Pecuniary Efficiency of Distributed Antenna Systems

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***Abstract*—In this letter, the gross expenditure, revenue, and profit of a distributed antenna system (DAS) are modeled and investigated. Using stochastic geometry framework, we evaluate the communications *pecuniary efficiency* (PE) that is defined as a ratio of average achievable bits per expenditure. In addition, we further investigate the ratio of gross revenue over expenditure, which is the PE in economics. From the evaluation of profit and PEs, we justify the economic benefit (profit) of DAS. The PE is expected to be an important decision factor for the deployment of new communication systems, along with typical metrics in communications, namely spectral efficiency and energy efficiency.**

***Index Terms*—DAS, pecuniary efficiency (PE), expenditure, profit, revenue, spectral efficiency, energy efficiency.**

I. INTRODUCTION

DISTRIBUTED antenna system (DAS), which consists of multiple distributed antennas (DAs) connected to a centralized unit through possibly optical fiber, has recently become a more tangible technology (e.g., for a stadium [1]) due to pervasive high-speed fiber broadband service. Recently, DAS's superiority in terms of spectral efficiency (SE) and energy efficiency (EE) has been vigorously studied [2], [3]. However, practical cost for deploying and operating DAS, namely the capital expenditure (CAPEX) and operating expenditure (OPEX), has not been considered in the previous studies. Furthermore, since the high SE and/or EE do not necessarily guarantee high profit, it is useful to study the financial efficiency and the SE/EE jointly. Such investigations will provide the operators a concrete comprehension on the tradeoff between achievable profits and actual cost.

In this study, using stochastic geometry results in [4]–[6], we derive the coverage probability and average achievable sum rate of DAS. Furthermore, we model gross expenditure, which is denoted by COPEX as it is the sum of CAPEX and OPEX, and derive the revenue and profit for DAS. We then introduce pecuniary (or cost) efficiency (PE) in communications (PEcom) and in economics (PEeco). PEcom and PEeco are defined as a *ratio of average achievable sum rate over COPEX*, and a *ratio of gross revenue over COPEX*, respectively. It is worth noting that PEcom is prevalently known as return on investment (ROI) in economics. From the fact that the number of DAs is a critical parameter that determines COPEX as well as communication performance, we formulate a profit/PEcom/PEeco maximization problem over the number of DAs. By solving the problem numerically, we can obtain the answers *whether* and *when* DAS really delivers economic benefit to an operator, and furthermore justify the merit of DAS. The PE models proposed in the letter can provide a systematic and rigorous reference for DAS deployment decision, by applying it to any network with actual cost and profit values evaluated by the operators.

II. STOCHASTIC GEOMETRY MODEL FOR DAS

We consider a DAS with N DAs and K user equipments (UEs) distributed according to homogeneous Poisson point processes (PPPs) Φ_d and Φ_u of intensity λ_d and λ_u, respectively, in the Euclidean plane. The intensities can be approximated for given area A km^2 as $\lambda_d = N/A$ and $\lambda_u = K/A$. Note that if UE intensity λ_u is insufficiently larger than λ_d, then some DAs may not possibly be connected to any UE, and thus, do not transmit signals. Therefore, we first investigate the transmission probability of each DA.

For tractability, from the results in [4], [6], we assume that the transmitting DAs are modeled as a homogeneous PPP $\widehat{\Phi}_d$ derived by thinning the DA PPP Φ_d with a DA activation probability ϵ. The DA activation probability ϵ is approximated as $\epsilon = 1 - (1 + \lambda_u/(3.5\lambda_d))^{-3.5}$ [4], under an infinite region assumption, i.e., $A \to \infty$. Note that the number of active DAs is the same as the number of UEs if there are sufficient DAs, i.e., as $\lambda_d/\lambda_u \to \infty$, $\epsilon \to \lambda_u/\lambda_d = K/N$. From [5, Proposition 1.8], the active DAs are distributed according to a homogeneous PPP with intensity $\epsilon\lambda_d$ being the density of effective communication node pairs in A. Furthermore, the user density equals to $\epsilon\lambda_d$ and the corresponding effective number of users will be $K_\epsilon = \lfloor \epsilon N \rfloor \le K$.

Using the active DA probability ϵ, we derive a coverage probability for a typical user k in Lemma 1.

Lemma 1: Given that active DAs are modeled as PPP $\widehat{\Phi}_d$ in an infinite region and that both desired and interfering signals experience Rayleigh fading, the coverage probability of a typical user k for a quality-of-service (QoS) rate R_k $\mathrm{bps/Hz}$ is derived as follow (see Appendix for the proof):

$$
\begin{aligned}
p_k &= \pi\lambda_d \int_0^\infty \exp\left(-\pi\lambda_d \left[1+\epsilon\rho(\mu, Q_k)\right] t - \frac{\sigma^2 Q_k \sqrt{t^\mu}}{P}\right) dt \qquad (1)\\
&\overset{(a)}{\approx} \pi\lambda_d \int_0^\infty \exp\left(-\pi\lambda_d \left[1+\frac{2\pi\epsilon\, Q_k^{2/\mu}}{\mu \sin(2\pi/\mu)}\right] t - \frac{\sigma^2 Q_k \sqrt{t^\mu}}{P}\right) dt,
\end{aligned}
$$

where $\rho(\mu, Q_k) = Q_k^{2/\mu} \int_{Q_k^{-2/\mu}}^\infty 1/(1 + u^{\mu/2}) du$; μ denotes a path loss exponent (refer to Table I); $Q_k = 2^{R_k} - 1$; σ^2 is the variance of additive white Gaussian noise at UE; and P is the fixed maximum transmit power of each[1] active DA. The approximation (a) in (1) is valid when $\mu > 2$, and it becomes more accurate as Q_k increases.

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[1] The equal power transmission is reasonable in DAS as i) significant signaling overhead for power control can be avoided, and ii) very low fixed power transmission is possibly implemented at low-price, large number of DAs [3].

TABLE I
SIMULATION PARAMETERS USED IN PERFORMANCE EVALUATION

path-loss model		$-38+10\log_{10}(d_{k,k'}^{-\mu})$, $\mu=3$ [18]
network power		$P_{\mathrm{net}}=46$ dBm
AWGN power		$\sigma^2=-174$ dBm/Hz
system bandwidth		$\Omega=10$ MHz
full loading time		$\eta=12$ hours/day
revenue		$v_k\le$ 60 \$/10 GB [19]
system parameter set $\mathcal{S}=\{N,K,A,P\}$	# of DAs	$N\le 800$, depending on λ_d
	# of users	K depending on user density low density: $\lambda_{u,L}=50/\mathrm{km}^2$ high density: $\lambda_{u,H}=150/\mathrm{km}^2$
	area/site	$A=1\ \mathrm{km}^2$
	DA Tx pow.	$P=P_{\mathrm{net}}/K_\epsilon$
QoS set $\mathcal{Q}$	service class	$R_k=1$ bit/sec/Hz, $\forall k$, [19]
cost parameter set $\mathcal{C}\triangleq\{\alpha_0,\alpha_1,\gamma_1\}$	low	$\mathcal{C}_L=\{1000,500,0.9\}$ in Fig. 1
	medium	$\mathcal{C}_M=\{2000,1000,1\}$ in Fig. 1
	high	$\mathcal{C}_H=\{3600,1500,1.1\}$ in Fig. 1

III. PECUNIARY COST, PROFIT, AND PE MODELS

A. Pecuniary Cost Model

The pecuniary cost can be generally modeled as any type of function, for example, as fractional polynomials [7] given by $g(N,A;\boldsymbol{\alpha};\boldsymbol{\beta};\boldsymbol{\gamma})=\sum_{i=0}^{s}\alpha_i H_i(N,A)$, where $H_0(N,A)=1$, $\beta_0=\gamma_0=0$, and for $i=1,\dots,s$,

$$H_i(N,A)=\begin{cases}A^{\beta_i}N^{\gamma_i}, & \text{if } \gamma_i\neq\gamma_{i-1},\\ H_{i-1}(N,A)\ln(NA), & \text{if } \gamma_i=\gamma_{i-1}.\end{cases}$$

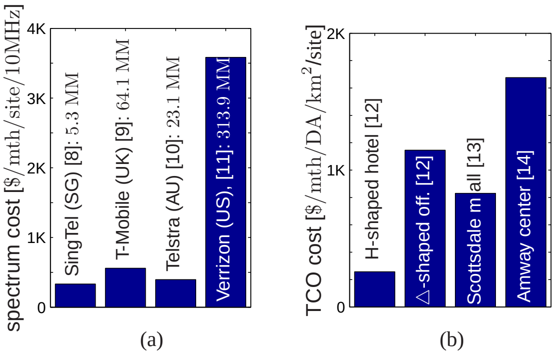
Here, $\boldsymbol{\alpha}=[\alpha_0\cdots\alpha_s]^T$ is a real-value linear weight vector; and $\boldsymbol{\beta}=[\beta_0\cdots\beta_s]^T$ and $\boldsymbol{\gamma}=[\gamma_0\cdots\gamma_s]^T$ are fractional polynomial power vectors for the cost, which are proportional to the area A, and the number of DAs N, respectively.

Since the CAPEX and OPEX are monotonically increasing functions over N and A[2], for simplicity, we set $s=1$ with $\boldsymbol{\alpha}=[\alpha_0\ \alpha_1]$, $\boldsymbol{\beta}=[0\ 1]$, and $\boldsymbol{\gamma}=[0\ \gamma_1]$, in this study, and obtain the simplified model with a cost parameter set $\mathcal{C}\triangleq\{\alpha_0,\alpha_1,\gamma_1\}$ as

$$\mathsf{COPEX}(N,A,\mathcal{C})=\alpha_0+\alpha_1 A N^{\gamma_1}. \tag{2}$$

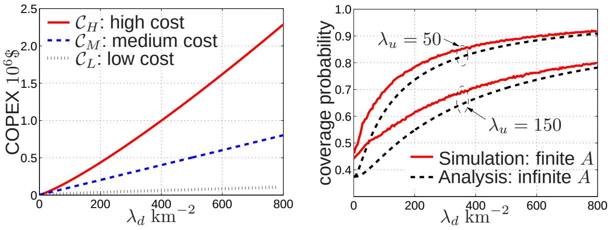
The cost parameters α_0, α_1, and γ_1 in $\mathcal{C}$ are precisely characterized as follows:

- α_0 [\$/mth/site/10 MHz]: $\alpha_0>0$ is a constant cost (in U.S. dollar \$) that is independent from N and A. It includes the cost for site/tower rent/acquisition, preparation work, advertisement, spectrum resource Ω Hz, and average electricity for fixed network power[3] P_{net}. The spectrum cost is dominant and shown in Fig. 1(a), which depends highly on population (million, MM) due to the auction-based pricing.
- α_1 [\$/mth/site/DA/km^2]: α_1 is determined according to the equipment, installation (laboring), and commissioning cost for attaching one DA per unit area (1 km^2), e.g., total cost of ownership (TCO) as shown in Fig. 1(b). Also, α_1 includes maintenance cost to operate DAs as well as the electricity expenditure regarding signal processing and transmission at each DA[4].
- γ_1 [no unit]: γ_1 is the power exponent depending on the network topology and architecture structure [12], e.g., $\gamma_1\le 1$ for star topology in simple architecture, while $\gamma_1>1$ for mesh topology in complex architecture.



(a) (b)

Fig. 1. Cost examples. (a) Spectrum cost in α_0. (b) TCO cost in α_1.



(a) (b)

Fig. 2. (a) COPEX with different parameters. (b) Coverage probability

The cost is averaged per month and normalized by the number of sites (10,000 LTE sites from [16]). The COPEX model (2) including CAPEX and OPEX is simple, yet it can be generally applied to various scenarios as shown in Fig. 2(a). The parameters $\mathcal{C}_L$, $\mathcal{C}_M$, and $\mathcal{C}_H$ are given in Table I.

B. Profit Model

In this subsection, we model a gross revenue function for DAS. If UE k's achievable rate is higher than its service class (or plan) R_k, the service provider obtains revenue v_k, and otherwise, no revenue is obtained, which is a conservative revenue model. Therefore, the expected gross revenue with a system parameter set $\mathcal{S}\triangleq\{N,K,A,P\}$ and a QoS set $\mathcal{Q}=\{Q_1,\dots,Q_{K_\epsilon}\}\triangleq\{2^{R_1}-1,\dots,2^{R_{K_\epsilon}}-1\}$ is derived as

$$\mathsf{REVENUE}(\mathcal{S},\mathcal{Q})=\boldsymbol{v}^T\left(\eta\Omega\boldsymbol{r}\odot\boldsymbol{p}\right)\ \$, \tag{3}$$

where $\boldsymbol{v}=[v_1\cdots v_{K_\epsilon}]^T\in\mathbb{R}^{K_\epsilon\times 1}$ is a revenue vector; the superscript T represents the vector/matrix transpose; v_k is the revenue for fixed amount data D_k of UE k, i.e., v_k \$/[D_k MB]; η is an effective full loading time (hours per day), which is obtained from a mobile traffic profile; $\odot$ represents element-wise product; $\boldsymbol{r}=[R_1\cdots R_{K_\epsilon}]^T\in\mathbb{R}^{K_\epsilon\times 1}$

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[2] The CAPEX is proportional to A as wider area costs more for wiring and extending the DAS. The OPEX is also proportional to A as, for example, maintenance cost increases as A increases.

[3] The network power is shared equally for all active DAs within a site and can be sustained through power grid or renewable energy.

[4] Overhead power consumption, e.g., from power amplifier's inefficiency and system's inefficiency [15], is captured by the electricity expenditure in α_1.

is a rate vector; and $\boldsymbol{p} = [p_1 \cdots p_{K_\epsilon}]^T \in \mathbb{R}^{K_\epsilon \times 1}$ is a coverage probability vector obtained from Lemma 1[5].

In (3), we assume that all UEs subscribe rationally a flat-rate mobile plan: if UE k expects to use D_k MB per month, he/she subscribes the plan providing D_k with payment associated to v_k \$, and otherwise, will subscribe other plan for more/less D_k yielding more/less v_k. Now, by subtracting COPEX (2) from the revenue (3), we get the profit of DAS as

$$\mathsf{PROFIT}(\mathcal{S}, \mathcal{Q}, \mathcal{C}) = \boldsymbol{v}^T \left(\eta \Omega \boldsymbol{r} \odot \boldsymbol{p}\right) - \alpha_1 A N^{\gamma_1} - \alpha_0 \ \$. \quad (4)$$

C. PE Models in Communications and Economics

We define PE in communications (PEcom) that is a *ratio of average achievable sum rate over gross pecuniary cost* as

$$\mathsf{PEcom}(\mathcal{S}, \mathcal{Q}, \mathcal{C}) = \frac{\Omega \boldsymbol{r}^T \boldsymbol{p}}{\mathsf{COPEX}(N, A, \mathcal{C})} \ \text{bits/\$},$$

where the return value is the average achievable rate in communications. Note that α_0 in $\mathcal{C}$ is proportional to Ω. The new metric PEcom can be used for service providers/operators to decide investment possibility by readily evaluating the required expenditure for target service quality, i.e.,

$$\text{required } \mathsf{COPEX} = \frac{\text{rate for the target service}}{\mathsf{PEcom}(\mathcal{S}, \mathcal{Q}, \mathcal{C})} \ \$.$$

In addition, we define a PE in economics (PEeco) that is a *ratio of gross revenue over gross pecuniary cost* as

$$\mathsf{PEeco}(\mathcal{S}, \mathcal{Q}, \mathcal{C}) = \frac{\mathsf{PROFIT}(\mathcal{S}, \mathcal{Q}, \mathcal{C})}{\mathsf{COPEX}(N, A, \mathcal{C})},$$

where the return value is profit, and thus, the PEeco is equivalent to the return on investment (ROI) in economics. The PEeco is directly used to evaluate or compare the efficiency of an investment for any type of wireless networks including DAS.

IV. PROBLEM FORMULATION AND UNDERSTANDING

A problem to find an optimal N^* that maximizes the revenue \$ of DAS is formulated for given area A and service classes $\mathcal{Q}$ as follows:

$$N^* = \arg\max_{N} \mathsf{obj}(\mathcal{S}, \mathcal{Q}, \mathcal{C}), \ \text{s.t.} \ K_\epsilon P \leq P_{\text{net}}, \quad (5)$$

where the objective function can be defined as one of the following metrics

$$\mathsf{obj}(\mathcal{S}, \mathcal{Q}, \mathcal{C}) \in \begin{cases} \mathsf{PROFIT}(\mathcal{S}, \mathcal{Q}, \mathcal{C}), \\ \mathsf{PEcom}(\mathcal{S}, \mathcal{Q}, \mathcal{C}), \\ \mathsf{PEeco}(\mathcal{S}, \mathcal{Q}, \mathcal{C}). \end{cases}$$

It is generally formidable to obtain an analytical solution of (5) due to the intractability of integral in the coverage probability (1), even for the specially simple cases when $\mu = 4$ or when there is no noise (see [17, (13)] and [17, (14)]). Nevertheless, the formulation in (5) with the closed-form expression of coverage probability allows us to obtain N^* numerically. Note that both net pecuniary cost (2) and expected gross profit (4) are functions of a single variable, i.e., the number of distributed antennas N. Therefore, we can readily find N^* through one-dimensional line search over integer N.

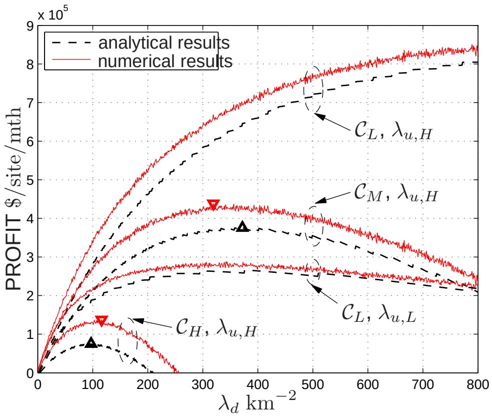

Fig. 3. Profit over DA density λ_d, equivalently the number of DAs.

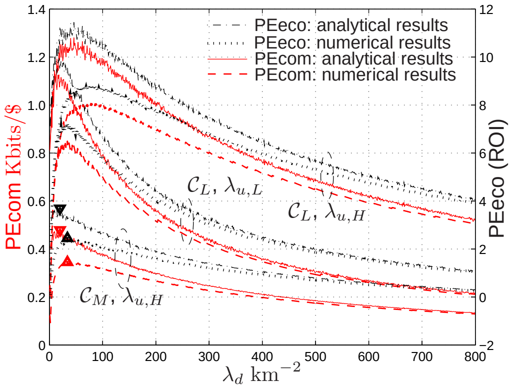

Fig. 4. PE, namely PEcom and PEeco, over DA density λ_d.

V. PERFORMANCE EVALUATION AND DISCUSSION

In this section, we evaluate the profit and PE of DAS in Figs. 3 and 4, respectively. The service area is set to be 1 km^2 (i.e., $A = 1$), the maximum number of DAs is limited by 800, and accordingly, the DA density $\lambda_d \leq 800 \text{ km}^{-2}$. The number of UEs in A corresponds to a UE density that is either low ($\lambda_{u,L} = 50 \text{ km}^{-2}$) or high ($\lambda_{u,H} = 150 \text{ km}^{-2}$). As described in Section II, the number of DAs is modeled as PPP with λ_d, while that of UEs is with $\lambda_{u,L}$ or $\lambda_{u,H}$. Since the distance $d_{k,k}$ between UE k and the allocated DA is typically short, the channels between DAs and UEs are realized following the parameters for small cell environment in [18]. The total transmit power of the network is bounded by P_{net}.

Other simulation parameters including the QoS set $\mathcal{Q}$ and cost parameter set $\mathcal{C}$ are given in Table I. Since the actual cost data for real-life DAS is limited and sometimes restricted to public, we set the cost parameters based on the survey in Fig. 1 by adding other extra expenditures, which are introduced in Section III-A. Note that the actual values of the parameters may vary with countries and operators, and finding them for

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[5] From Slivnyak's theorem [5], p_k in Lemma 1 for typical user k is applicable for all other users, $k \in \mathcal{K}$ assuming an infinite service region $A \to \infty$. Furthermore, as shown in Fig. 2(b), p_k obtained from Monte Carlo simulation (refer to Table I for the simulation parameters) matches well with our analytical result in Lemma 1 when $\lambda_d \to \infty$ for fixed A. Therefore, ϵ and (1) are asymptotically applicable to our study with finite A and high λ_d.

this study is out of scope of our work. Our key emphasis is on the insights from the proposed model and the new metric PE.

As shown in Fig. 3, the analytical and numerical results of the DAS profit match well within marginal discrepancy and have an identical trend. For numerical results, users and DAs are realized $1,000$ times independently, and each UE selects the nearest DA in each realization. If there are more than one UE choosing the same DA, only one UE will be selected for the communications. As expected, in general, the larger profit is obtained as either the user density increases or the cost parameter decreases, when the DA density is sufficiently larger than the user density. From the results, we see that bisection search can be used to effectively find the optimal DA density, i.e., the optimal number of DAs for given service area. For the case with $\{\mathcal{C}_M, \lambda_{u,H}\}$ and $\{\mathcal{C}_H, \lambda_{u,H}\}$, we mark the maximum PE points which are obtained from bisection search with analytical results (marked by $\triangle$) and numerical results (marked by $\triangledown$). As verified in simulation, the optimal density points based on analytical and numerical results match well with each other. Note that the variation around the optimal DA density (or equivalently the optimal DA number N^*) is not severe. For example, with $\{\mathcal{C}_M, \lambda_{u,H}\}$, the optimal DA number $N^* = 372$ from our analysis gives a near-optimal profit in the numerical result, which is obtained when $N^* = 319$.

In Fig. 4, we plot PEcom and PEeco with y-axes on both left and right side, respectively. Interestingly, the optimal DA density in terms of PE is different from that of profit. For example, $N^* = 319$ is optimal for the profit, while $N^* = 20$ is for PEs when $\{\mathcal{C}_M, \lambda_{u,H}\}$. The reason follows intuitively from the fact that the investment benefit remains with higher profit as long as PEeco (i.e., ROI) is greater than one. Similar to the profit, the larger PE is generally obtained as the user density increases, or the cost parameter decreases.

The mismatch between our analytical and simulation results in Figs. 3 and 4 is caused by our approximations that $\widehat{\Phi_d}$ is a homogeneous PPP and $\widehat{\lambda_d} = \epsilon\lambda_d$.

VI. CONCLUSION

In this letter, we have investigated and justified the economic benefit of large-scale DAS using stochastic geometry and a pecuniary cost model. The proposed PE model can be generally applied to any type of networks with actual cost and profit, which are evaluated by the operators. It thus offers a systematic and rigorous framework for DAS deployment decision. Results reported in this study provide a good reference to motivate DAS research and accelerate its implementation in real-life communications.

APPENDIX

Letting L be the distance between a typical user and an assigned DA from the *original DA PPP* Φ_d, the probability density function of the random variable L is given by [17]

$$f_L(\ell) = 2\pi\lambda_d e^{-\lambda_d \pi \ell^2} \ell. \tag{A.1}$$

Conditioning on the distance ℓ between a typical user k and nearest DA, the coverage probability p_k is derived as

$$\begin{aligned} p_k &= \int_0^\infty \Pr\left(\mathsf{SINR}_k \geq 2^{R_k} - 1 | \ell\right) f_L(\ell) d\ell \\ &\overset{(a)}{=} \int_0^\infty \mathbb{E}_{I_\ell}\Big[\exp\left(-\ell^\mu Q_k P^{-1}\left(I_k + \sigma^2\right)\right)\Big|\ell\Big] f_L(\ell) d\ell \\ &= \int_0^\infty \exp\left(-\ell^\mu Q_k \sigma^2 P^{-1}\right) \mathcal{L}_{I_\ell}\left(\ell^\mu Q_k P^{-1}\right) f_L(\ell) d\ell \\ &\overset{(b)}{=} \int_0^\infty \exp\left(-\ell^\mu Q_k \sigma^2 P^{-1} - \pi\epsilon\lambda_d \rho(\mu, Q_k)\ell^2\right) f_L(\ell) d\ell \end{aligned}$$

where $\mathsf{SINR}_k = \left|h_{k,k}\right|^2 P\ell^{-\mu}(I_k + \sigma^2)^{-1}$ is a signal-to-noise-plus-interference ratio of user k; $h_{k,k} \sim \mathcal{CN}(0,1)$ is the Rayleigh fading channel between user k and the assigned DA and $I_k = \sum_{k'=1,k'\neq k}^{K} \left|h_{k,k'}\right|^2 P d_{k,k'}^{-\mu}$ is the interference; $d_{k,k'}$ is the distance between user k and DAs allocated to other users. Here, (a) follows from the fact that I_ℓ is the cumulative interference of all other DAs *after thinning*, i.e., all DAs other than DA k which belong to the *thinned PPP* $\widehat{\Phi_d}$ with intensity $\lambda_u = \epsilon\lambda_d$; $\mathcal{L}_X(s)$ denotes Laplace transform of a random variable X; and (b) follows from [17, (22)]. Using (A.1) into p_k and substituting variable ℓ^2 by t, we obtain (1).

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