Propagation-Invariant Space-Time Wave Packets from Free Electron Radiation

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Abstract: We present Smith-Purcell radiation as a promising platform for the generation of space-time wave packets that are ultra-broadband and highly tunable from terahertz to X-ray frequencies.

Light waves designed to propagate without diffractive spreading have attracted much attention due to their unique physical properties [1], and potential applications that include optical microscopy [2], micromanipulation of particles [3] and laser micromachining [4]. Among these nondiffracting light waves are wave packets whose propagation invariance is enforced through strong correlations between their constituent spatial and temporal frequencies [5]. These space-time wave packets feature pulse-like intensity patterns that travel without changing and are noteworthy not least because they can be synthesized with user-designed group velocities, across the subluminal and superluminal ranges [6].

Here, we show that free electrons are a promising means of generating intense, broadband space-time wave packets, due to inherent space-time correlations in free electron radiation. Using Smith-Purcell radiation [7] as an example, we show that the diffraction of an electron's evanescent waves off a periodic grating is a versatile source of space-time wave packets (Fig. 1). In particular, the frequency content, spatial profile and propagation speed of these free-electron-based space-time wave packets can be tailored by controlling the electron speed, electron bunch distribution, as well as the grating material and geometry. Using experimentally realizable electron bunches, we show the feasibility of generating space-time wave packets at exotic frequencies, including terahertz and X-ray regimes. Our findings should pave the way to an even broader range of pulse shaping techniques available for generating propagation-invariant light.

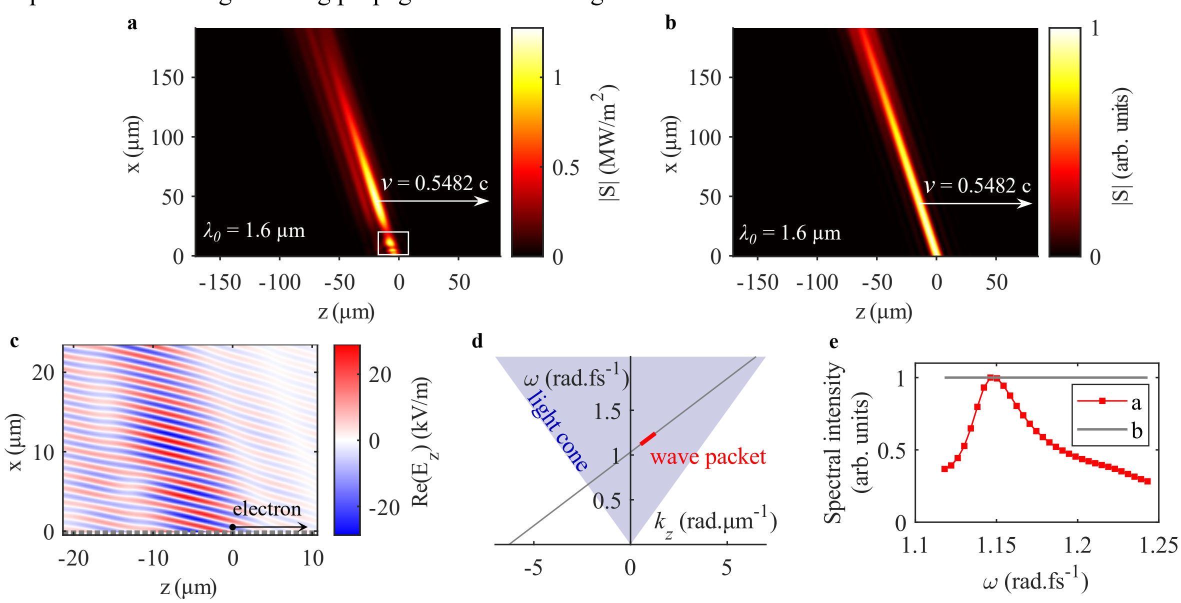

Figure 1. Tunable, broadband space-time wave packets from Smith-Purcell radiation. (**a**) Intensity profile of Smith-Purcell radiation with a central wavelength of 1.6 μm and 170 nm bandwidth. The radiation source is an array of 100 keV electron bunches (with a total charge of 3.2 fC) interacting with a 1 μm period silicon grating (ϵ_r = 11.7). (**b**) A superposition of plane waves representing a space-time wave packet, with the same spatiotemporal correlation as in (**a**), that could be generated, for instance, by using spatial light modulators. (**c**) Electric field profile near the surface of the grating. (**d**) Dispersion relation of the 1st order Smith-Purcell radiation (gray) and the space-time wave packet (red). (**e**) Spectral intensity distribution of the space-time wave packets.

Fig. 1a shows the intensity profile of an infrared space-time wave packet generated from Smith-Purcell radiation. We see a striking similarity between the intensity profile of a space-time wave packet generated from Smith-Purcell radiation and a space-time wave packet generated by superposing plane waves of the same amplitude and phase (Fig. 1b), such as might be obtained through wave-shaping using optical elements like spatial light modulators [2]. To restrict the free electron radiation to our frequency range of interest, we applied a frequency filter of central wavelength 1.6 μm and bandwidth 170 nm [8]. This yields a space-time wave packet of peak intensity 1.27 MW/m^2. Fig. 1d shows the inherent correlation between temporal frequency (ω) and spatial frequency (k_z) in Smith-Purcell radiation through the relation $k_{zm} = \omega/v + 2\pi m/d$, where k_{zm} is the wave-vector corresponding to diffraction order m, v is the electron velocity (also the group velocity of the resulting space-time wave packet) and d is the grating period. The directionality and frequency tunability of Smith-Purcell radiation can therefore be tailored by controlling the electron velocity, grating period and frequency filter bandwidth. In Fig. 1, only the m = -1 diffraction mode exists due to our choice of the filtered frequency range. The intensity profile of the space-time wave packet from free electron radiation can also be user-designed through the electron bunch configuration, leading, for instance, to the intensity hotspot (situated about 50 µm away from the grating surface) in Fig. 1a which is absent in Fig. 1b. Tailoring the electron bunch configuration thus allows us to tune the properties of the space-time wave packet (without changing its group velocity, which is determined solely by the electron velocity). In Fig. 1, we used 0.2 fC, 100 keV electron bunches (0.25 µm spot size and 1.5 fs pulse duration) uniformly spaced across 16 µm in the y direction, traveling 0.5 µm above the grating surface. Such electron bunches can be achieved from existing nanotip emitters and bunch compression techniques [9].

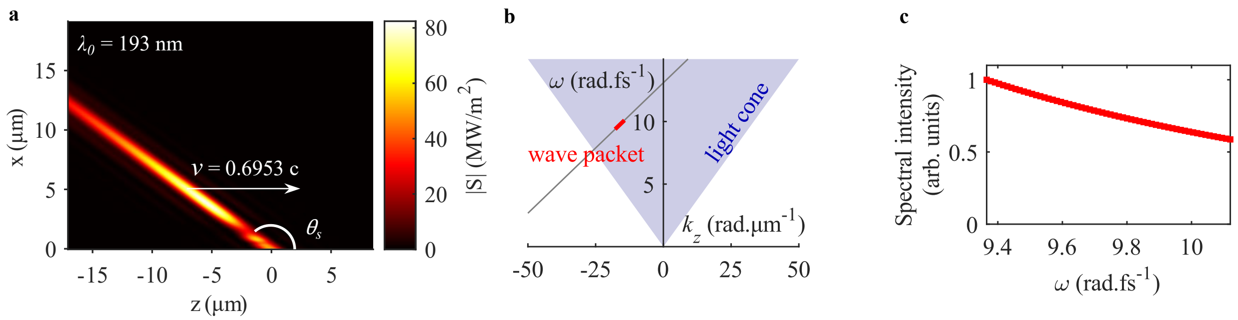

Figure 2. Broadband ultraviolet space-time wave packet from Smith-Purcell radiation. (**a**) Intensity profile of a space-time wave packet of central wavelength 193 nm and bandwidth 15 nm generated from 200 keV electron bunches (50 nm spot size and 0.24 fs pulse duration) interacting with an amorphous silicon grating of period 100 nm. The electron bunches carry a total charge of 0.64 fC and are uniformly spaced across 1.6 µm in the transverse y direction, 100 nm above the grating surface. (**b**) Dispersion relation of the 1st order Smith-Purcell radiation (gray) and the space-time wave packet (red). (**c**) Spectral intensity distribution of the space-time wave packet.

Figure 2 shows the generation of an ultraviolet space-time wave packet from Smith-Purcell radiation. By employing frequency filters of varying bandwidth, we can control the intensity profile of the propagation-invariant wave and tune it from terahertz to X-ray frequencies. For example, our calculations predict the generation of a space-time wave packet of central frequency 0.187 THz, bandwidth 20 GHz and peak intensity 3.82 kW/m^2, from 20 pC, 200 keV electron bunches (0.5 mm spot size and 2.4 ps pulse duration) interacting with a 1 mm period silicon grating; and the generation of a soft X-ray space-time wave packet of central wavelength 6 nm, bandwidth 0.6 nm and peak intensity 2.94 kW/m^2, from 0.2 fC, 5 MeV electron bunches (50 nm spot size and 0.02 fs pulse duration) interacting with a 4 nm period silicon grating. Due to the subluminal nature of free electrons, it should be noted that only space-time wave packets with subluminal group velocities can be generated via this method. We also obtain a formula for the slant angle θ_s in Fig. 2a, described by $\cos(\pi - \theta_s) = (v/c - k_{zm}/k)[1 + (v/c)^2 - 2(v/c)(k_{zm}/k)]^{-1/2}$ where c is the speed of light in vacuum and $k = \omega/c$.

In conclusion, we have presented Smith-Purcell radiation as a promising platform for generating broadband, tunable space-time wave packets. We showed that intense, broadband infrared and ultraviolet space-time wave packet, with peak intensities 1.27 MW/m^2 and 82.4 MW/m^2 respectively, can be generated from 100 keV and 200 keV electron bunches available from existing electron emitters [9]. Our results also reveal the promise of free electron radiation to realize intense, broadband space-time wave packets in extreme frequency regimes – such as terahertz and X-ray – by controlling the electron energy, grating geometry, and using existing frequency filters. Such propagation-invariant waves have potential applications in laser-electron interactions where velocity matching between light pulses and electrons are important [10]. While we have focused on Smith-Purcell radiation here, our findings indicate that other free electron radiation mechanisms involving electrons in uniform motion, such as Cherenkov radiation and transition radiation, are potentially interesting as sources of spatiotemporally coupled light. Our findings thus pave the way to an even broader range of techniques for generating propagation-invariant light.

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