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Spectrally resolved quantum tomography of polarization-entangled states

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Abstract. We studied the broadband polarization entangled states generated within the linewidth of type-II spontaneous parametric down conversion (SPDC). Applying a complete quantum polarization tomography protocol at an arbitrary combination of frequency and angular sideband modes, we reveal the complex structure of non-degenerate non-collinear polarization-entangled states generated within the SPDC linewidth. We demonstrate that the simultaneous compensation of longitudinal and transverse walk-off leads to homogenization of the structure of polarization entanglement.

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1. Introduction

The concept of quantum entanglement, which was considered as a subject of purely theoretical interest some decades ago, is nowadays accessible in many experimental laboratories [1]. Exploiting the process of spontaneous parametric down conversion (SPDC) allows, one can obtain a broad class of photon-entangled states in both continuous and discrete degrees of freedom [2]. The feasibility of such experiments has opened up a range of promising applications spanning from fundamental tasks, such as tests of the concepts of quantum mechanics [3], to nearly practical applications, such as quantum cryptography [4], absolute metrology [5], quantum imaging [6] and quantum computation [7].

Within a broad class of entangled states generated via SPDC, polarization-entangled states are of particular interest. To date, there are many developed and readily accessible methods for generating such states. The most straightforward method relies on type-II frequency degenerate SPDC in the collinear or non-collinear regime, when a photon of a laser pump decays into a pair of orthogonally polarized photons that are superimposed in well-defined spatial modes [8]–[11]. Other widely implemented methods are based on the interference of orthogonally polarized downconverted pairs produced in two type-I SPDC crystals [12]–[15] and a double pass of the pump through a single type-I crystal [16, 17].

In the present paper, we consider the generation of polarization entangled states in frequency degenerate collinear type-II SPDC from a continuous wave pump [8, 9]. Due to the limited length of the SPDC crystal, the phase-matching condition imposes a spread of correlated frequency and angular modes in the vicinity of exact degeneracy, henceforth referred to as the *SPDC linewidth*. In this case, the polarization-entangled two-photon state that occupies correlated sideband modes A and B, henceforth referred to as *sidebands*, can be written as $|\Psi\rangle \propto |H_A V_B\rangle + e^{i\phi} |V_A H_B\rangle$, where indices H and V denote horizontal and vertical polarizations of photons, respectively, and ϕ is a relative phase. Considering the birefringence and chromatic dispersion of the SPDC crystal, it can be shown that polarization states at different angular and frequency sidebands acquire different relative phases ϕ [18]. This effect, also referred to as longitudinal (for frequency modes) and transverse (for angular modes) walk-off, leads to the generation of an inhomogeneous broadband entangled state within the SPDC linewidth, which prevents observation of high-visibility two-photon polarization interference. Several techniques have been suggested to restore the two-photon interference in type-II SPDC. Some are based on filtering the SPDC spectrum, but significantly reducing the brightness of the source [9]. Others are based on compensation of the walk-off by means of a specifically tailored birefringent material placed in a downconversion beam [9, 10].

Although the majority of experiments aim at eliminating the walk-off in order to optimize the generation of a given entangled state, it is highly intriguing that in the case where the walk-off is not compensated, a single non-linear crystal can be used as a natural source of different entangled states. In particular, it allows the simultaneous generation of slightly mismatched but completely orthogonal entangled states. Note also that the above effect should always be accurately considered in the observation of two-photon interference with narrowband filters as it would define the purity and entanglement of post-selected states.

The structure of polarization entangled states generated within the SPDC linewidth has been considered earlier in [18]. However, the experiments were performed separately for frequency degenerate and strictly collinear regimes and did not allow access to any non-degenerate non-collinear states. Apart from this, characterization of the states was limited only

to observation of basic polarization dependences, which, as it will be shown later, do not allow a complete state characterization. In the present work, we make an effort to fill these gaps and present a complex approach, where by using quantum polarization tomography we are able to reconstruct a density matrix of the entangled state in an arbitrary combination of angular and frequency SPDC sideband modes.

2. Theory

Let us consider type-II SPDC in a critically phase-matched birefringent crystal of length L , pumped by a continuous wave laser with frequency Ω_p and wave vector $\vec{k}_p$. The orientation of the optical axis is set in such a way that orthogonally polarized signal and idler photons (s,i) have nearly degenerate frequencies ($\Omega_s \approx \Omega_i \approx \Omega_p/2$) and propagate collinearly ($\vec{k}_s \parallel \vec{k}_i \parallel \vec{k}_p$), yielding energy and momentum conservation,

$$\Omega_s + \Omega_i = \Omega_p; \quad \vec{k}_s + \vec{k}_i \approx \vec{k}_p.$$

Further, it is assumed that the exact momentum conservation is fulfilled in the transverse direction of the crystal (orthogonal to $\vec{k}_p$), provided that the pump diameter is larger than the transverse walk-off, given by $L \tan \theta$, where θ is a typical scattering angle. In this case, the finite length of the SPDC crystal in the longitudinal direction (along $\vec{k}_p$) weakens the momentum conservation condition and allows the generation of correlated photons in non-degenerate sideband modes. Expanding the longitudinal component of the momentum mismatch Δ_z up to linear terms in frequency ($\omega = \Omega_s - \Omega_p/2$) and angular (θ) sidebands, one obtains [19]²

$$\begin{aligned} \vec{\Delta} &\equiv \vec{k}_p - \vec{k}_s - \vec{k}_i; \quad \Delta_z \approx D\omega + B\theta, \\ D &= (dk_e/d\omega - dk_o/d\omega); \quad B = dk_e/d\theta, \end{aligned} \quad (1)$$

where $k_{o,e}$ is the wave vector for ordinary (horizontally polarized) and extraordinary (vertically polarized) waves in the crystal, respectively. In this case, the two-photon polarization-entangled state, generated within the whole SPDC linewidth, can be written as

$$|\Psi\rangle \propto \int \int d\omega d\theta F(\omega, \theta) (|H_\omega^\theta V_{-\omega}^{-\theta}\rangle + e^{i\phi} |V_\omega^\theta H_{-\omega}^{-\theta}\rangle), \quad (2)$$

where $|H_\omega^\theta V_{-\omega}^{-\theta}\rangle$ ($|V_\omega^\theta H_{-\omega}^{-\theta}\rangle$) is a two-photon Fock state with one horizontally (vertically) polarized photon in mode $(\omega; \theta)$ and one vertically (horizontally) polarized photon in mode $(-\omega; -\theta)$ and $\phi \equiv \Delta_z L$ is a relative phase. The function $F(\omega, \theta)$ represents the spectral amplitude of the type-II SPDC process, which is given as [9]

$$F(\omega, \theta) = \sin(\Delta_z L/2)/(\Delta_z L/2) \equiv \text{sinc}(\Delta_z L/2). \quad (3)$$

For future discussion, it is of crucial importance to consider the dependence of the relative phase ϕ in (2) on the frequency and angular sidebands. In figure 1(a), we present the calculations of the SPDC spectral amplitude (3) near exact degeneracy and in figure 1(b) a corresponding phase dependence, using (1). From figure 1(b), one can see that each combination of correlated frequency and angular sidebands $(\omega; \theta)$ defines a specific value of the relative phase ϕ in (2) provided that a continuum of non-degenerate non-collinear entangled states is generated within

² Here we consider the scattering in the plane, containing the optical axis of the crystal. In the case of the scattering in the isotropic plane of the crystal, a uniform entangled state is generated within the linewidth. See [18] for more details.

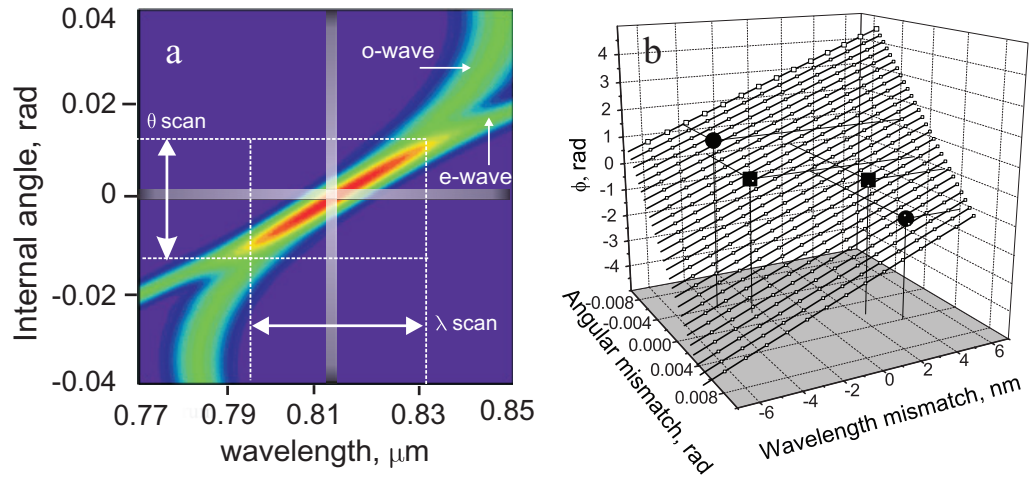


Figure 1. (a) Calculation of the spectral distribution according to (3) with the parameters corresponding to the experiment described in section 4. The shaded stripes and arrows schematically show the detection bandwidths and tuning ranges of the detectors in the angular (horizontal white shaded) and frequency (vertical white shaded) sidebands. (b) The dependence of the relative phase in (2) on the wavelength and angular mismatch. The pairs of black squares and black circles depict non-degenerate correlated modes that were chosen for tomography reconstruction in section 5.

the SPDC linewidth. Thus, for instance, even for non-degenerate modes, when a choice of specific sidebands results in $\phi = 0$, one of the triplet $\Psi^+ \equiv (|H_\omega^\theta V_{-\omega}^{-\theta}\rangle + |V_\omega^\theta H_{-\omega}^{-\theta}\rangle) / \sqrt{2}$ Bell states is generated. At the same time, when the combination of frequency and angular sidebands results in $\phi = \pi$, the singlet Bell state $\Psi^- \equiv (|H_\omega^\theta V_{-\omega}^{-\theta}\rangle - |V_\omega^\theta H_{-\omega}^{-\theta}\rangle) / \sqrt{2}$ is generated.

Let us mention that the above effect clearly explains the difficulty of observing two-photon interference in the case of broadband detection. Indeed, from (1) and (2) it follows that a resultant state would represent a statistical mixture of pure states with different relative phases. In this case, the entropy and entanglement of the generated mixed state would depend on the corresponding bandwidths of spatial and frequency filters, opening, however, the possibility of studying mixed entangled states [20]. In order to restore two-photon interference, the phase dependence on scattering parameters should be erased. This can be done, for instance, by placing a piece of birefringent material after the SPDC crystal, which adds a phase shift ϕ_{comp} to the initial relative phase ϕ . If the introduced phase shift flattens the dependence of the total phase $\phi_{\text{total}} \equiv \phi + \phi_{\text{comp}}$ on both the scattering parameters within the detection band, then two-photon interference can be clearly observed.

Let us now consider the general representation of a two-photon polarization state. The following set of four orthogonal basis states can be defined for two distinguishable photons in the horizontal-vertical polarization basis: $|H_\omega^\theta H_{-\omega}^{-\theta}\rangle$; $|H_\omega^\theta V_{-\omega}^{-\theta}\rangle$; $|V_\omega^\theta H_{-\omega}^{-\theta}\rangle$; $|V_\omega^\theta V_{-\omega}^{-\theta}\rangle$ [21]. Therefore, an arbitrary two-photon polarization state in the given sidebands can be written as a state of a four-level quantum system (ququart) of the following form,

$$|\Psi\rangle = c_1 |H_\omega^\theta H_{-\omega}^{-\theta}\rangle + c_2 |H_\omega^\theta V_{-\omega}^{-\theta}\rangle + c_3 |V_\omega^\theta H_{-\omega}^{-\theta}\rangle + c_4 |V_\omega^\theta V_{-\omega}^{-\theta}\rangle, \quad (4)$$

where $c_i = |c_i| \exp(i\phi_i)$ is the complex probability amplitude of the corresponding two-photon state, fulfilling the normalization condition $\sum |c_i|^2 = 1$. In the ququart basis, the state (2) has the following complex amplitudes: $c_1 = 0$; $c_2 = 1/\sqrt{2}$; $c_3 = \exp(i\phi)/\sqrt{2}$; $c_4 = 0$. Note that (2) can be easily transformed into an arbitrary state (4) by polarization rotations on separate photons and in particular into the triplet Bell states $\Phi^\pm \equiv (|H_\omega^\theta H_{-\omega}^{-\theta}\rangle \pm |V_\omega^\theta V_{-\omega}^{-\theta}\rangle) / \sqrt{2}$ [10]. The generation, transformation and measurement of ququarts have been extensively studied in earlier works in strongly non-frequency degenerate [21, 22] and in non-collinear regimes [23, 24]. At the same time, exploiting polarization entanglement within the SPDC linewidth allows, for the first time to the best of our knowledge, one to obtain ququart states in non-frequency degenerate, non-collinear modes.

3. Measurement procedure

The polarization entangled state (2) can be experimentally characterized with a simple setup that consists of a 50/50 non-polarizing beam splitter (NPBS), two polarization analyzers installed at its output ports and two photodetectors [10]. Two complimentary measurements of coincidences in the configuration when one of the analyzers is oriented at 45° to the laboratory basis and the other analyzer is flipped between 45° and -45° would allow us to measure the relative phase ϕ according to the formulae [18]

$$\begin{aligned} R_c^{45/45} &= \text{sinc}^2(\phi/2) \cos^2(\phi/2), \\ R_c^{45/-45} &= \text{sinc}^2(\phi/2) \sin^2(\phi/2). \end{aligned} \quad (5)$$

Although the above measurements provide a comprehensive and rapid estimation of the state, they do not allow complete state reconstruction. When one considers an entangled state (2) as a particular case of a ququart state (4), a more sophisticated procedure is needed that would allow reconstruction of 16 density matrix elements in (4).

In the present work, different entangled states at arbitrary combinations of angular and frequency sidebands are characterized by means of a complete protocol of quantum polarization tomography. The quantum polarization tomography is an indispensable tool for the characterization of polarization-entangled states, allowing complete reconstruction of the density matrix via correlation measurements in various polarization bases [23, 25, 26]. Among the variety of suggested protocols, we exploit a particular one by James *et al* [23], which is referred to as J16 [27]. The choice of J16 is motivated by its experimental implementation being convenient for studying slightly mismatched entangled states. Thus, in contrast to the method of Bogdanov *et al* [25], it does not require simultaneous wavelength-dependent transformations on both photons. According to the chosen protocol, a wide-aperture broadband 50/50 non-polarizing beamsplitter is placed into the downconverted beam with two sets of polarization analyzers in its output ports, followed by single-photon detectors. Each polarization analyzer consists of broadband quarter- and half-wave plates and a linear polarization filter, which allows the projection of a polarization state of a single photon onto an arbitrary state on a Poincare sphere. Coincidence measurements between two detectors at 16 pre-defined orientations of the wave plates allow a complete reconstruction of the density matrix of an unknown state (4).

To allow an arbitrary choice of frequency–angular mode within the SPDC linewidth, a traditional tomography setup has been upgraded in the following way. One detector selects a narrow band in a frequency spectrum and, at the same time, accepts a whole angular spectrum. Similarly, another detector performs a narrow angular selection, while the frequency spectrum is

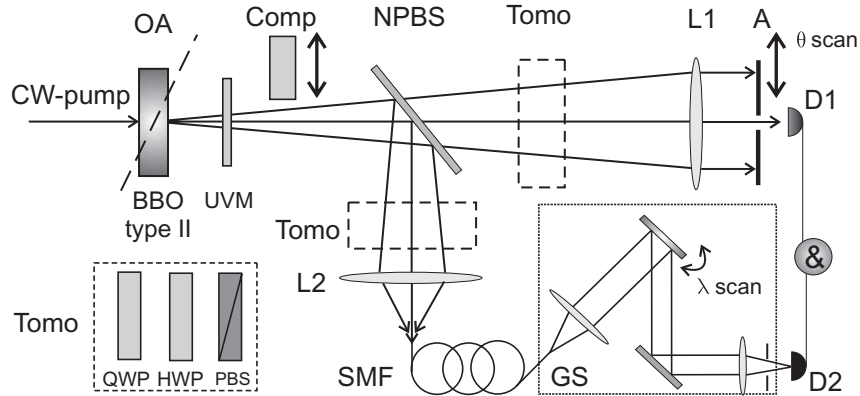


Figure 2. Experimental setup: a 406 nm cw laser is focused into 1 mm long type-II BBO and filtered out by a UVM. Broadband SPDC radiation is split by a 50/50 NPBS and sent into two tomography analyzers (Tomo), consisting of quarter- and a half-wave plates (QWP, HWP) and a PBS. One branch of the NPBS is used for angular scanning and has a 500 mm lens (L1) focused at the crystal and a 1 mm aperture (A), mounted onto a translation stage. Another branch of the NPBS accepts a broad angular spectrum in a SMF, which is connected to a GS. D1 and D2 are avalanche photodiodes, connected to a coincidence scheme (&). A quartz compensator (Comp) was used in some experiments to demonstrate the homogenization of polarization entanglement.

not filtered. In figure 1(a), we schematically represent the corresponding bandwidths and tuning ranges of two detectors by horizontal white shaded (angular selective detector) and vertical white shaded (frequency selective detector) stripes. For the case of a non-strongly focused continuous wave pump, such a configuration allows the filtering of an arbitrary polarization state by tuning a frequency filter in one arm and a spatial filter in the other arm. Note that broadband detection of a conjugated mode allows one to avoid a drastic decrease in the signal and thus reduces the contribution of the dark counts of the detectors.

4. Experiment

The experimental setup on polarization tomography of SPDC sidebands is shown in figure 2. A pump beam from a cw diode laser with power 50 mW at a wavelength of 406 nm and a spectral full-width at half-maximum (FWHM) of 0.3 nm was mode cleaned by a single-mode fiber (SMF) and was focused by a lens ($f = 500$ mm) into a 1 mm long BBO crystal cut for type-II SPDC. The width of the pump beam in the crystal was $65 \mu\text{m}$, large enough to have no significant effect on the broadening of the SPDC spectrum. The crystal was set for the collinear frequency degenerate regime of SPDC with its axis at 47.6° to k_p , so that signal and idler photons had the same central wavelength of 812 nm and propagated collinearly. After passing through the crystal, the pump was reflected by a UV mirror (UVM); SPDC radiation smoothly passed through it and was then divided into two paths by a 50/50 NPBS. In each output port of the NPBS, a set of broadband quarter- and half-wave plates (HWP) projected the state onto a pre-defined linear polarization state (the horizontal one), selected by a polarizing

beam splitter (PBS). In one output arm of the NPBS, the whole SPDC angular linewidth, estimated as 0.01 rad FWHM (see figure 1), was imaged with a demagnification ($M = 12$) at the face of a single mode fiber (SMF) with a numerical aperture of 0.12. The SMF led to a grating spectrometer (GS) with a resolution of 0.35 nm. In another port of the NPBS, a lens L1 ($f = 500$ mm) was placed in such a way that the SPDC crystal was at its front focus. Thus, the lens performed a projection of scattering angles into physical space, where they were filtered with a 1 mm diameter translatable aperture (A). This scheme provided an angular resolution of 1×10^{-3} rad. A broadband interference filter (FWHM = 100 nm) was used to pass the whole spectral band of SPDC while cutting off the undesirable optical noise. Perkin-Elmer SPCM-14FC avalanche photodiodes (D1, D2) were used to detect photons in each port. Signals from both photodiodes were then sent to a coincidence unit (&) with a time window of 5 ns.

For the reconstruction of the density matrix, we used the above-mentioned tomography protocol J16 [23], where 16 sets of orientations of waveplates provided projections at *a priori* defined polarization states. At each set, 20 measurements were performed with the acquisition time of 10 s. The number of coincidences obtained at each setting corresponded to the linear combination of components of the density matrix. Thus, after the tomography procedure, one obtains a system of 16 equations for 16 unknown parameters³.

In experiments devoted to study the walk-off compensation, a specially prepared compensator was introduced just after the UVM. The parameters of the compensator, i.e. the length and orientation of its optical axis, were chosen in such a way that the introduced phase ϕ_{comp} being added to the original phase ϕ at any combination of angular and frequency sidebands resulted in $\phi_{\text{total}} = \phi + \phi_{\text{comp}} \approx 0$, thus providing a homogeneous preparation of the quantum state. A 6.5 mm long piece of crystalline quartz was used for this purpose with its optical axis orientated at 49.6° with respect to the wave vector of the pump.

5. Results and discussion

First, we have checked that the setup allows broadband registration and has a good enough spectral resolution to study entangled states generated within the frequency and angular SPDC linewidths. Angular and frequency scans were performed in natural (filtering H/V polarizations by the tomography block) and diagonal polarization bases (filtering $45^\circ/-45^\circ$ polarizations) in the frequency degenerate and collinear cases, respectively. The interference pattern presented in figure 3 clearly demonstrates the inhomogeneous structure of polarization entanglement in angular (a,b) and frequency (c,d) sidebands. This confirms that a broad range of states is generated with a relative phase determined by both the frequency and the angular offset from exact degeneracy. By introducing the quartz compensator, the relative phase in (2) changes to $\phi_{\text{total}} = \phi + \phi_{\text{comp}} \approx 0$ for any sideband. Thus, the simultaneous homogenization of the interference pattern in both angular and frequency spectra was observed, providing for the generation of the Bell Ψ^+ state within the whole spectrum.

In order to stress that complex polarization analysis can be performed on the setup, one can choose the state to reconstruct in such a way that it should not be at trivial settings such as a collinear or frequency-degenerate regime. Firstly, the state is considered at a pair of conjugated wavelengths 814.5 and 809.5 nm and at angular modes $\theta = \pm 0.002$ rad. According

³ While elaborating experimental data, we became aware of a useful real-time quantum tomography tool available online at <http://research.physics.illinois.edu/QI/Photonics/Tomography>.

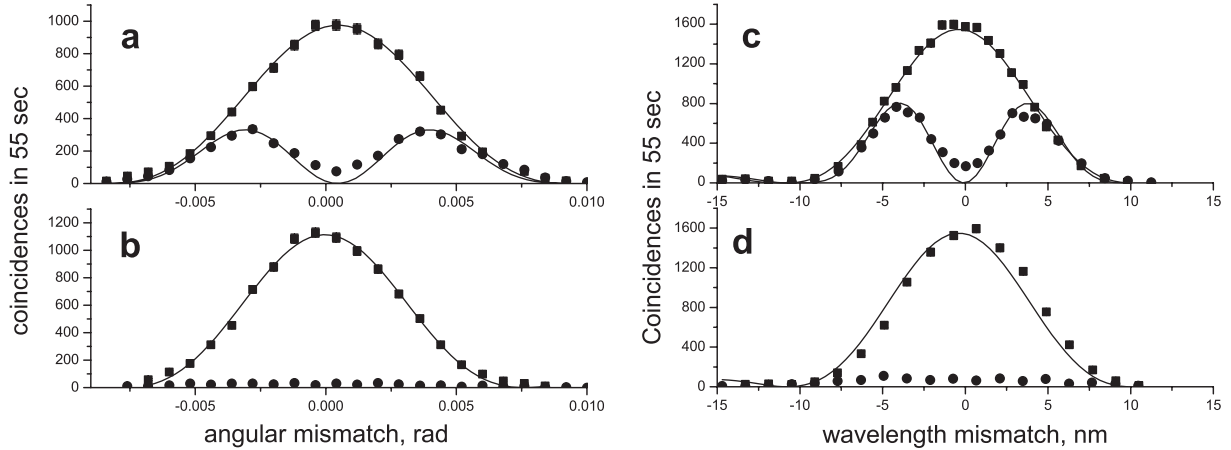


Figure 3. Angular (a) and frequency (c) spectra without the compensator in natural H/V (squares) and diagonal $45^\circ/-45^\circ$ bases (circles). The inhomogeneous structure of polarization entanglement is clearly seen, and it is due to variation in phase in (2). Compensating for longitudinal and transverse walk-off by means of a specially designed compensator allows the homogenization of polarization entangled states in angular (b) and frequency (d) spectra, so that Ψ^+ state is generated within the whole SPDC linewidth. The solid curves are fitted according to equations (3) (squares) and (5) (circles).

to the theoretical predictions in figure 1(b), where the chosen sidebands are shown as two squares, this arrangement corresponds to the triplet Bell Ψ^+ state to be filtered out of the SPDC linewidth. Note that in this case, the observed signal is of the same level as in the degenerate mode, despite being detuned from the exact degeneracy. The polarization tomography of the state has been performed at this particular frequency–angular sideband. After elaboration of the results, it was found that the raw reconstructed density matrix did not satisfy the basic physical requirements; for example, the positivity of its eigenvalues. A maximum likelihood estimation (MLE) algorithm was applied to find the maximally close physical state [23, 28] (see footnote 3). Thus, the following density matrix was obtained,

$$\rho_0 = \begin{pmatrix} 0.0068 & -0.0356 - 0.0127i & -0.0370 + 0.0004i & -0.0003 + 0.0005i \\ -0.0356 + 0.0127i & 0.4615 & 0.4369 - 0.0258i & 0.0095 + 0.0225i \\ -0.0370 - 0.0004i & 0.4369 + 0.0258i & 0.5275 & 0.0057 + 0.0243i \\ -0.0003 - 0.0005i & 0.0095 - 0.0225i & 0.0057 - 0.0243i & 0.0042 \end{pmatrix}. \quad (6)$$

The experimental results on the reconstructed state are presented in table 1 and the real and imaginary parts of the density matrix (6) are presented in figures 4(a) and (b). For all the experiments, the fidelity between the reconstructed state ρ_{exp} and the theoretical state ρ_{th} was calculated according to $F = (\text{Tr}(\sqrt{\sqrt{\rho_{\text{th}}}\rho_{\text{exp}}\sqrt{\rho_{\text{th}}}}))^2$.

The above protocol has been accomplished in this setup by placing a birefringent compensator after the UVM, which provides for the generation of the Bell Ψ^+ state, as well. The corresponding data on the reconstructed state showed the fidelity of the experimental results $F = 0.92 \pm 0.01$, which is in agreement with the results obtained without the compensator.

Table 1. Experimental results on the tomography of polarization entangled states in different sidebands of the SPDC linewidth. $\lambda_{s,i}$ are the selected wavelengths and $\theta_{s,i}$ are the internal scattering angles of signal and idler photons, respectively. The fidelity is calculated between the experimentally reconstructed state and state (2) with a corresponding value of the relative phase ϕ , defined by the selected sidebands.

Symbol in figure 1(b)	Selected state			Tomography results	
	$\lambda_s; \lambda_i$ (nm)	$\theta_s; \theta_i$ (rad)	ϕ (rad)	Fidelity	$\text{Tr}(\rho)^2$
■	809.5; 814.5	± 0.002	0	0.931 ± 0.015	0.8824
●	808.2; 815.8	± 0.004	$-6\pi/15$	0.926 ± 0.015	0.8634

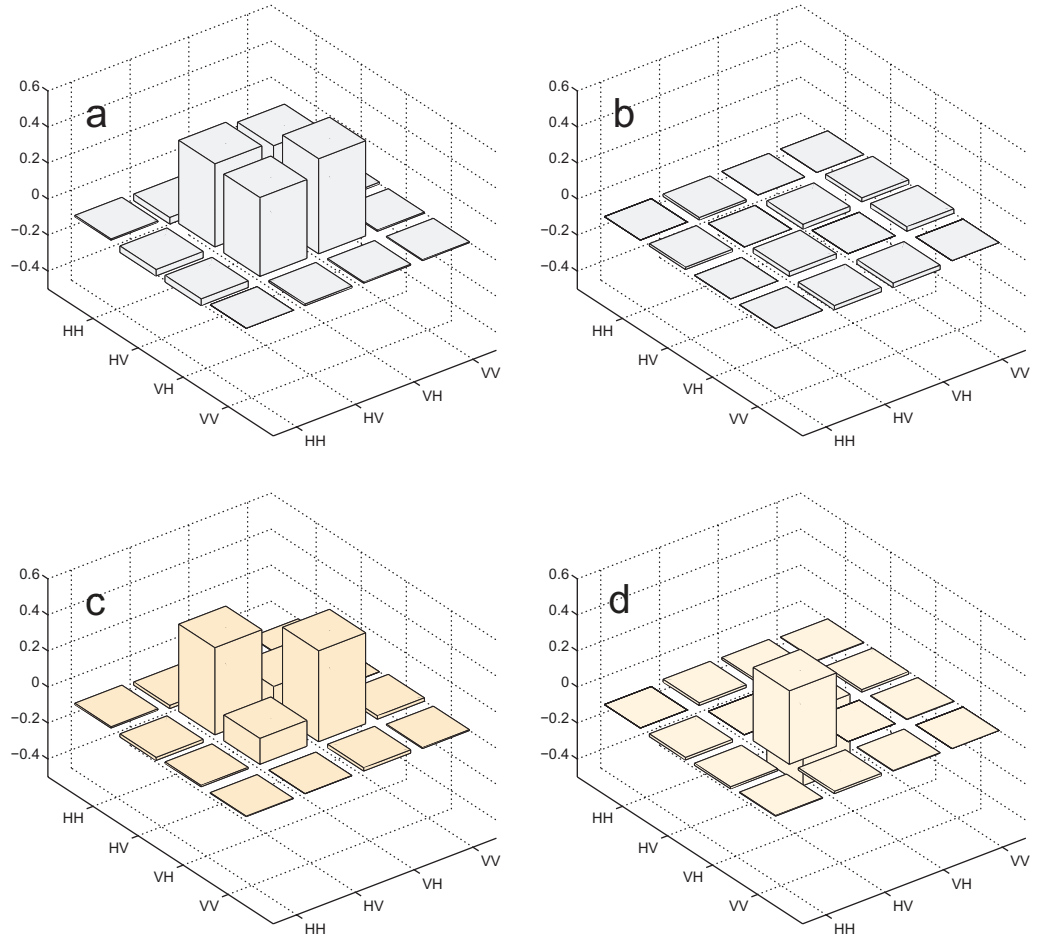


Figure 4. Graphical representation of experimentally reconstructed density matrices. (a, b) The real and imaginary parts of (6), respectively; (c, d) the real and imaginary parts of (7), respectively.

Furthermore, states at an arbitrary combination of frequency and angular modes have been studied with the tomography setup. We have chosen the state at an angular mismatch of $\theta = \pm 0.004$ rad, while a pair of correlated wavelengths has been changed to 815.8 and 808.2 nm.

The chosen sidebands, which are shown in figure 1(b) by a pair of black circles, correspond to the filtered state (2) with a phase $\phi = -6\pi/15$. The result of the state reconstruction after application of MLE is presented below, plotted in figures 4(c) and (d) and analyzed in table 1.

$$\rho_{-6\pi/15} = \begin{pmatrix} 0.0073 & -0.0170 + 0.0146i & 0.0074 + 0.0108i & -0.0028 - 0.0030i \\ -0.0170 - 0.0146i & 0.4840 & 0.1430 - 0.4071i & -0.0010 - 0.0128i \\ 0.0074 - 0.0108i & 0.1430 + 0.4071i & 0.5043 & 0.0181 - 0.0024i \\ -0.0028 + 0.0030i & -0.0010 + 0.0128i & 0.0181 + 0.0024i & 0.0044 \end{pmatrix}. \quad (7)$$

With independent measurements (not shown), a slight difference was observed in the spectral widths of orthogonally polarized photons for highly detuned angular sidebands. The observations from these experiments explain the inequality of diagonal components of the density matrix that leads to degradation of the fidelity of the reconstructed state.

The analysis of the experimental uncertainties of the tomography procedure revealed that for the presented results, the value of fidelity and the purity of the state were partially compromised by the final spectral resolution of angular and frequency scans and by the purity of the source itself. The finite resolution of the spectrometer and angular filter contributed to the non-homogeneity of a chosen phase, launched into the tomography setup. In turn, the purity of the state, produced by the source, was affected by the final frequency bandwidth of the pump and diffraction of SPDC angular modes. Therefore, the chosen experimental parameters were optimized to find a reasonable compromise between the mentioned factors and a reasonable collection efficiency, to allow a reliable tomography reconstruction.

6. Conclusions

In conclusion, in this work, non-degenerate non-collinear polarization-entangled states, generated within the natural linewidth of type-II SPDC, have been studied with a quantum tomography setup. It has been demonstrated that due to the multi-mode nature of the SPDC process, the structure of polarization entanglement is inhomogeneous. Our results suggest that in experiments with type-II SPDC, an accurate analysis of downconverted modes and their spectral bandwidths is essential for ensuring the high purity and entanglement of produced states. In addition, it has been demonstrated that the inhomogeneity of polarization entanglement can be effectively tailored with a birefringent compensator. In this regard, it is of practical interest when polarization entanglement is simultaneously homogenized in spatial and frequency domains, providing for the generation of a given broadband entangled state at high photon fluxes. It is also worth mentioning that the developed technique allows one to obtain a limited class of mixed states with their entropy being controlled by the settings of the experiment (e.g. by the filter bandwidth). This is of considerable interest for future experimental studies of mixed entangled states [20].

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