Adaptable Load Scheduling for Smart Grids

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***Abstract*—We propose a new concept of adaptable load scheduling by introducing the so-called adaptable load that can work as either a fixed load or a controllable load at different times of the day. Under the right conditions, rather than provide a fixed but unnecessary high electricity or energy to one fixed load while achieving a diminishing utility return, it is more worthwhile to share and schedule the same energy to more than one controllable loads. We expect that the total utility achieved by the latter case is likely to be higher than the former case with the same total energy consumption. Motivated by this purpose, we formulate an optimization problem that include the fixed loads, controllable loads, and the proposed adaptable loads. We want to optimally determine when each of the adaptable load should be in a fixed load state or a controllable load state, and how much energy consumption level should be scheduled to each of them at each time. New optimal solutions are derived for the optimal load states and energy consumption levels for the adaptable loads that maximize the total utility. The same optimization problem can also be solved using standard mathematical packages. Simulation results are presented to confirm that the proposed adaptable load scheduling scheme achieves a higher total utility than the energy scheduling scheme without adaptable load.**

I. INTRODUCTION

In general, recent research works on the energy scheduling for residential users and pricing control for smart grids can be categorized as either from the energy provider's or the users' perspective. From the energy provider's perspective, several prior studies on energy scheduling and real-time pricing control for multiple residential users have adopted the objective that maximizes the total social welfare across the users [1] and the energy provider [2] [3] [4]. From users' perspective, the commonly adopted objective is either by maximizing the user's utility, minimizing the total electricity bill or waiting time based on predicted electricity pricing. The common goal is to enable proper energy scheduling and pricing control, by adapting to the energy consumption patterns of various electrical appliances or devices, while maximizing users' utility or welfare and minimizing their electricity bills.

In [5], the authors study distributed energy scheduling to multiple cooperative users by formulating the energy scheduling problem using game-theoretic approach. By modeling the utility as the logarithm function, a water-filling based energy scheduling scheme which requires centralized computation is proposed in [1]. In [2] [6], the authors maximize the social welfare, which is defined in terms of the difference between the total utility and the total cost of electricity of all users. More recently, distributed energy scheduling or load control problem for multiple users with energy storage is studied in [4] for the scenario of incomplete advance metering infrastructure messages.

In all of the above works on energy scheduling, the loads at the demand-side users are generally classified as fixed loads and controllable loads.

- Fixed loads include critical loads are basic and essential loads, and other nonelastic loads that cannot be shifted.
- Controllable loads that can be further divided into interruptible loads and deferrable loads that can be turned off or be scheduled to a later time, respectively.

Based on the above load classification, only the energy consumption level and time schedule of the controllable load can be changed or shifted. However, the benefits that the user receives in terms of utility or welfare is likely to be limited, especially if there is less number of controllable loads or they consume less energy.

In this paper, we propose a new concept of adaptable load scheduling by introducing the so-called adaptable load that can work as either a fixed load or a controllable load at different times of the day. Throughout this paper, we shall use the terminology of adaptable loads to refer to loads or appliances that have two load states (either fixed or controllable). We derive optimal solutions that determine the optimal load states (either fixed or controllable) and energy consumption levels for the adaptable loads that maximize the total utility. Numerical and simulation results are presented to show that adaptable load scheduling scheme can achieve a higher total utility.

II. ADAPTABLE LOAD SCHEDULING: A NEW PROPOSAL

In this section, we present the concept of adaptable load scheduling and explain how it enables the users to achieve a higher total utility. We first explain the motivations behind this works as follows.

1) Adaptable load classification for optimal energy scheduling: In general the classification of loads into fixed and controllable loads are not as clear cut as has been done in prior works. A typical example is the air-conditioning units in office buildings, which are normally unavailable for energy scheduling during a sunny or hot working hours, and should be set as fixed loads. However, the same air-conditioning units can also be treated as controllable load during cold hours, or as an interruptible load during weekends and holidays, because the energy demand for cooling is less during such periods. The same reasoning is applicable to other appliances or loads like heaters or refrigerators. Based on such observation, we propose a new concept of adaptable load scheduling, which jointly optimizes the adaptable load classification and optimal energy scheduling. An adaptable load can be in either a fixed state with a fixed time schedule and energy consumption level, or a controllable state with a controllable time schedule and energy consumption level, depending on which state gives a higher total utility, as well as the other conditions such as users' requirements.

2) Potential for a higher maximum total utility: We expect that a demand-side user can achieve a higher total utility if the same high power that is supposed to be used by one fixed load can be scheduled or shared with another or more controllable loads. This observation is derived from our understanding on the diminishing property of the utility with respect to the increasing energy consumption. We speculate that given the appropriate conditions, rather than provide a fixed but unnecessary high energy to one fixed load while achieving a diminishing utility return, it is more worthwhile to split and schedule the same total energy to more than one controllable loads. The proposed adaptable load is equipped with such an option to optimize between a fixed and a controllable state. The total utility achieved by several controllable loads, each with a lower energy consumption, will likely to be higher than the single fixed load with the same total energy consumption.

The cooling or heating energy consumption constitute a significant percentage of the total energy consumption of any household, or building. Due to this reason, the amount of electricity or energy that can be optimized through adaptable load scheduling to achieve a higher total utility can be very significant.

A. Adaptable load classification

In Figure 1, we show the commonly used load classification approach using the fixed load, controllable load, and interruptible load. Once a load is being specified as a particular load class, it cannot switch to become another class of load.

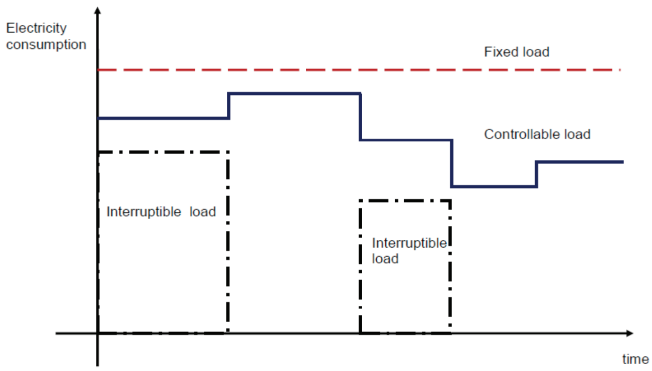

Fig. 1. Commonly used load classification method using the fixed load, controllable load, and interruptible load. Once a load is being specified as a particular load class, it cannot switch to become another class of load.

In Figure 2, we use the air-conditioning unit as an example to illustrate the proposed adaptable load scheduling concept. An adaptable load is given the option to switch between more than one states. In this particular example, the air-conditioner is in a fixed load state during the hot hours, but it is in a controllable state during the cold hours.

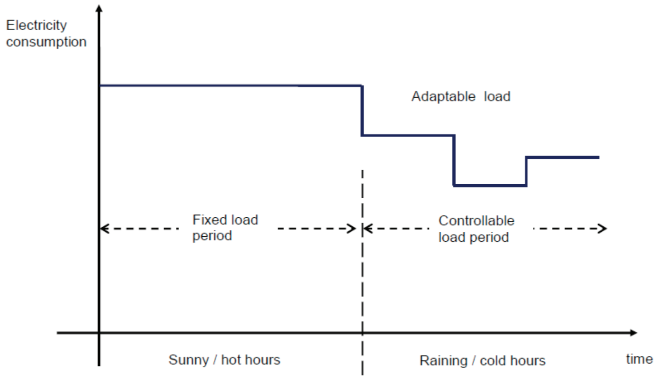

Fig. 2. Proposed adaptable load classification method in this work by using the air-conditioner as an example. An adaptable load has the option to switch between more than one class of load.

To facilitate the implementation of our proposed scheme in later sections, we consider that the loads are classified into three classes of load: fixed loads, controllable loads and adaptable loads,

1) Fixed loads that cannot be shifted, interrupted or controlled in any circumstance.
2) Controllable loads that can be shiftable or interruptible whenever the operating conditions allow it to be shifted or interrupted, or whenever there is a need.
3) Adaptable loads that can be in a either fixed or controllable state depending on the operating conditions, such as weather condition in the case of air-conditioners.

Throughout this paper, we use the term adaptable load scheduling to cover the following:

1) Under normal conditions, the adaptable loads are classified and treated as fixed loads by the proposed scheme. A given example is the air-conditioning units operating during a sunny and hot working hours. Only the controllable loads are subject to optimal energy scheduling.
2) Under favorable conditions, the adaptable loads are classified and treated as controllable loads by the proposed scheme. A given example is the air-conditioning units operating during a raining and cold hours. Both controllable loads and adaptable loads are subject to optimal energy scheduling.

We expect that when the adaptable loads switch to become interruptible or controllable loads more often, the utility or welfare that the user receives is also increased.

III. System Model

We consider a time period (for example, within a day) $\mathcal{T} = \{0, 1, \ldots, |\mathcal{T}|-1\}$ which is divided into $|\mathcal{T}|$ time slots of equal interval. The electricity or energy scheduling is stationary over at least one time slot interval. Assume there are $\mathcal{K}$ users who draw electricity from the smart grid to support their own set of appliances. In order to introduce the concept of adaptable load scheduling as presented in previous sections, we shall categorize the loads into 3 classes of loads: fixed loads $\mathcal{M}$, shiftable loads $\mathcal{N}$, and adaptable loads $\mathcal{L}$, and $\mathcal{A} = \mathcal{M} \cup \mathcal{N} \cup \mathcal{L}$ denotes the total set of loads. Let x_{klt} be the energy consumption of user $k \in \mathcal{K}$'s appliance $a \in \mathcal{A}$ at time slot $t \in \mathcal{T}$.

Note that for each appliance, the index a can be m, n, or l depending on its purpose or electricity consumption pattern. For each fixed load x_{kmt}, we further introduce its total energy consumption E_{km} and working time duration $\tau_{km} = t^e_{km} - t^s_{km}$. Therefore we get $x_{kmt} = \frac{E_{km}}{\tau_{km}}$ for $t \in [t^s_{km}, \ldots, t^e_{km}]$. For each controllable load x_{knt}, its electricity consumption at each time slot is limited by a maximum value $x^{\max}_{kn}$.

An adaptable load x_{klt} can be in either a fixed state with a fixed energy scheduling $\bar{x}_{klt}$, or a controllable state with a controllable energy scheduling $\tilde{x}_{klt}$, depending on which state gives a higher total utility. To model an adaptable load with two states (fixed or controllable), we define the state index, $z_{klt} \in \{0, 1\}$, that is associated with the k-th user's l-th adaptable load at the t-th hour,

$$z_{klt} = \begin{cases} 1; & \text{it becomes a fixed load,} \\ 0; & \text{it becomes a controllable load.} \end{cases} \quad (1)$$

In addition, we model the response of each user's appliance or device using the utility function [2] [6], that provides a reasonable indication of user's satisfaction. Our derivations and results throughout this paper are applicable to any valid utility function, denoted by $U_{klt}(x_{klt}, z_{klt})$, which is assumed to be a non-decreasing function of x_{klt}, and its first derivative is also non-decreasing.

To provide incentives to promote user cooperation in energy usage, the objective function is defined as the total utility from all users' appliances over the time period $\mathcal{T}$,

$$U(\boldsymbol{X}_k, \boldsymbol{Z}_k, \forall k \in \mathcal{K}) = \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{M}\cup\mathcal{N}\cup\mathcal{L}} U_{klt}(x_{klt}, z_{klt}). \quad (2)$$

For the k-th user, $\boldsymbol{X}_k$ is a matrix consists of the elements x_{klt}, whereas $\boldsymbol{Z}_k$ is a matrix consists of the elements z_{klt}, $l \in \mathcal{M} \cup \mathcal{N} \cup \mathcal{L}$.

IV. Optimization of Adaptable Load Scheduling

In this section, we formulate the problem by incorporating the concept of adaptable load scheduling. Our objective is to maximizing the total utility in (2). The optimal solutions for the adaptable state index, z_{klt}, and the energy scheduling x_{knt} are derived.

A. Problem Formulation

We formulate the problem as follows,

[Max_Utility]

$$\begin{aligned}
\max \quad & U(\boldsymbol{X}_k, \boldsymbol{Z}_k, \forall k \in \mathcal{K}) \\
\text{subject to} \quad & \sum_k \sum_m x_{kmt} + \sum_k \sum_n x_{knt} \\
& + \sum_k \sum_l z_{klt} \bar{x}_{klt} \\
& + \sum_k \sum_l (1 - z_{klt}) \tilde{x}_{klt} \le G_t, \quad \forall t \\
& \sum_{t=t^s_{km}}^{t^e_{km}} x_{kmt} = E_{km}, \quad \forall k, m \\
& \sum_{t=t^s_{kl}}^{t^e_{kl}} \bar{x}_{klt} = E_{kl}, \quad \forall k, l \\
& 0 \le x_{knt} \le x^{\max}_{knt}, \quad \forall k, n, t \\
& 0 \le \tilde{x}_{klt} \le x^{\max}_{klt}, \quad \forall k, l, t \\
& z_{klt} \in \{0, 1\}. \quad \forall k, l
\end{aligned}$$

The above problem formulation is a mixed integer programming problem because both binary variables z_{klt} and real variables x_{knt} are involved. It is difficult to solve due to the integer constraints which involves z_{klt}.

Given that $x_{kmt} = \frac{E_{km}}{\tau_{km}}$ and $\bar{x}_{klt} = \frac{E_{kl}}{\tau_{kl}}$ and each of their sum is fixed as in the second and third constraints, the x_{kmt} and $\bar{x}_{klt}$ can be treated as constants. Therefore, without loss of generality, we do not need to consider the second and third constraints in the subsequent optimization because they do not affect the final optimization results.

Next, we relax the last (integer) constraint in **Max_Utility** by using box constraints. The original formulation in Problem **Max_Utility** can be simplified as:

[Relax_Max_Utility]

$$\begin{aligned}
\max \quad & U(\boldsymbol{X}_k, \boldsymbol{Z}_k, \forall k \in \mathcal{K}) \\
\text{subject to} \quad & \sum_k \sum_n x_{knt} + \sum_k \sum_l (1 - z_{klt}) \tilde{x}_{klt}, \\
(\lambda_t) \quad & + \sum_k \sum_l z_{klt} \bar{x}_{klt} \\
& \le G_t - \sum_k \sum_m x_{kmt}, \quad \forall t \\
& 0 \le x_{knt} \le x^{\max}_{knt}, \quad \forall k, n, t \\
& 0 \le \tilde{x}_{klt} \le x^{\max}_{klt}, \quad \forall k, l, t \\
(\mu_{kl}) \quad & z_{klt} \ge 0, \quad \forall k, l \\
(\nu_{kl}) \quad & z_{klt} \le 1, \quad \forall k, l
\end{aligned}$$

Note that λ_t, μ_{kl}, and ν_{kl} in parentheses on the left are nonnegative dual variables associated with their respective constraints on the right.

Next, we present a dual analysis on the Problem **[Relax_Max_Utility]**, and derive the optimal solutions.

B. Derivations of optimal solutions for adaptable load scheduling

The Lagrangian of **Relax_Max_Utility** can be written as

$$\begin{aligned} \mathbf{L} &= U(\boldsymbol{X}_k, \boldsymbol{Z}_k, \forall k \in \mathcal{K}) - \sum_t \lambda_t \Big(\sum_k \sum_n x_{knt} \\ &+ \sum_k \sum_l z_{klt} \bar{x}_{klt} \\ &+ \sum_k \sum_l (1 - z_{klt}) \tilde{x}_{klt} - G_t + \sum_k \sum_m x_{kmt} \Big) \\ &+ \sum_k \sum_l \mu_{kl} z_{klt} - \sum_k \sum_l \nu_{kl} (z_{klt} - 1) \end{aligned} \quad (3)$$

1) Optimal solution for energy scheduling: By substituting (2) into (3) and evaluating $\frac{\partial \mathbf{L}}{\partial x_{klt}} = 0$, we obtain the following conditions that can be used to solve for optimal x_{knt} of the shiftable loads,

$$\frac{\partial U_{klt}(x_{klt}, z_{klt})}{\partial x_{klt}} = \lambda_t. \quad (4)$$

Likewise for optimality $\tilde{x}_{klt}$ of the adaptable loads,

$$\frac{\partial U_{klt}(x_{klt}, z_{klt})}{\partial \tilde{x}_{klt}} = \lambda_t (1 - z_{klt}). \quad (5)$$

Note that an adaptable load can either be in a fixed state ($z_{klt} = 1$) or in a controllable state($z_{klt} = 0$), but only when it is in the controllable state its energy consumption $\tilde{x}_{klt}$ can be optimized. Therefore, the optimal $\tilde{x}_{klt}$ of the adaptable loads can be solved from the following conditions,

$$\frac{\partial U_{klt}(x_{klt}, z_{klt})}{\partial \tilde{x}_{klt}} = \lambda_t, \quad \text{when} \quad z_{klt} = 0. \quad (6)$$

The left hand side of (4) to (6) are the marginal benefit function. The Lagrange multiplier λ_t can be interpreted as the price per unit (kWh) energy at time slot t as charged by the energy provider. These conditions for optimality has the following intuitive interpretation: To maximize the utility, the optimal energy of shiftable loads x^*_{knt} and adaptable loads $\tilde{x}^*_{klt}$ should make the marginal benefit equal to the price at each time slot t. Given any utility function, the optimal x^*_{knt} and $\tilde{x}^*_{klt}$ can be solved directly from (4) and (5).

2) Optimal solution for the adaptable load state: Let $B = \partial U_{klt}(x_{klt}, z_{klt}) / \partial z_{klt}$ denote the marginal benefit obtained by adapting from a controllable state to a fixed state, and let Δ denote the incremental price to adapt from a controllable state to a fixed state,

$$\Delta = \sum_t \lambda_t (\bar{x}_{klt} - \tilde{x}_{klt}). \quad (7)$$

The optimal solutions for z_{klt} should satisfy $\frac{\partial \mathbf{L}}{\partial z_{klt}} = 0$, from which we derive the optimal solution for z_{klt} of the adaptable load as follow,

$$z^*_{klt} = \begin{cases} 1; & \text{(in a fixed state) if } B > \Delta, \\ 0; & \text{(in a controllable state) if } B < \Delta. \end{cases} \quad (8)$$

The solution in (8) provides the following interpretation: If adapting to a fixed state brings a marginal benefit which is higher than the incremental price, the adaptable load should be in a fixed state ($z_{klt} = 1$). Otherwise, if being in a fixed state brings a marginal benefit which is lower than the incremental price, the adaptable load should be in a controllable state ($z_{klt} = 0$).

PROOF. The optimal solution for z_{klt} should satisfy $\frac{\partial \mathbf{L}}{\partial z_{klt}} = 0$. We obtain the following condition for optimal z_{klt},

$$\frac{\partial U_{klt}(x_{klt}, z_{klt})}{\partial z_{klt}} = \sum_t \lambda_t (\bar{x}_{klt} - \tilde{x}_{klt}) - \mu_{kl} + \nu_{kl}. \quad (9)$$

To derive the solution of the state index z_{klt}, we first determine the activeness of the constraint $z_{klt} \leq 1$ in **Relax_Max_Utility** while ignoring the other constraint $z_{klt} \geq 0$. Therefore, we can set the Lagrange multiplier $\mu_{kl} = 0$ in Eqn. (3), and Eqn. (9), which gives,

$$\frac{\partial U_{klt}(x_{klt}, z_{klt})}{\partial z_{klt}} - \sum_t \lambda_t (\bar{x}_{klt} - \tilde{x}_{klt}) = \nu_{kl} \geq 0. \quad (10)$$

By applying the condition of complementary slackness, we have the following condition for optimal z_{klt} (≤ 1),

$$\frac{\partial U_{klt}(x_{klt}, z_{klt})}{\partial z_{klt}} > \sum_t \lambda_t (\bar{x}_{klt} - \tilde{x}_{klt}), \quad (11)$$

$$\text{or } B > \Delta \Rightarrow z_{klt} = 1, \quad (12)$$

which implies that the adaptable load should be in a fixed state ($z_{klt} = 1$) if the marginal benefit $B > \Delta$.

In a similar way, we can determine the activeness of the constraint $z_{klt} \geq 0$ in **Relax_Max_Utility** while ignoring the other constraint $z_{klt} \leq 1$. For this case, we obtain the following condition for optimal z_{klt} (≥ 0),

$$\frac{\partial U_{klt}(x_{klt}, z_{klt})}{\partial z_{klt}} < \sum_t \lambda_t (\bar{x}_{klt} - \tilde{x}_{klt}), \quad (13)$$

$$\text{or } B < \Delta \Rightarrow z_{klt} = 0, \quad (14)$$

which implies that the adaptable load should be in a controllable state ($z_{klt} = 0$) if the marginal benefit $B < \Delta$.

By combining the results in (12) and (14), we arrive at the solution in (8).

V. EXAMPLES AND NUMERICAL RESULTS

In this section we present an example to illustrate how the adaptable load scheduling can achieve a higher total utility.

For simplicity, we consider 3 loads which require a proper energy scheduling scheme over a 4-time slot period. The first load is a fixed load with energy consumption pattern given by $\{1, 1, 2, 0\}$, which means that it consumes 1 kWh, 1 kWh, 2 kWh, and 0 kWh in 4 consecutive time slots. The second load is the proposed adaptable load which can either be fixed or controllable, and the third load is a shiftable load. Both the second and third loads are with the same energy consumption pattern given by $\{0, 1, 1, 2\}$. Moreover, the total of these 3 loads has to meet the maximum total energy generation capacity of 3 kWh per time slot. The utility function is $U(x) = 2x - \frac{x^2}{2}$, and therefore its marginal benefit function is a linear function given by $\frac{\partial U}{\partial x} = 2 - x$.

To highlight the how the proposed adaptable load scheduling can achieve a higher total utility, we present and compare the following two cases:

- Case 1: Without adaptable load scheduling. The second load is set as a fixed load.
- Case 2: With adaptable load scheduling. The second load is an adaptable load.

Case 1: Without adaptable load scheduling. The second load is set as a fixed load.

The energy scheduling results are shown in Table I. Note that because the second load is now set as a fixed load, the shiftable (third) load has to be shed in the second time slot in order to meet the maximum total energy generation capacity constraint.

TABLE I
ENERGY SCHEDULING RESULTS WITH THE SECOND LOAD BEING SET AS A FIXED LOAD.

Time slot	0	1	2	3
fixed load a	1	1	2	0
fixed load b	0	1	1	2
shiftable load	0	1	0	1

The total utility in this case is $U = 1.5 \times 6 + 2 \times 2 = 13$.

Case 2: With adaptable load scheduling. The second load is an adaptable load.

This case corresponds to the proposed adaptable load scheduling scheme, and the energy scheduling results are shown in Table II.

TABLE II
ENERGY SCHEDULING RESULTS WITH THE SECOND LOAD BEING AN ADAPTABLE LOAD.

Time slot	0	1	2	3
fixed load	1	1	2	0
adaptable load	0	1	0.5	1.5
shiftable load	0	1	0.5	1.5

Note that because the adaptable load is now set to be controllable, both of the adaptable load and shiftable load can be shed if necessary in order to meet the maximum total energy generation capacity constraint.

In this case, the total utility achieved is $U = 0.875 \times 2 + 1.5 \times 4 + 1.875 \times 2 + 2 \times 1 = 13.5$, which is higher than the Case 1.

This is because a higher energy consumption very often results in a diminishing return in terms of utility. We observe that in the 3rd time slot of Case 2, the total utility that is achieved by the adaptable load and shiftable load, each with 0.5 kWh, is higher than the single second fixed load in Case 1 with the same total (1 kWh) energy consumption. Similar observation hold for the second load in the 4th time slot.

Therefore, the proposed adaptable load scheduling has a potential to achieve a higher total utility whenever there is a diminishing return in terms of utility at higher energy consumption. In the next section, we present simulation results to provide a more realistic assessment of the proposed concept.

VI. SIMULATION RESULTS AND DISCUSSIONS

We present simulation results to give further insights into the proposed load selection and energy scheduling scheme. The scheduling interval follows half-hour time slots. The time horizon in the simulation is 24 hours, corresponding to 700 am, 7.30 am, 8.00 am, 8.30 am, etc., until 6 am of the next day. The maximum total energy generation capacity is 30 kWh per time slot unless specified otherwise. The optimization is solved as the Mixed Integer Quadratic Programming (MIQP) problem using the CVX and Gurobi optimization packages.

TABLE III
PARAMETERS USED IN THE SIMULATIONS. THE UNIT kWh IS KILOWATT HOURS.

	Parameters generated for the simulations
x_{klt}^{max}(kWh)	Uniform on {0.5, 0.6, ..., 1.5}
$\bar{x}_{klt}$(kWh)	$0.75x_{klt}^{max}$
t_{kl}^{s}	Uniform on {7am, 8am, ..., 6pm}
t_{kl}^{e}	Uniform on {t_{kl}^{s}, ..., min(t_{kl}^{s}+5, 24)}
ω_{klt}	$\alpha_{ka}x_{km}^{max} + \delta$, $\delta \in$ Uniform on {0,1}
α_{klt}	Uniform on {0, 1}

In order to compute numerical results, we consider a specific utility function [2] [7],

$$U_{klt} = \begin{cases} \omega_{klt}x_{klt} - \frac{\alpha_{klt}}{2}x_{klt}^2 & 0 \leq x_{klt} \leq \frac{\omega_{klt}}{\alpha_{klt}} \\ \frac{\omega_{klt}^2}{2\alpha_{klt}} & x_{klt} \geq \frac{\omega_{klt}}{\alpha_{klt}} \end{cases}. \quad (15)$$

The value of ω_k is private information of the customer and are unknown to the utility. The α_{kn} is a constant that determines the saturation point of the utility for the n-th device, where a higher α_{kn} implies a lower power to reach the saturation point.

Its marginal benefit that is described by a linear function of x_{klt},

$$\frac{\partial U_{klt}(x_{klt}, z_{klt})}{\partial x_{klt}} = \begin{cases} \omega_{klt} - \alpha_{klt}x_{klt} & 0 \leq x_{klt} \leq \frac{\omega_{klt}}{\alpha_{klt}} \\ 0 & x_{klt} > \frac{\omega_{klt}}{\alpha_{klt}} \end{cases} \quad (16)$$

In Figure 3, we compare the total utility versus the number of loads for the case of fixed load and adaptable load scheduling. As expected, the total utility for both cases increase linearly with the total number of loads. As explained in the previous example, the proposed adaptable load scheduling scheme always achieve a higher total utility than the fixed load scheduling, no matter what the total number of loads.

The effect of various percentage of adaptable loads on the total utility is highlighted in 4. The 0% on the x-axis corresponds to the case where all the loads are fixed, whereas the 100% on the x-axis corresponds to the case where all the loads are subject to adaptable load scheduling. This result confirm our speculation that a higher percentage of adaptable loads can indeed achieve a higher total utility. In this simulation setup, the total utility is increased by 23% if the percentage of adaptable load is increased from 0% to 100%.

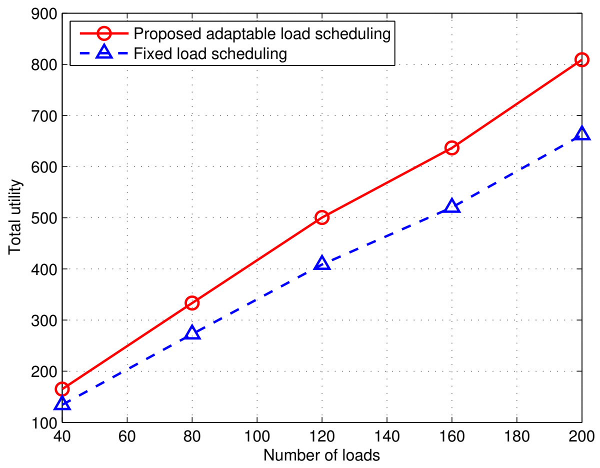

Fig. 3. Comparison on the total utility versus the number of adaptable loads and fixed loads.

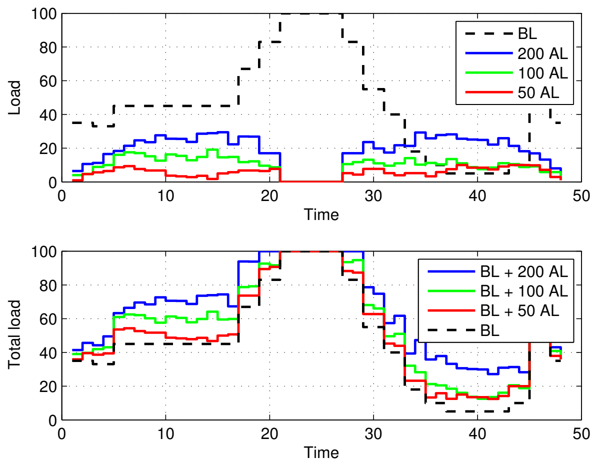

Fig. 5. Optimal adaptable load and energy scheduling at different hours for the simulation setup with 50 residential users. Top figure: Sum of adaptable loads (AL). Bottom figure: Sum of base load (BL) and adaptable loads. The maximum total energy generation capacity is 100 kWh per time slot.

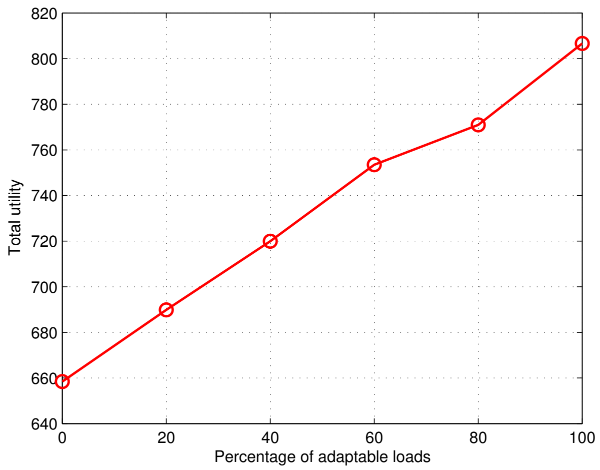

Fig. 4. Comparison on the total utility versus the percentage of adaptable loads.

In Figure 5, we present energy scheduling results for different hours of the day, that have been optimized by the adaptable load scheduling scheme. The base load (BL) line is plotted in both figures as a common reference. The top figure is plotted for various number of adaptable loads (AL) only without the base load. The bottom figure is plotted using the sum of base load and various number of adaptable loads. As expected, the adaptable loads are scheduled mostly at the time periods when the base load is low, due to the relatively low electricity demand and low electricity price at those periods. In addition, with the increase in the number of adaptable loads, the proposed adaptable load scheduling scheme enables a higher total energy consumption only during the low base load periods.

VII. Conclusions

We have presented optimal solutions for the optimal load states and energy consumption levels for the proposed adaptable loads that maximize the total utility. Based on the optimal solutions, if adapting to a fixed state brings a marginal benefit which is higher than the incremental price, the adaptable load should be scheduled in according to a fixed load. Otherwise, the adaptable load should be scheduled in according to a controllable load. Simulation results show that a higher percentage of adaptable loads can indeed achieve a higher total utility. Moreover, the adaptable loads are scheduled mostly at the time periods with relatively low electricity demand and low electricity price.

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