-mean complex Gaussian random variables with a unit variance. System bandwidth is $5\,\text{MHz}$. $R_1 = 40\,\text{Mbps}$ and $R_2 = 60\,\text{Mbps}$. Power related parameters are as follows [2], [3], [16]: $c = 2.63$, $P_{\text{cc}} = 66.4\,\text{W}$, $P_{\text{fix}} = 36.4\,\text{W}$, $P_{\text{sp1}} = 1.82\,\mu\text{W}/\text{Hz}$ and $P_{\text{sp2}} = 3.32\,\mu\text{W}/\text{Hz}$. For high EE, we assume equal power output capability [9], i.e., $\eta_m = 0.3$ and $P_n^{max} = P$. In AS algorithm of Step 2, we fix α' as its maximum, i.e., $\alpha' = \overline{\alpha}(\boldsymbol{S})$ for maximum (worst case) and conservative power allocation for QoS.

In Figs. 2(a), (b), and (c), we compare throughput, EE, and outage probability over maximum transmit power of TXs. First, we compare three AS algorithms with optimal PC method. This comparison verifies well the tradeoff between AS complexity and EE performance. For example, PCB-greedy algorithm can achieve the highest EE compared to the other algorithms, yet its complexity is obviously highest. Next, comparing the optimal and heuristic PC methods, we see that the optimal PC method can always obtain the higher EE than the heuristic PC method, for the same AS algorithm with a cost of higher complexity. The above results are well expected. However, one interesting result is observed by comparing the EEs of different AS algorithms with a heuristic PC method. In contrast with an optimal PC method, the heuristic PC method with a CNB-greedy AS algorithm achieves the highest EE.



(a) (b) (c)

Fig. 2. Numerical comparison when $U = 2$, $N = 4$, and $M_u = 2$. a) Average transmit power. b) Outage probability. c) Energy efficiency (EE).

This is because of the initial value of α' in Step 2. Since the AS algorithms focus on minimization of $G(\boldsymbol{S})$ in (14) in Step 2, the $\alpha' = \overline{\alpha}(\boldsymbol{S})$ is desired to be as small as possible in the AS optimization. The smaller $\overline{\alpha}(\boldsymbol{S})$ yields the smaller upper bound α_{UB} in Step 3, resulting in the lower EE with higher probability in the heuristic PC. One interesting remark is as follows. Depending on an initialization of PC in AS optimization, the tradeoff between AS complexity and EE performance may be violated. In our simulation setup, we can conclude that the CNB-greedy AS algorithm requires the lowest complexity, and at the same time it is the most efficient suboptimal method in terms of EE.

VIII. Summary and Future Work

In this paper, we have considered energy efficiency (EE) maximization for a distributed transmitter (D-TX) system, in which each TX has different number of transmit antennas with individual power constraint. A three-step decomposition approach involving multiuser multiple-input multiple-output precoding matrix weight minimization, antenna selection, and power control has been introduced to solve the EE maximization problem sub-optimally. Numerical results have validated the potential of our proposed approach in EE maximization. Remaining work for further study includes a decomposition approach through: i) introducing an iterative approach between Steps 2 and 3; and ii) including an additional step for power initialization in Step 2.

Appendix

Proof: Noting the properties for given $\boldsymbol{S}$ and $\boldsymbol{P}$ that $\boldsymbol{SS} = \boldsymbol{S}$, $\boldsymbol{SS}^{+} = \boldsymbol{S}$, $\boldsymbol{S}^{H} = \boldsymbol{S}$, and $\boldsymbol{P}^{H} = \boldsymbol{P}$, we can further derive the objective function in (7) as

$$\begin{aligned}
&\mathrm{tr}\Big[\sqrt{\boldsymbol{P}}\boldsymbol{H}^{+H}\boldsymbol{S}\left(\boldsymbol{SH}^{+}\sqrt{\boldsymbol{P}} + \boldsymbol{S}\mathrm{null}(\boldsymbol{HS})\boldsymbol{S}\sqrt{\boldsymbol{P}}\right) \\
&\quad + \sqrt{\boldsymbol{P}}\boldsymbol{A}^{H}\left(\mathrm{null}(\boldsymbol{HS})\right)^{H}\boldsymbol{S} \\
&\qquad \times \left(\boldsymbol{SH}^{+}\sqrt{\boldsymbol{P}} + \boldsymbol{S}\mathrm{null}(\boldsymbol{HS})\boldsymbol{A}\sqrt{\boldsymbol{P}}\right)\Big] \\
&= \mathrm{tr}\Big[\sqrt{\boldsymbol{P}}\boldsymbol{H}^{+H}\boldsymbol{SH}^{+}\sqrt{\boldsymbol{P}} \\
&\quad + \sqrt{\boldsymbol{P}}\boldsymbol{A}^{H}\left(\mathrm{null}(\boldsymbol{HS})\right)^{H}\boldsymbol{S}\mathrm{null}(\boldsymbol{HS})\boldsymbol{A}\sqrt{\boldsymbol{P}}\Big] \\
&= \left\|\boldsymbol{SH}^{+}\sqrt{\boldsymbol{P}}\right\|_F^2 + \left\|\boldsymbol{S}\mathrm{null}(\boldsymbol{HS})\boldsymbol{A}\sqrt{\boldsymbol{P}}\right\|_F^2,
\end{aligned}$$

and obtain the lower bound when $\boldsymbol{A}$ is a zero matrix. ■

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