

Barrier regularization for nonlinear isogeometric topology optimization with large mesh distortion

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Abstract:

In topology optimization involving large nonlinear deformations, excessive mesh distortion in void regions often leads to ill-conditioned stiffness matrices and convergence failure. In isogeometric topology optimization (ITO), this issue is further exacerbated by the non-zero support of NURBS basis functions across multiple knot spans (elements): (i) distortion in voids can cause unrealistic deformation in neighboring solid regions, and (ii) conventional remedies for mesh distortion, built on C^0 inter-element continuity, are not readily applicable. This paper introduces a barrier regularization method to robustly stabilize nonlinear ITO under extreme mesh distortion. The proposed regularization term penalizes the system's potential energy at quadrature points where the Jacobian degenerates and the element tends to invert, providing a geometrically actuated and computationally simple mechanism for suppressing element irregularities. The method is first validated using isogeometric analysis on a C-shaped beam example, demonstrating improved mesh quality and enhanced numerical stability. It is subsequently applied to several benchmark problems (including a cantilever beam, a compliant mechanism, and a classical FEM-based case),

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exhibiting strong mesh control, robust optimization performance under diverse loading conditions, and seamless extensibility to FEM frameworks. The results verify the method's effectiveness for large-deformation topology optimization, its broad applicability across different types of basis functions, and its ease of implementation within the standard optimization workflow.

Keywords: Nonlinear; Isogeometric topology optimization; Barrier regularization; Large deformation; Mesh distortion

1. Introduction

1.1 Background

Topology optimization, as a material distribution design tool, has been widely adopted to enhance the performance of various engineering structures [1]. However, most existing topology optimization studies are based on linear mechanical assumptions, making it difficult to accurately capture the nonlinear behaviors found in many engineering applications. This often results in optimized designs that deviate from actual responses or even fail under operational conditions. With the increasing demands for structural performance in advanced fields such as aerospace [2] and 4D printing [3], the critical role of nonlinear mechanical effects in topology optimization has become increasingly significant [4].

Current research primarily focuses on three categories of nonlinearities: material nonlinearity, geometric nonlinearity, and coupled material and geometric nonlinearities. Material nonlinearity [5-8] refers to a nonlinear constitutive response, typically characterized by a non-constant material tangent modulus, even when the strain remains small. In contrast, geometric nonlinearity [9-13] arises from nonlinear kinematic effects, with large displacements and rotation, regardless of whether the material model itself is linear or nonlinear. Coupled material and geometric nonlinearities [14-19] occur when both nonlinear material behavior and finite deformation effects are simultaneously considered. Despite these advancements, traditional FEM still face significant challenges in large deformation nonlinear analysis, including imprecise geometric descriptions, sensitivity to mesh distortion, and low computational efficiency.

1 These limitations mark the need for a unified, efficient, and stable numerical framework
2 to facilitate nonlinear topology optimization applications.

3 The emergence of isogeometric analysis (IGA) [20], offers a unified framework
4 for geometry and analysis by employing spline basis functions, thereby enhancing
5 accuracy and robustness in nonlinear structural simulations [21]. With the growing
6 adoption of IGA, isogeometric topology optimization (ITO) has gained attention [22,
7 23]. Seo et al. [24] first applied trimmed splines in ITO, and Qian [25] introduced a B-
8 spline density method to suppress numerical artifacts. While early studies focused on
9 linear problems, recent works have extended ITO to elastoplasticity and geometric
10 nonlinearity [26-28] Nevertheless, under extreme deformations, ITO still suffers from
11 mesh distortion and convergence failure, limiting its application in complex nonlinear
12 scenarios.

13 *1.2 Mesh distortion in large-deformation nonlinear topology optimization*

14 Scherz et al. [29] attributed this numerical instability primarily to the formation of
15 ill-conditioned global stiffness matrices under large deformation conditions.
16 Specifically, during geometric nonlinear modeling, iterative solvers often encounter
17 convergence difficulties due to severe ill-conditioning of the stiffness matrix, ultimately
18 leading to analysis failure. To address this issue, researchers have proposed various
19 solutions, including: (1) **Relaxed convergence criteria method** [30, 31] alleviates non-
20 convergence issues by ignoring degrees of freedom surrounded by void elements,
21 though it may increase solver iterations or even cause solution divergence. (2) **Element**
22 **[32, 33] and redundant degree of freedom removal method** [34, 35] effectively
23 eliminates degrees of freedom in low-density regions that may cause singular stiffness
24 matrices, thereby improving numerical stability. However, this approach may overlook
25 physically relevant elements and introduce solution discontinuities. (3) **Element**
26 **connectivity parameterization method** [36, 37] offers an alternative approach using
27 virtual spring stiffness modeling in a non-density framework, though it cannot
28 guarantee physical accuracy of results. Similarly, (4) **artificial stiffness method** [38,
29 39] avoids numerical instability by artificially raising the stiffness lower bound in void
30 regions, but may compromise the physical fidelity of the optimization problem. (5)

Polyconvex constitutive model method [40, 41] applies hyperelastic material models to topology optimization. For larger strains, this constitutive model type generally demonstrates better numerical stability, though its applicability is limited to specific material models. Subsequent research has primarily focused on hyperelastic materials. (6) **Artificial enhancement methods**, including additive hyperelastic techniques [42] and element deformation control techniques [43], can effectively stabilize the analysis process. (7) **The third medium method** [44, 45] stabilizes low-density elements by adding void regularization terms to the hyperelastic strain energy potential of the void material. Additionally, (8) Wang et al. [46] proposed an elegant and effective energy interpolation method that significantly improves numerical stability by interpolating the elastic energy density in the solid and void regions. Owing to its conceptual clarity and practical robustness, this approach has been widely adopted and further extended in subsequent studies [47-50].

1.3 Challenge for adopting FEM-based solutions to isogeometric frameworks

Element or nodal removal strategies are generally not applicable to IGA due to the non-local support of its basis functions (e.g., NURBS), where each control point influences a region spanning multiple elements. This makes it difficult to directly assign distinct material states (e.g., solid or void) at the element level, unlike in conventional FEM (see Fig. 1). In addition, severe fluctuations of the Jacobian within elements may degrade numerical integration accuracy and lead to ill-conditioned tangent stiffness matrices, significantly affecting convergence behavior and topological evolution paths. Certain numerically driven approaches, such as the energy interpolation method and the third medium method, can be adapted to IGA, as they operate at the formulation level rather than relying on strict element-wise material assignments. However, these strategies typically improve stability by introducing artificial stiffness into void regions. While this enhances convergence, it remains a numerical remedy that may compromise variational consistency. This motivates the exploration of geometry-driven strategies that directly regulate element distortion, offering a more consistent foundation for nonlinear IGA.

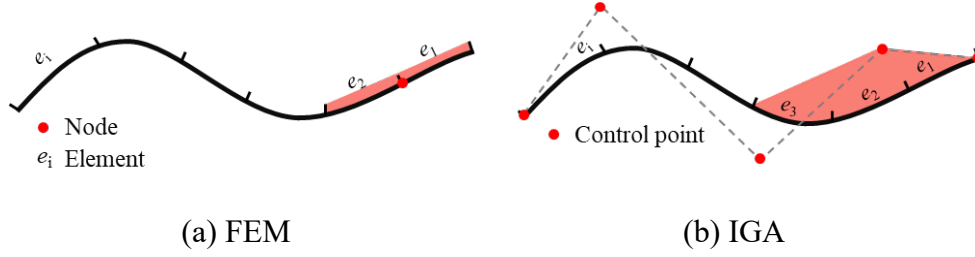


Fig. 1 The influence of nodes or control points on the element under the FEM and IGA frameworks.

To address these limitations from a geometric perspective, recent works have explored mesh regularization strategies that act directly on element shape quality and geometric mappings. Sastry et al. [51] proposed a log-barrier interior point method for mesh quality improvement and untangling, effectively correcting inverted elements and achieving superior worst-element quality compared to existing methods. Satheesh et al. [52] proposed a distortion-potential-based mesh regularization method that effectively suppresses element distortion in large-deformation nonlinear problems without requiring costly remeshing. Similar challenges arise in fluid-structure interaction problems. Inspired by Tezduyar [53], who proposed enhancing mesh quality by treating the fluid domain as a virtual elastic body with locally adjusted stiffness, we noted that assigning higher stiffness in distorted regions, specifically by scaling with the inverse of the Jacobian determinant, can effectively resist element degradation near interfaces. Furthermore, Hughes et al. [20, 54] emphasized the variation in the Jacobian within an element is a more accurate measure of element distortion than the numerical value itself. These insights directly motivated the design of our Jacobian-based barrier function for controlling mesh distortion in nonlinear analysis.

1.4 Scope and focus

To address the common issues of mesh distortion and numerical instability in topology optimization under large deformations, this paper proposes a barrier regularization method based on each element's Jacobian and an explicitly penalty for abnormal deformation behavior. This approach introduces a deformation gradient-dependent barrier regularization function into the potential energy when the element Jacobian approaches zero or loses positivity, thereby effectively mitigating the effects

of distorted elements on stiffness matrices and integration accuracy. This study develops one type of logarithmic barrier regularization function with clear geometric interpretation and consistency, suitable for ITO of highly nonlinear, large-deformation structures. Compared with stiffness modification approaches, the present study pursues a geometric perspective: the proposed log-barrier regularization avoids introducing fictitious stiffness into void regions, while enforcing a unified nonlinear formulation across both solid and void regions. In this way, the method ensures variational consistency, provides a clear geometric interpretation in terms of mesh distortion control, and significantly improves numerical stability and optimization convergence. The methodology is systematically validated in nonlinear isogeometric analysis and topology optimization, with extensions to finite element frameworks to demonstrate its effectiveness and universality.

The remainder of this paper is organized as follows. Section 2 introduces the NURBS basis function and the proposed barrier regularization function, and establishes the nonlinear IGA framework. Section 3 formulates the SIMP-based nonlinear topology optimization model and derives its sensitivity expressions. Section 4 verifies the effectiveness of the barrier regularization in nonlinear IGA through a benchmark example. Section 5 applies the proposed method to ITO and conducts numerical validations. Finally, some conclusion remarks are given in Section 6.

2. Nonlinear isogeometric analysis with barrier regularization

2.1 NURBS concept

Non-Uniform Rational B-Splines (NURBS) are widely used in computer-aided design and computational mechanics for their ability to accurately represent complex geometries with high stability [55]. By incorporating weight factors into B-splines, NURBS can model both polynomial and conic curves (e.g., arcs, ellipses), making them advantageous for engineering analysis [56].

B-splines are defined based on a knot vector, which is a non-decreasing sequence of real numbers in the parametric space, defined as $\Xi = \{\xi_0, \xi_1, \dots, \xi_{n+p}\}$, where n represents the number of basis functions and control points, and p denotes the order of

the polynomials. In one-dimensional space, the B-spline basis functions for a given knot vector can be defined by the well-known Cox-de Boor recursion formula [57].

$$B_{i,0}(\xi) = \begin{cases} 1, & \text{if } \xi_i \leq \xi < \xi_{i+1} \\ 0, & \text{Otherwise} \end{cases}$$

$$B_{i,p}(\xi) = \frac{(\xi - \xi_i)B_{i,p-1}(\xi)}{\xi_{i+p} - \xi_i} + \frac{(\xi_{i+p+1} - \xi)B_{i+1,p-1}(\xi)}{\xi_{i+p+1} - \xi_{i+1}} \quad \text{if } \xi_i \leq \xi < \xi_{i+1}$$

Moreover, NURBS basis functions can be obtained through rationalization of B-spline basis functions. By assigning a positive weight w_i to each B-spline basis function, the NURBS basis function is defined as:

$$N_{i,p}(\xi) = \frac{B_{i,p}(\xi)w_i}{\sum_j^n B_{j,p}(\xi)w_j}$$

Based on the tensor product property, two-dimensional NURBS basis functions are constructed as:

$$N_{i,j}(\xi, \eta) = \frac{B_{i,p}(\xi)B_{j,q}(\eta)w_{i,j}}{\sum_{k=0}^n B_{k,p}(\xi)B_{k,q}(\eta)w_{k,l}}$$

Compared with traditional Lagrange basis functions, NURBS basis functions demonstrate unique advantages, including non-negativity, partition of unity, local support, and differentiability [58].

2.2 Neo-Hookean material model

To characterize the mechanical response of materials under large deformations, this study employs the compressible Neo-Hookean hyperelastic material model [17]. This model effectively captures elastic behavior under finite deformations while ensuring energy consistency and geometric rationality. The strain energy density function is defined as:

$$W = \frac{\kappa}{2}(\text{tr}(\mathbf{C}) - 3) - \kappa \ln J + \frac{\chi}{2}(\ln J)^2$$

where J denotes the Jacobian determinant of the deformation gradient; κ and χ are Lamé constants, respectively defined through shear modulus and bulk modulus as:

$$\kappa = \frac{E}{2(1+\mu)}, \quad \chi = \frac{\mu E}{(1+\mu)(1-2\mu)}$$

with E and μ representing Young's modulus and Poisson's ratio, respectively. $\mathbf{C}$ is the right Cauchy-Green tensor corresponding to the Green-Lagrange strain defined as:

$$\mathbf{E} = \frac{1}{2}(\mathbf{C} - \mathbf{I}) \quad (6)$$

where $\mathbf{I}$ is the identity matrix.

By differentiating the strain energy density function with respect to the Green-Lagrange strain, we obtain the second Piola-Kirchhoff stress tensor, defined as:

$$\mathbf{S} = \frac{\partial W}{\partial \mathbf{E}} = 2 \frac{\partial W}{\partial \mathbf{C}} \quad (7)$$

Additionally, the tangential elasticity tensor can be derived as:

$$\mathbf{D} = \frac{\partial^2 W}{\partial \mathbf{E}^2} = 4 \frac{\partial^2 W}{\partial \mathbf{C}^2} \quad (8)$$

2.3 Modified potential energy functional formulation with barrier regularization

This section first constructs a new energy formulation by introducing barrier regularization function into the conventional potential energy expression. For a hyperelastic structure (particularly under large deformation), the total potential energy functional is generally expressed as:

$$\Pi(\mathbf{u}) = \int_{\Omega} W(\mathbf{F}(\mathbf{u})) dV - \int_{\Omega} \mathbf{b} \cdot \mathbf{u} dV - \int_{\Gamma_t} \mathbf{t} \cdot \mathbf{u} dA \quad (9)$$

where, Ω represents the material volume domain in the reference configuration, $\mathbf{F}$ is the deformation gradient, $\mathbf{b}$ denotes the body force per unit reference volume, $\mathbf{u}$ is the displacement vector, $\mathbf{t}$ indicates the traction per unit reference area, and Γ_t represents the boundary portion where surface tractions are applied.

To enhance overall numerical stability during large deformations, we incorporate a barrier regularization function into the potential energy functional framework. This modification addresses the issues of degraded numerical integration accuracy and stiffness matrix singularity caused by severe fluctuations of the Jacobian within elements. The modified potential energy functional is defined as:

$$\tilde{\Pi}(\mathbf{u}) = \Pi(\mathbf{u}) + \Pi_{\text{bar}}(\mathbf{u}) \quad (10)$$

where

$$\Pi_{\text{bar}}(\mathbf{u}) = \int_{\Omega} \psi(J(\mathbf{u})) dV \quad (11)$$

where ψ represents the barrier regularization function.

By taking the second variation of this potential energy functional, we obtain the tangent stiffness matrix:

$$\mathbf{K}_{\text{tan}} = \frac{\partial^2 \tilde{\Pi}}{\partial \mathbf{u}^2} = \mathbf{K}_{\text{mat}} + \mathbf{K}_{\text{geo}} + \mathbf{K}_{\text{bar}} \quad (12)$$

where $\mathbf{K}_{\text{mat}}$ and $\mathbf{K}_{\text{geo}}$ represent the material and geometric stiffness matrices, respectively, defined as:

$$\mathbf{K}_{\text{mat}} = \int_{\Omega} \mathbf{B}^T \mathbf{D} \mathbf{B} dV, \quad \mathbf{K}_{\text{geo}} = \int_{\Omega} \mathbf{G}^T \mathbf{S} \mathbf{G} dV \quad (13)$$

Here, $\mathbf{B}$ is constructed from the deformation gradient-related term matrix $\mathbf{F}_G$ and the shape function derivative matrix $\mathbf{G}$. For two-dimensional problems, they are defined as:

$$\mathbf{B} = \mathbf{F}_G \mathbf{G} = \begin{bmatrix} \frac{\partial u}{\partial X} + 1 & \frac{\partial v}{\partial X} & 0 & 0 \\ 0 & 0 & \frac{\partial u}{\partial Y} & \frac{\partial v}{\partial Y} + 1 \\ \frac{\partial u}{\partial Y} & \frac{\partial v}{\partial Y} + 1 & \frac{\partial u}{\partial X} + 1 & \frac{\partial v}{\partial X} \end{bmatrix} \begin{bmatrix} \frac{\partial N_1}{\partial X} & 0 & \frac{\partial N_2}{\partial X} & 0 & \dots & \frac{\partial N_n}{\partial X} & 0 \\ 0 & \frac{\partial N_1}{\partial X} & 0 & \frac{\partial N_2}{\partial X} & \dots & 0 & \frac{\partial N_n}{\partial X} \\ \frac{\partial N_1}{\partial Y} & 0 & \frac{\partial N_2}{\partial Y} & 0 & \dots & \frac{\partial N_n}{\partial Y} & 0 \\ 0 & \frac{\partial N_1}{\partial Y} & 0 & \frac{\partial N_2}{\partial Y} & \dots & 0 & \frac{\partial N_n}{\partial Y} \end{bmatrix} \quad (14)$$

where (u, v) represent the displacement field, (X, Y) denote the initial coordinates, and N represents the NURBS basis functions. For quadratic basis functions, $n = 9$ is the number of control points.

$\mathbf{K}_{\text{bar}}$ is the barrier stiffness matrix, an additional stiffness matrix derived from the variation of the barrier regularization function. As a symmetric positive semi-definite matrix, it provides local reinforcement when element distortion occurs, defined as:

$$\mathbf{K}_{\text{bar}} = \frac{\partial^2 \psi(J)}{\partial \mathbf{u}^2} = \int_{\Omega} \phi(J) \mathbf{B}^T \mathbf{B} dV \quad (15)$$

where

$$\phi(J) = \frac{\partial^2 \psi}{\partial J^2} \quad (16)$$

The core characteristic of this method lies in its direct action on the potential energy functional, essentially introducing a penalty mechanism for geometric mapping distortion. This approach exhibits excellent locality and geometric consistency. Moreover, it naturally extends to modeling systems with high-order continuity like IGA without introducing additional numerical parameters or artificial criteria.

2.4 Barrier regularization function

Mesh distortion of the void regions can cause the Jacobian to become zero or even negative, leading to numerical integration errors, ill-conditioned stiffness matrices, and convergence failure. To directly suppress distortion from a geometric perspective, this study introduces Jacobian-based barrier regularization. Firstly, the deformation gradient is used to describe the mapping relationship of material points before and after deformation, defined as:

$$\mathbf{F} = \mathbf{I} + \frac{\partial \mathbf{u}}{\partial \mathbf{X}} \quad (17)$$

where $\mathbf{X}$ denotes the initial position vector.

Then, one type of logarithmic barrier regularization function is proposed. When severe element distortion occurs, the barrier function rapidly increases, imposing a strong penalty on abnormal geometric deformation over corresponding void regions, effectively enhancing numerical stability and convergence during iterative analysis. As part of the implementation of the barrier regularization, a numerical threshold δ (typically chosen such that $0 < \delta \ll 1$) is introduced to avoid numerical singularities when $J \rightarrow 0$. This threshold is not a formulation-level parameter but a numerical safeguard to ensure stable computation. When the Jacobian determinant $J = \det(\mathbf{F})$ falls below δ , it is set to $J = \delta$, that is,

$$J = \max(J, \delta) \quad (18)$$

Subsequently, based on the Jacobian determinant, the Log-barrier regularization function is defined as follows:

$$\psi(J) = -\gamma \log(J) \quad (19)$$

where γ is a formulation-level penalty coefficient that governs the intensity of the barrier function, typically ranging from 0.001 to 0.1.

For numerical implementation, to maintain consistency with the energy formulation, we approximate its second derivative (Hessian approximation) to construct the stiffness matrix term:

$$\phi(J) = \frac{\gamma}{J^2} \quad (20)$$

As shown in **Fig. 2**, the barrier value increases sharply as the Jacobian determinant approaches barrier threshold δ , effectively penalizing highly distorted or inverted elements. For regular elements, the barrier function remains negligible, ensuring that the optimization remains unaffected in reasonable regions.

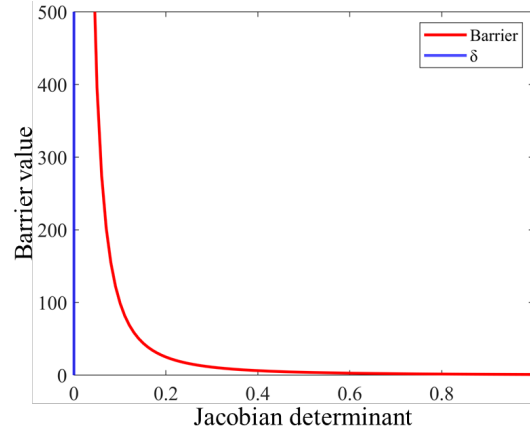


Fig. 2 Barrier regularization as a function of the Jacobian determinant.

From a mechanical perspective, although the logarithmic barrier regularization function is primarily introduced to enhance numerical stability under large deformations, it can be interpreted as a penalty mechanism embedded at the energy level. Specifically, the barrier term increases rapidly as the Jacobian determinant approaches zero, effectively imposing a resistance to element collapse or inversion. This behavior is conceptually analogous to a virtual spring system within each finite element, where the spring stiffness increases as the local volume shrinks abnormally. Such a formulation provides a localized repulsive effect that counters excessive distortion. In this sense, the barrier function not only serves a numerical mitigation but also embodies a geometrically actuated energy regularization, ensuring that the mapping from reference to deformed configuration remains well-behaved. This intrinsic geometric consistency makes the method particularly suitable for isogeometric frameworks.

2.5 Incremental-iterative solution

In nonlinear analysis, the structural response does not scale linearly with the applied load, making it unsuitable to apply the entire external load in a single step. Therefore, the incremental-iterative strategy is adopted, where the total load is divided

into multiple load increments. At each increment, the system is brought into equilibrium based on the known configuration from the previous step.

The Newton-Raphson method is employed for incremental solution of the nonlinear isogeometric analysis problem. In each iteration, the displacement increment $\Delta \mathbf{u}$ is solved by:

$${}^t K_{\tan} \Delta \mathbf{u} = {}^t \mathbf{R} \quad (21)$$

The variable t refers to the index of current load increment, and the equilibrium at $t + \Delta t$ is established based on the solution obtained at increment t . The residual vector ${}^t \mathbf{R}$ is defined as:

$${}^t \mathbf{R} = {}^{t+\Delta t} \mathbf{f}_{\text{ext}} - {}^t \mathbf{f}_{\text{int}} \quad (22)$$

where, $\mathbf{f}_{\text{ext}}$ and $\mathbf{f}_{\text{int}}$ denote the external and internal force vectors, respectively, given by:

$$\begin{aligned} \mathbf{f}_{\text{ext}} &= \int_{\Omega} {}^{t+\Delta t} \mathbf{b} \Delta \mathbf{u} dV + \int_{\Gamma_t} {}^{t+\Delta t} \mathbf{t} \Delta \mathbf{u} dA \\ \mathbf{f}_{\text{int}} &= \int_{\Omega} \mathbf{B}^T {}^t \mathbf{S} dV \end{aligned} \quad (23)$$

And ${}^{\Delta t} \mathbf{f}_{\text{ext}}$ represents the load increment. The iterative index t used here refers solely to the iterations within the nonlinear analysis and should not be confused with the iterative index associated with the optimization loop. In each Newton-Raphson iteration, convergence is assessed based on the relative L₂-norm of the displacement increments between two consecutive steps:

$$e_{\text{tol}} = \frac{\|\Delta \mathbf{u}\|_2}{\|\mathbf{u}\|_2} \quad (24)$$

where e_{tol} represents the convergence tolerance.

3 Nonlinear topology optimization problem

3.1 Material interpolation method

This work employs the basic Solid Isotropic Material with Penalization (SIMP) interpolation scheme:

$$E(x) = \rho_e^\beta E_0 \quad (25)$$

where, ρ_e represents element density (i.e. design variable), β is penalization factor, and

E_0 represents Young's modulus of the base material.

Unlike the modified SIMP method [59], this study does not assign a minimum modulus ($E_{\min}$) to void regions. While both $E_{\min}$ -based methods and the barrier regularization method introduce additional terms at the implementation level, the key difference lies in their formulation. TIt penalizes excessive mesh distortion without altering the constitutive model. Moreover, it avoids trial-and-error tuning and is more mesh-independent than traditional $E_{\min}$ -based strategies.

3.2 Topology optimization mathematical model

With material interpolation defined, the topology optimization problem is formulated for two objectives under material and geometric nonlinearity: minimizing structural compliance and maximizing output displacement in compliant mechanisms. Both are implemented within the nonlinear IGA framework.

A generalized load vector $\mathbf{L}$ unifies the two problems stated above. For compliance minimization: $\mathbf{L} = \mathbf{f}_{\text{ext}}$ is the external load vector. For compliant mechanism: $\mathbf{L}$ is a column vector with 1 at the output degree of freedom and 0 elsewhere. The unified nonlinear topology optimization problem is:

$$\begin{aligned} \min_{0 \leq \rho_e \leq 1} : O(\boldsymbol{\rho}) &= \mathbf{L}^T \mathbf{u} \\ \text{s.t.: } \mathbf{R}(\boldsymbol{\rho}, \mathbf{u}) &= \mathbf{0} \\ g &= \sum_{e=1}^{n_e} \rho_e v_e - V \leq 0 \end{aligned} \quad (26)$$

where O represents the objective function value to be optimized, $\boldsymbol{\rho}$ denotes the design variable vector, n_e indicates the total number of discretized elements in the design domain, g signifies the volume constraint function, v_e corresponds to the volume of an individual element, V stands for the prescribed target volume of the structure.

The sensitivity expression of the objective function, formulated using the Lagrange multiplier method for the compliance minimization problem, is given as follows:

$$\frac{\partial O}{\partial \rho_e} = \mathbf{L}^T \frac{\partial \mathbf{u}}{\partial \rho_e} + \boldsymbol{\lambda}^T \left(\frac{\partial \mathbf{R}}{\partial \rho_e} + \frac{\partial \mathbf{R}}{\partial \mathbf{u}} \frac{\partial \mathbf{u}}{\partial \rho_e} \right) \quad (27)$$

Combining SIMP and the adjoint method yields:

$$\frac{\partial O}{\partial \rho_e} = -\boldsymbol{\lambda}^T (\beta \rho_e^{\beta-1} E_0) \mathbf{f}_{\text{int}} \quad (28)$$

The adjoint variable $\boldsymbol{\lambda}$ is solved from:

$$\mathbf{K}_{\text{tan}} \boldsymbol{\lambda} = \mathbf{L} \quad (29)$$

where the tangent stiffness matrix $\mathbf{K}_{\text{tan}}$ includes not only the material and geometric stiffness components, but also the barrier stiffness matrix $\mathbf{K}_{\text{bar}}$ as derived in Section 2.2.2. This ensures that the regularization effect of the barrier function is fully embedded in the nonlinear finite element analysis and reflected in both the state and adjoint equations. Since the barrier function depends solely on the deformation gradient, it acts independently of the material interpolation, allowing "true void" modeling without introducing artificial stiffness.

3.3 Sensitivity filtering and design variable update

To suppress numerical artifacts [60], a sensitivity filter [61] is applied:

$$\frac{\partial O}{\partial \rho_e} = \frac{1}{\max(\vartheta, \rho_e) \sum_{i \in N_e} H_{ei}} \sum_{i \in N_e} H_{ei} \rho_e \frac{\partial O}{\partial \rho_e} \quad (30)$$

where, ϑ is a small positive value to prevent division by zero, N_e represents the set of elements within the filtering neighborhood of element e , H_{ei} denotes the filtering weight coefficient between elements e and i , defined as:

$$H_{ei} = \max(0, r_{\min} - \text{dist}(i, e)) \quad (31)$$

where $r_{\min}$ is the filter radius and $\text{dist}(i, e)$ represents the distance between centers of elements e and i .

Furthermore, the Optimality Criteria (OC) method is employed to update design variables:

$$\rho_e^{\text{new}} = \begin{cases} \max(0, \rho_e - m) & \text{if } \rho_e B_e^\theta \leq \max(0, \rho_e - m) \\ \min(0, \rho_e + m) & \text{if } \rho_e B_e^\theta \geq \min(1, \rho_e + m) \\ \rho_e B_e^\theta & \text{otherwise} \end{cases} \quad (32)$$

where m is the move limit controlling the maximum allowable change of design variables per iteration, θ represents the damping coefficient used to stabilize convergence, and the parameter B_e is calculated as:

$$B_e = -\frac{\partial O}{\partial \rho_e} \bigg/ \lambda \frac{\partial g}{\partial \rho_e} \quad (33)$$

3.4 Implementation workflow

To better connect the analysis formulation in Section 2 and the topology optimization model in Section 3, this subsection outlines the overall implementation workflow of the proposed barrier regularization for nonlinear ITO framework, as shown in **Fig. 3**. The key objective is to demonstrate that the proposed method can be seamlessly embedded into a standard topology optimization loop, while preserving geometric consistency and ensuring ease of implementation.

The workflow begins with constructing the isogeometric model using NURBS basis functions, followed by SIMP-based initialization of material distribution and interpolation of Neo-Hookean material properties. Nonlinear analysis is then carried out incrementally using the Newton-Raphson method, where the tangent stiffness matrix includes material, geometric, and barrier contributions. The barrier function stabilizes the analysis, particularly in regions susceptible to element inversion. After reaching equilibrium, the objective function and its sensitivities, naturally incorporating the barrier effect, are evaluated via the adjoint method. Sensitivities are filtered to suppress numerical artifacts and used to update design variables using the OC method. This process iterates until convergence is achieved.

This workflow illustrates the tight coupling between the barrier regularization and the nonlinear ITO process. Since the barrier function is introduced at the energy level and does not require modifying the SIMP formulation or optimization algorithm, it offers a robust yet simple way to enhance numerical stability while maintaining modeling fidelity.

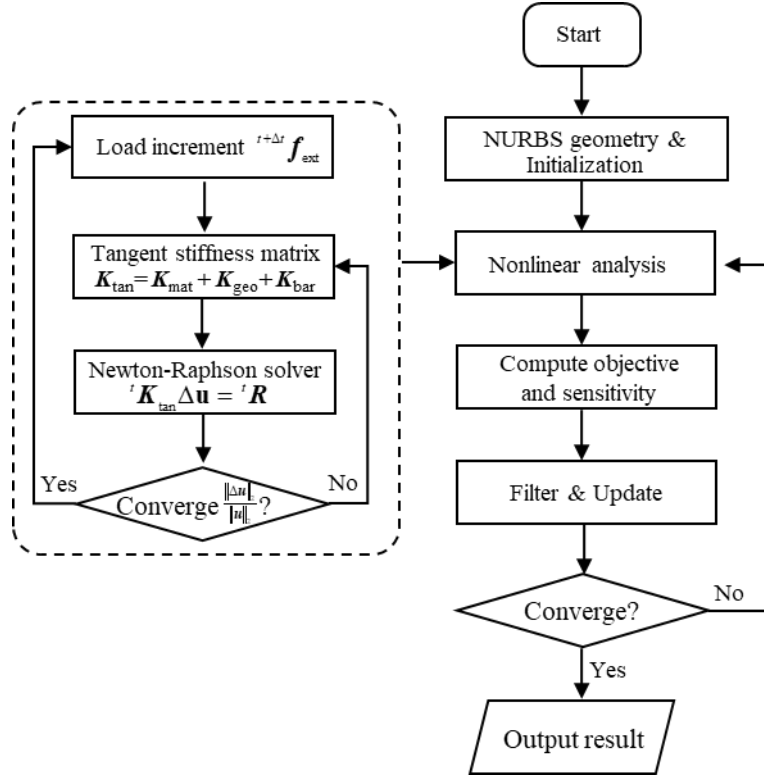


Fig. 3 The workflow of the proposed barrier regularization for nonlinear ITO framework

4. Benchmark and numerical investigation

To validate the feasibility and effectiveness of the proposed Jacobian-based barrier regularization method in large-deformation geometric nonlinear IGA, this section employs a classic C-shaped beam structure as a numerical example, as illustrated in **Fig. 4**. The beam dimensions are 10 mm (length) $\times$ 10 mm (width) $\times$ 1 mm (thickness), with a fixed left boundary and subjected to two forces, f_1 and $f_2 = 2$ N. The Neo-Hookean material has an elastic modulus of 300 MPa and a Poisson's ratio of 0.4. Through numerical experiments under different loads, we systematically evaluate the barrier regularization term's effectiveness in suppressing mesh distortion, improving numerical stability, and enhancing computational accuracy. All examples were executed on a personal computer equipped with a 12th Gen Intel (R) Core (TM) i7-12700F 2.10 GHz CPU and 16 GB RAM.

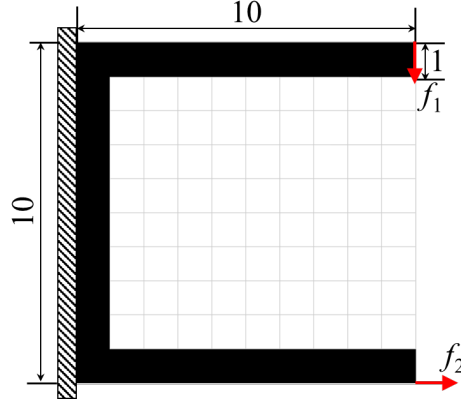


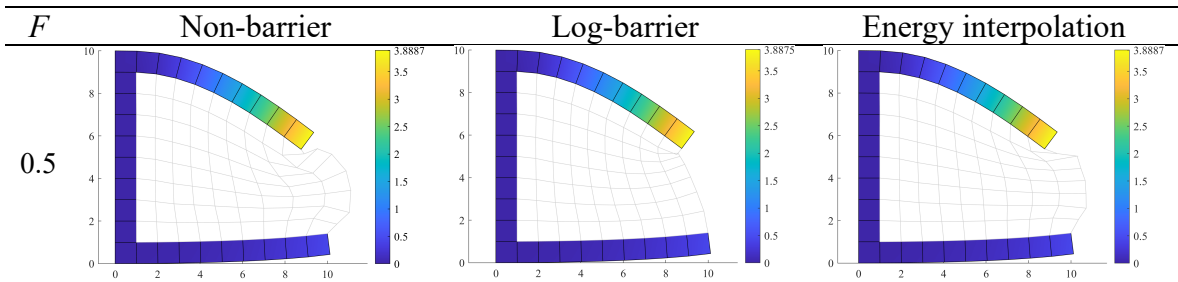
Fig. 4 Schematic of the C-shaped beam.

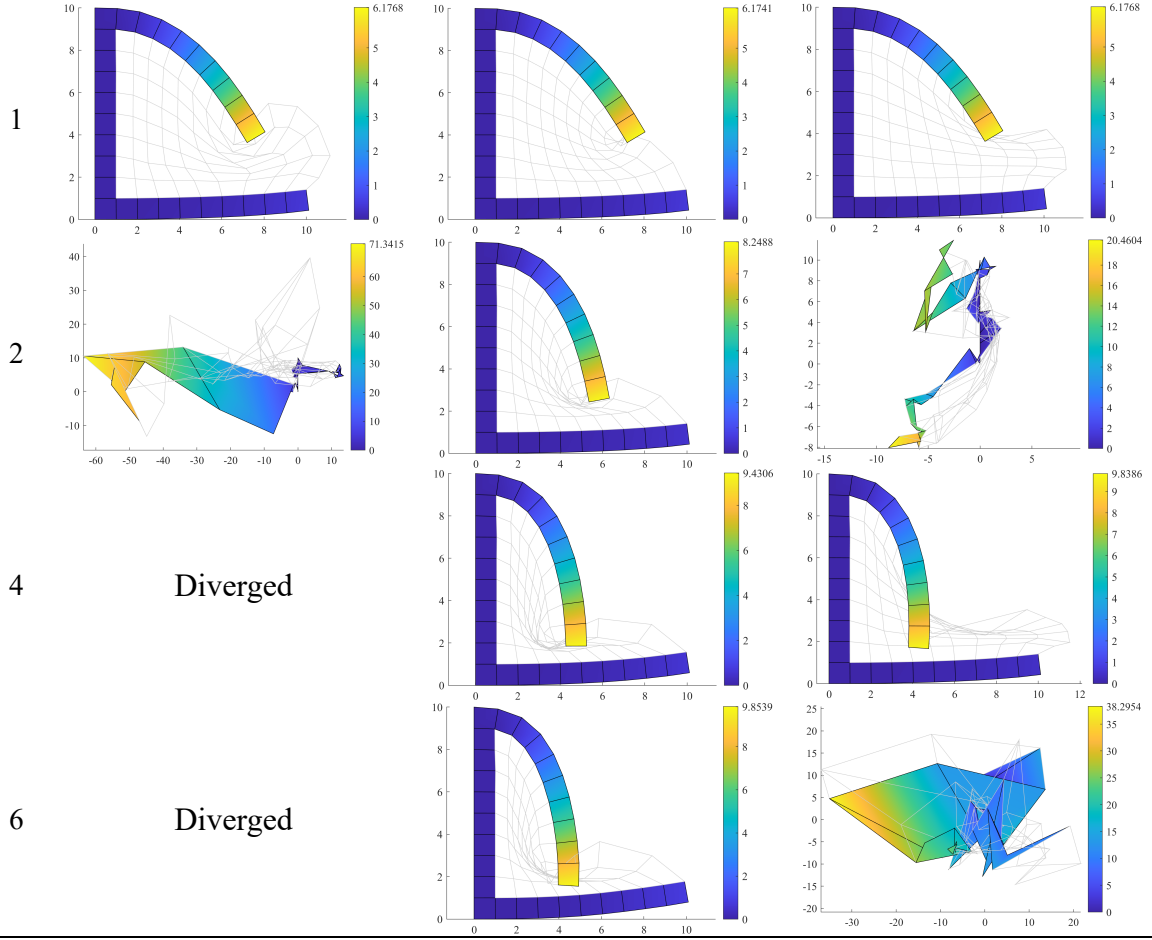
4.1 Comparative analysis under different loads

This section compares the analysis results of three methods, logarithmic barrier regularization (Log-barrier) method, without regularization barrier (Non-barrier) method, and energy interpolation method under different loads. For Non-barrier and energy interpolation method: A small Young's modulus ($E_{\min} = 10^{-9}$ MPa) is assigned to void regions. For Log-barrier: Zero Young's modulus is applied to void regions. The objective is to assess the barrier regularization term's role in enhancing numerical stability and analysis convergence under large deformations and to investigate the sensitivity of barrier effectiveness to load magnitude.

The penalty coefficient is set to $\gamma = 0.1$, and the barrier threshold to $\delta = 10^{-6}$. Displacement contour plots when the horizontal loads $f_1 = 0.5$ N, 1 N, 2 N, 4 N and 6 N are respectively explored, as shown in **Table 1**. The number of load increments is set to 10, with maximum 10 iterations per load increment and convergence tolerance $e_{\text{tol}} = 10^{-3}$.

Table 1 Displacement contour plots under three methods for different loads





As shown in the table, when the applied loads are 0.5 N and 1 N, both methods are capable of completing the analysis, and the maximum displacements are comparable, demonstrating the feasibility of the proposed barrier regularization method. However, as the load increases, the Non-barrier method fails to produce valid results due to severe mesh distortion, whereas the Log-barrier method continues to converge.

Moreover, at $F = 2$ N and 6 N, the energy interpolation method failed to converge. This may be attributed to its mixing of geometrically linear operators in low-density regions with nonlinear operators in solid regions. Such mixing can lead to inconsistent tangent stiffness matrices and near-singular modes along certain load paths, making the method sensitive to convergence tolerance, load increment number, and mesh resolution. This observation is similar that observed in a prior study [29]. **Fig. 5** shows that with parameter relaxation in the $F = 2$ N case, where the convergence tolerance was set to $e_{\text{tol}} = 2 \times 10^{-3}$, the number of load increments increased to 20, and the mesh refined to 40×40 elements, respectively, both the Log-barrier and energy interpolation methods achieved successful numerical analysis.

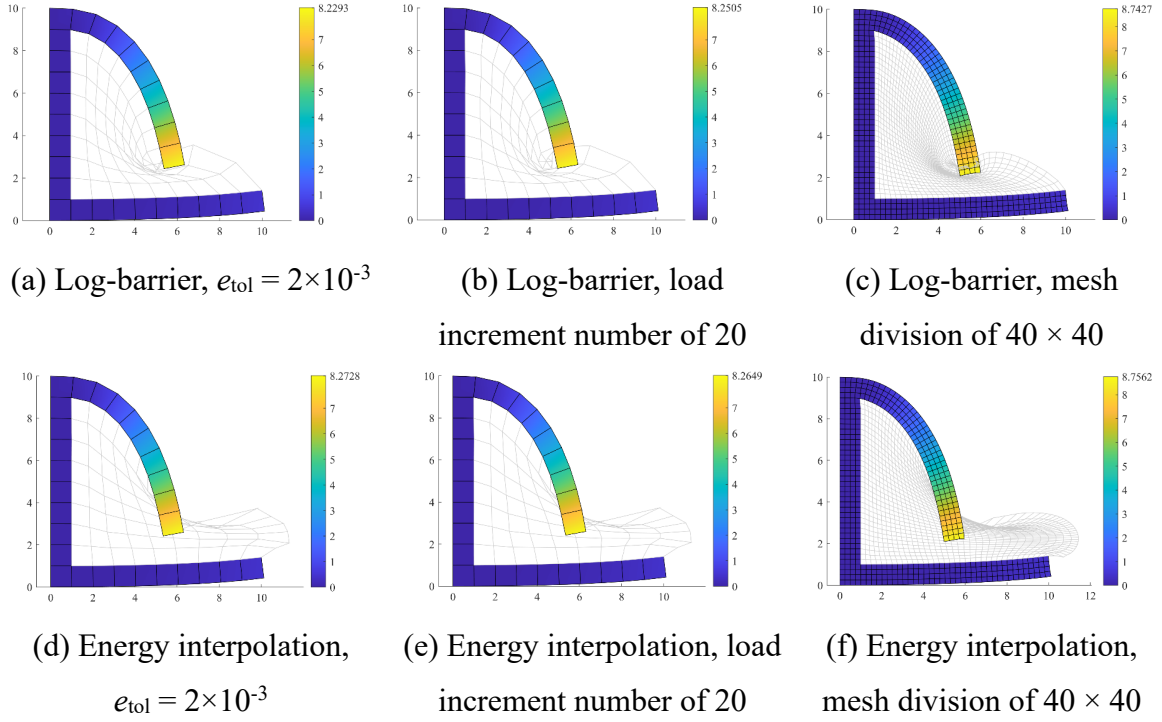


Fig. 5 The displacement contour plot under a load $f_1 = 2$ N

1 Additionally, to verify the accuracy of mesh deformation under the above Log-
2 barrier method in the presence of large deformations, we present displacement contour
3 plots for the Non-barrier method under a load of 2 N, with load increment numbers of
4 20 and 40, along with the corresponding analysis times and convergence iterations for
5 comparison, as shown in **Fig. 6**.

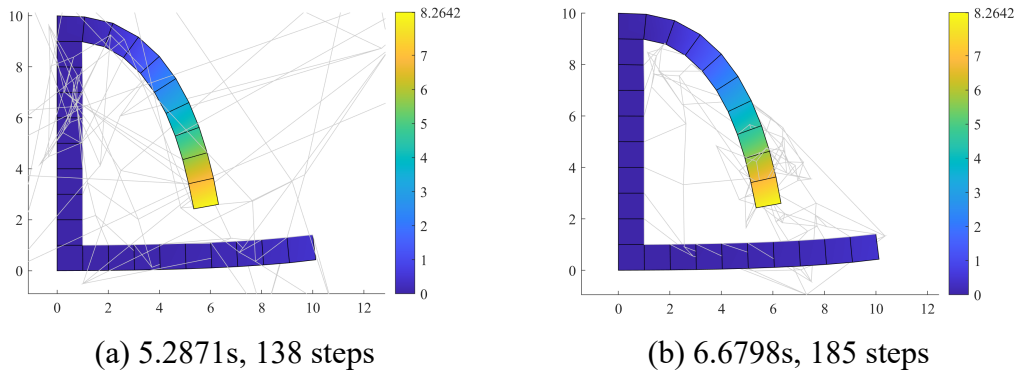


Fig. 6 Displacement contour plots using the Non-barrier method: (a) load increment number of 20 and (b) load increment number of 40.

6 It can be observed that as the load increment number increases, the nonlinear
7 analysis of the Non-barrier method exhibits feasibility, with the maximum displacement
8 consistently reaching 8.2642 mm with load increment numbers of 20 and 40.
9 Comparatively, the maximum displacements obtained using the Log-barrier method

with load increment number of 10 is 8.2488 mm, as shown in **Table 1**. Compared with the Non-barrier method result, the relative error is only 0.186%. However, under the Non-barrier method, severe mesh distortion occurs in void region at 20 load increments, and distortion is also relatively evident in the void region at 40 load increments. Additionally, increasing the load increment number undoubtedly leads to longer analysis times and a higher number of convergence iterations.

Table 2 summarizes the iteration numbers (Iter), computation time (Time), and maximum displacements ($U_{\max}$) for the three methods with load increment number of 10. When the applied loads are 0.5 N and 1 N, the Log-barrier method, due to the inclusion of the barrier function, requires slightly more computation time. In contrast, the energy interpolation method is more efficient because linear elements are used in void regions. But, as the load continues to increase, the Log-barrier method demonstrates better stability. One point to note is that at $F = 6$, the analysis time of the log-barrier method decreases because the barrier term grows rapidly with deformation, effectively stiffening severely distorted regions and thereby reducing the number of nonlinear iterations and the total computation time.

Table 2 Performance comparison for three methods with load increment number of 10 under different loads

f_i	Non-barrier			Log-barrier			Energy interpolation		
	Iter	Time	$U_{\max}$	Iter	Time	$U_{\max}$	Iter	Time	$U_{\max}$
0.5	40	1.46s	3.8887	65	1.55s	3.8875	40	1.20s	3.8887
1	45	1.59s	6.1768	73	1.72s	6.1741	42	1.31s	6.1768
2	91	3.18s	71.113	82	1.91s	8.2488	91	2.83s	20.4604
4	/	/	/	96	2.42s	9.4360	53	1.64s	9.8386
6	/	/	/	70	1.75s	9.8539	97	3.16s	38.2954

4.2 Comparison of different penalty coefficients and barrier threshold

This example aims to evaluate the effect of penalty coefficient and barrier threshold magnitude on the analysis stability of the Log-barrier regularization. Under an applied load of 2 N and the barrier threshold of $\delta = 10^{-6}$, displacement contour plots corresponding to penalty coefficients $\gamma = 0.001, 0.01$ and 1 are shown in **Fig. 7**.

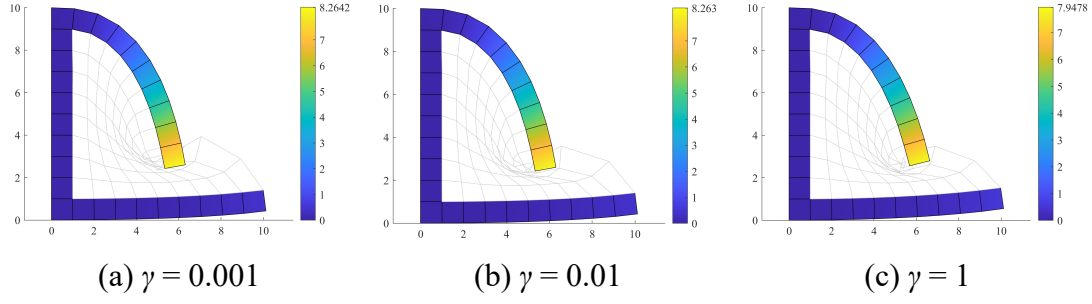


Fig. 7 Displacement contour plots under different penalty coefficients

It can be observed that as the penalty coefficient increases, the contribution of the barrier regularization function also increases, leading to a decreasing trend in the maximum displacement of the C-shaped beam. Additionally, when $\gamma = 0.001$, the maximum displacement under the Log-barrier method matches that of the Non-barrier method with load increment numbers of 20 and 40, further confirming the effectiveness of the proposed method.

To investigate the influence of the penalty coefficient on convergence, we provide a comparison of iteration counts, computation times, maximum displacements, and displacement errors (relative to $\gamma = 0.001$) for $\gamma = 0.001, 0.01, 0.1$ and 1 , as shown in **Table 3**.

Table 3 Performance comparison of the Log-barrier method under different penalty coefficients

γ	Iter	Time	$U_{\max}$	Error/%
0.001	50	1.30s	8.2642	/
0.01	47	1.21s	8.2630	0.0145
0.1	82	1.91s	8.2488	0.1863
1	100	2.04s	7.9478	3.8288

As shown in the table, the number of iterations and computation time first decrease and then increase with the increasing penalty coefficient, indicating that convergence is most favorable when $\gamma = 0.01$. Meanwhile, as the penalty coefficient increases, the displacement error gradually grows, while the convergence behavior initially improves and then deteriorates. This indicates that the penalty coefficient also affects the accuracy of the analysis, and a larger value is not necessarily better.

Similarly, under an applied load of 2 N and the penalty coefficient of $\gamma = 0.1$, displacement contour plots corresponding to barrier thresholds $\delta = 1e-5, 1e-4$ and $1e-3$ are shown in **Fig. 8**. Combined with Table 2, when δ is taken as $1e-6, 1e-5, 1e-4$ and

1e-3, the corresponding maximum displacements are 8.2488, 8.2485, 8.2460 and 8.2531, respectively. The maximum displacements for the three new barrier thresholds are 8.2485 (1e-5), 8.2460 (1e-4), and 8.2531 (1e-3), respectively. It is evident that when the value is $0 < \delta \ll 1$, the difference in displacement is not obvious.

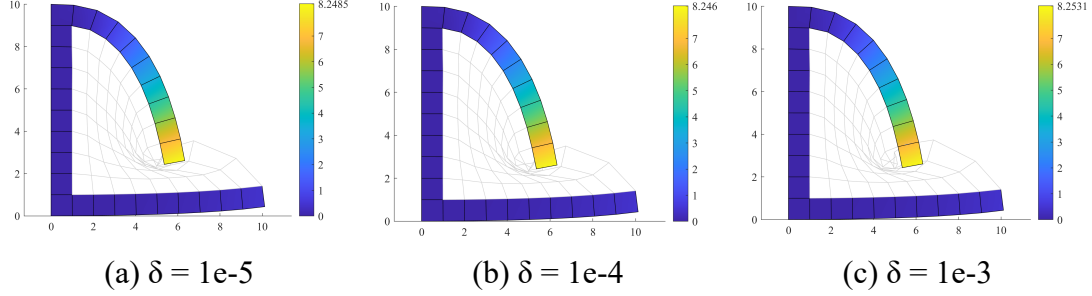


Fig. 8 Displacement contour plots under different barrier thresholds

4.3 Analysis of Jacobian variation and stiffness matrix condition number

In this example, we present the minimum and maximum values of the Jacobian determinant throughout the entire analysis process for the four cases described in Section 4.2. For better comparison, we also provide the corresponding maximum and minimum Jacobian determinant values obtained from the Non-barrier method under a load of 2 N, where $J_{\max} = 3.1446\text{e}9$ and $J_{\min} = -3.1446\text{e}9$. **Table 4** summarizes the Jacobian determinant values for the Log-barrier regularization method.

Table 4 Comparison of Jacobian determinant values under different penalty coefficients

	0.001	0.01	0.1	1
$J_{\min}$	-0.524	-0.286	-0.126	-0.019
$J_{\max}$	2.552	3.250	2.428	2.242

Without regularization, the extreme values of the Jacobian determinant reach $\pm 3.1446\text{e}9$, indicating severe mesh distortion that leads to computational failure. In contrast, from the table, it is evident that the Log-barrier method effectively controls the Jacobian.

Condition number is a manifestation of numerical stability. To systematically evaluate the numerical stability of the barrier regularization method, we compare the condition number for two methods: Non-barrier and Log-barrier, under a load of 2 N and a penalty coefficient of 0.01, as shown in **Fig. 9**. The number of load increments is 20 for the Non-barrier method and 10 for the Log-barrier method.

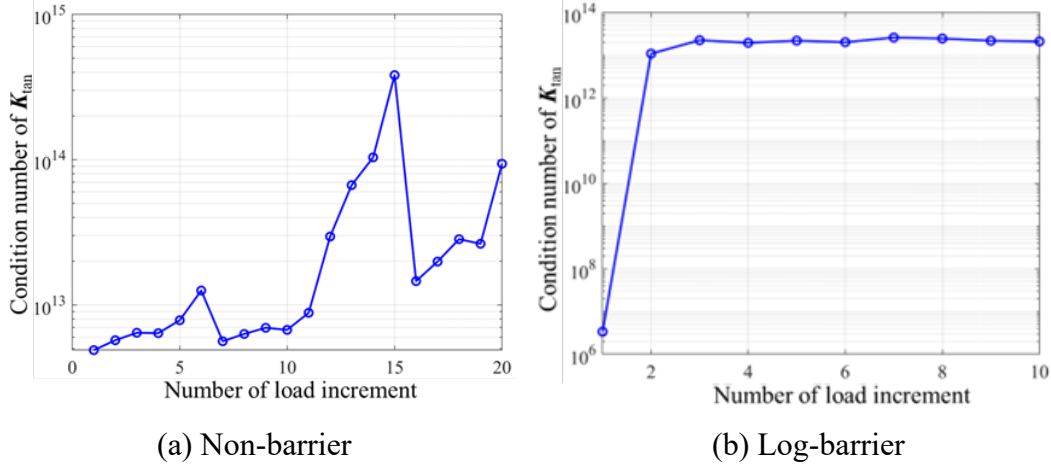


Fig. 9 Tangent stiffness matrix condition number for different methods

The results indicate that although the Non-barrier method can complete the analysis with 20 load increments, the condition number of the stiffness matrix deteriorates significantly during loading, reaching the order of 10^{15} . In contrast, the Log-barrier method with 10 load increments exhibits more stable analysis processes. However, although the Log-barrier method effectively prevents element inversion, it can lead to a sharp increase in the barrier function when the local Jacobian determinant approaches the barrier threshold. This results in extremely large entries in the stiffness matrix and a significant increase in its condition number, reaching the order of 10^{13} .

In summary, this example verifies the advantage of the Log-barrier method in controlling mesh distortion. Especially under large deformation conditions, it effectively maintains the Jacobian within a reasonable range, thereby ensuring numerical stability for subsequent topology optimization. These findings provide theoretical support for the engineering application of nonlinear topology optimization.

5. Numerical Examples

To further validate the effectiveness, robustness, and versatility of the proposed barrier regularization for large-deformation nonlinear topology optimization, three sets of numerical examples are presented in this section. First, a cantilever beam structure is used to demonstrate the feasibility of material and geometric nonlinear topology optimization within the IGA framework, with a particular focus on comparing the topological configurations obtained under different loading conditions and two methods

(Non-barrier, and Log-barrier). Next, a compliant mechanism (Inverter) optimization problem is employed to evaluate mesh quality and topological convergence during the optimization process. Finally, a comparison with an open-source FEM-based geometric topology optimization method is conducted to further demonstrate the advantages of the barrier regularization approach in enhancing optimization performance across different numerical platforms.

5.1 Cantilever beam design under nonlinear deformation

This example considers a classical cantilever beam, where topology optimization is performed within the IGA framework by incorporating a Neo-Hookean material model and geometric nonlinearity. The feasibility of large deformation ITO is examined by introducing the proposed barrier regularization function. The geometry, loading, and boundary conditions of the cantilever beam are illustrated in **Fig. 10**, with a mesh discretization of $120 \text{ mm} \times 32 \text{ mm}$. The Neo-Hookean model uses a Young's modulus of 300 MPa and a Poisson's ratio of 0.4, and no minimum stiffness is applied. The barrier regularization parameters are set as follows: penalty coefficient $\gamma = 0.01$, barrier threshold $\delta = 10^{-5}$. In the nonlinear analysis, the number of load increments is set to 10, with a convergence tolerance $e_{\text{tol}} = 2e-3$, and the Newton-Raphson iteration is capped at 5 per. The volume fraction, filter radius, and SIMP penalty factor are set to 0.5, 1.5, and 3, respectively. The termination criterion for optimization iterations is set to 10^{-5} .

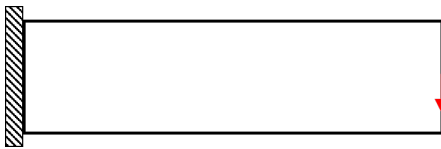


Fig. 10 Schematic of the cantilever beam

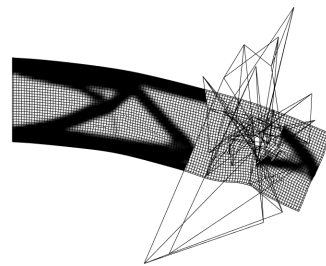


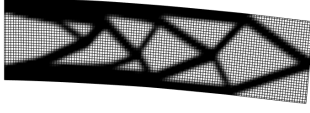
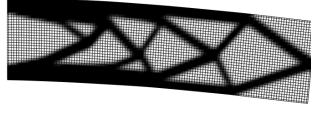
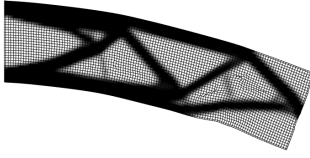
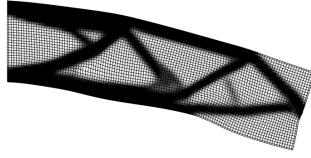
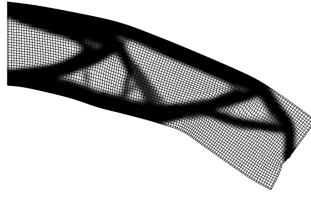
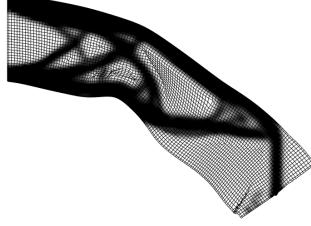
Fig. 11 Mesh configuration with Non-barrier

The topological mesh configurations obtained under different loading conditions and two methods, as well as the corresponding structural compliance (objective) and optimization time (given as O - T, for example 46.89 - 1153s under a load of 10 and Non-barrier condition), are summarized in **Table 5**.

The numerical results firstly demonstrate that the Non-barrier method can

effectively validate the feasibility of ITO under both material and geometric nonlinearities. When the applied load reaches 30, the structure undergoes significant large deformations, yet the optimization can be achieved even without introducing a minimum stiffness parameter. This is partly due to the inherent convergence characteristics of the Neo-Hookean material, and partly due to the inherent numerical stability provided by IGA.

Table 5 Mesh configuration, structural compliance (O), and optimization time (T)

	under different loading conditions and different methods	
	Non-barrier	Log-barrier
$f = 10$		
O - T	46.89 - 1153s	46.89 - 1177s
$f = 30$		
O - T	411.40 - 1501s	414.14 - 1529s
$f = 50$	/	
O - T	/	997.93 - 1711s
$f = 70$	/	
O - T	/	1648.28 - 2050s

However, it should be noted that during the optimization process with Non-barrier, mesh distortion as shown in **Fig. 11** occurred multiple times, leading to potential numerical divergence. By introducing the barrier regularization function, such mesh distortion issues are effectively suppressed. As shown in the final mesh configuration at load $f = 30$ in **Table 5**, local mesh distortion is evident in the Non-barrier method,

whereas the Log-barrier method maintain good mesh quality, further validating the effectiveness of barrier regularization in mitigating mesh irregularities. Moreover, when the applied load is 10 and 30, the topology, compliance, and computation time show negligible differences among the two methods, indicating the effectiveness of nonlinear ITO with barrier regularization.

When the load increases to $f = 50$, the Non-barrier method suffers from severe numerical divergence and computational failure. In contrast, the Log-barrier method successfully completes the optimization while maintaining reasonable mesh quality. As the load further increases to $f = 70$, the Log-barrier method experiences significant mesh distortion.

To further investigate the influence of the penalty coefficient on the topology optimization, we present the mesh configuration and final topology configuration obtained at the load of $f = 70$ with the penalty coefficient $\gamma = 0.1$ for the Log-barrier method, as shown in **Fig. 12**. The corresponding structural compliance and optimization time are 1601.19 and 1857s.

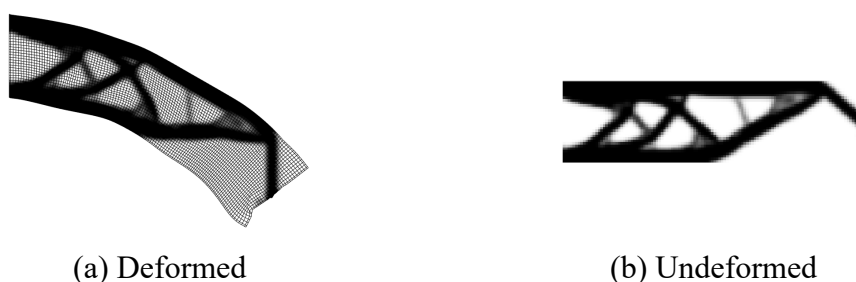


Fig. 12 Mesh and topology configuration under the Log-barrier method

It can be observed that with an increased penalty coefficient, both mesh distortion and optimization time are significantly decreased, while the structural compliance is moderately improved. These results confirm the observation from Section 4 that an increase of γ causes the structure to become stiffer, resulting in a reduction in structural compliance.

5.2 Compliant mechanism design under nonlinear deformation

To further evaluate the performance of barrier regularization in compliant mechanism optimization problems, this section considers a compliant inverter, focusing on mesh quality and topology evolution stability during the optimization process.

Owing to the symmetry of the inverter, the design domain geometry and boundary conditions are designed as shown in **Fig. 13**, with a mesh resolution of $120 \text{ mm} \times 80 \text{ mm}$. As illustrated, the top-left corner corresponds to the input force and the attached spring, with magnitudes of 250 N and 10 N/mm , respectively. The top-right corner represents the output displacement point and an additional spring, with a spring stiffness of 10 N/mm . The objective is to maximize the displacement u_{out} for a given input force f_{in} . To ensure consistency with [62], the Neo-Hookean material is assigned a Young's modulus of 3000 MPa and a Poisson's ratio of 0.4 . The barrier regularization parameters are configured as follows: penalty coefficient $\gamma = 0.01$, and barrier threshold $\delta = 10^{-5}$.

For nonlinear analysis, the load increment number is 20, the convergence tolerance $e_{tol} = 10^{-3}$, and the Newton-Raphson iterations are limited to a maximum of 5 per. The volume fraction, filter radius, and penalty factor are set to 0.25, 1.5, and 3, respectively.

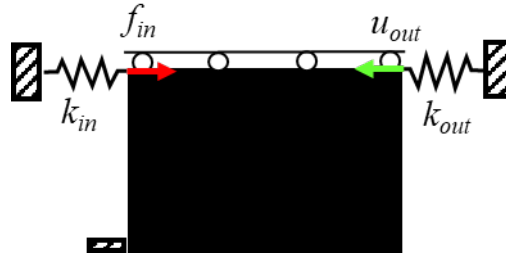
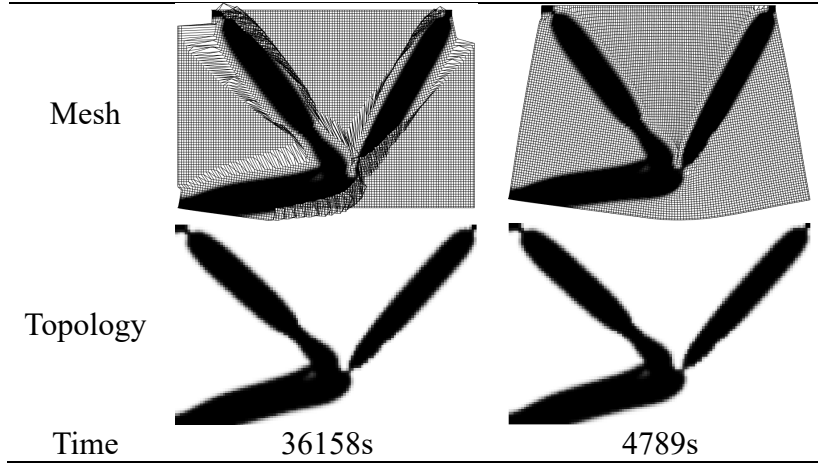


Fig. 13 Schematic of the inverter

The final mesh configurations, topology configurations, and optimization times under the Non-barrier and Log-barrier methods are summarized in **Table 6**. It is worth noting that in the Non-barrier case, the optimization process fails to converge due to element singularity if no additional minimal stiffness is introduced. Therefore, in this numerical example, we additionally add a small value of 10^{-6} to the tangent stiffness matrix. In contrast, the Log-barrier method does not require this modification, thereby preserving geometric consistency.

Table 6 Final mesh, topology configurations and optimization time under different

methods	
Non- barrier	Log-barrier



As shown in **Table 6**, the final topologies obtained from both methods are largely consistent, indicating the good feasibility of barrier regularization in compliant mechanism optimization problems. However, in terms of mesh quality, the Log-barrier method exhibits smoother and more natural mesh deformations, whereas the Non-barrier method results in noticeable element distortions, even with artificially added value to the tangent stiffness.

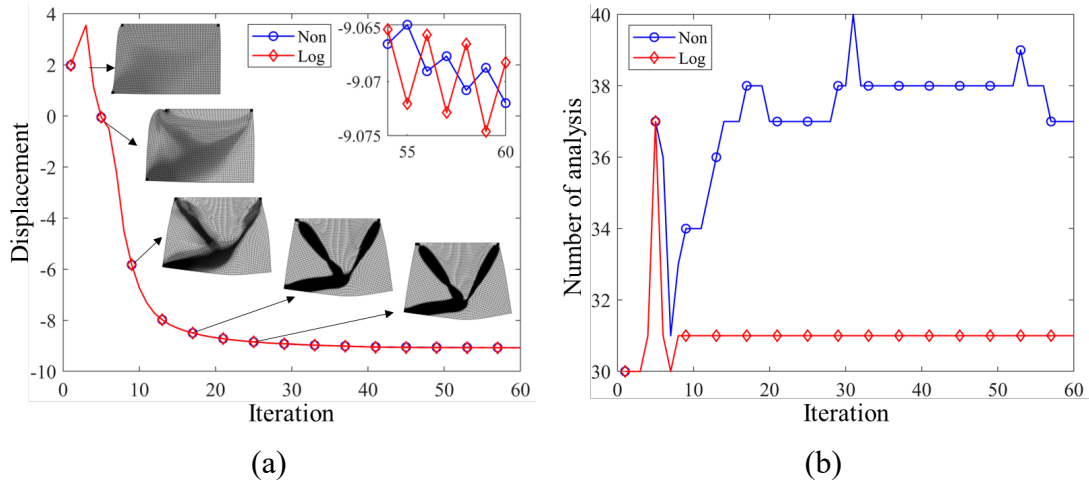


Fig. 14 Iteration curves: (a) Displacement and topology evolution; (b) Number of analysis iterations

Furthermore, the objective function convergence curves of different methods and the topology evolution process under the Log-barrier method are illustrated in **Fig. 14(a)**. It can be observed that both methods achieve rapid convergence, with minimal differences in objective displacement and no significant oscillations during the optimization process and mesh distortion, demonstrating the robustness of the proposed topology optimization method. **Fig. 14(b)** presents the number of Newton-Raphson

iterations at each optimization step, showing that the Log-barrier method converges faster and more stably during the analysis phase. In contrast, the Non-barrier method exhibits greater fluctuations in iteration numbers, reflecting numerical instabilities caused by mesh distortion, which in turn causes a significantly longer overall optimization time.

In summary, barrier regularization not only preserves the accuracy of the optimized results but also significantly enhances numerical stability and computational efficiency in compliant mechanism topology optimization.

5.3 Extending the Barrier Regularization Method to Traditional Finite Element Frameworks

Although Sections 5.1 and 5.2 focus on isogeometric topology optimization, the proposed barrier regularization has not yet been validated within a conventional FEM setting. Given the fundamental differences between IGA (using smooth NURBS basis functions) and FEM (based on C^0 Lagrange elements), it is important to assess the method's generality. To this end, we implement the barrier function in the open-source FEM-based nonlinear topology optimization MATLAB code TOP_Geo_Non [63], which supports geometric nonlinearity, offers algorithmic flexibility, and facilitates reproducible testing.

A cantilever beam is used as the example problem, with the design domain discretized into $120 \text{ mm} \times 30 \text{ mm}$. The left edge is fixed, and a downward load is applied at the midpoint of the right boundary. All other parameters follow the default settings in [63] as: the volume fraction is set to 0.5, the filter radius is set to 3, Young's modulus is 3000 MPa and Poisson's ratio is 0.4. The nonlinear settings also follow the default values: the number of load increments is set to 1, the Newton-Raphson iteration number is limited to 50, and the convergence tolerance is 10^{-3} . The deformation gradient using the element strain components (e_x , e_y , and e_{xy}) is given as:

$$\mathbf{F} = \begin{bmatrix} 1 + e_x & \frac{e_{xy}}{2} \\ \frac{e_{xy}}{2} & 1 + e_y \end{bmatrix} \quad (34)$$

in which e_x and e_y are normal strains and e_{xy} is the shear strain.

The barrier regularization parameters are set as follows: penalty coefficient $\gamma = 0.01$, and barrier threshold $\delta = 10^{-5}$. The resulting topologies, structural compliance, and optimization iterations under different loads for two methods are summarized in **Table 7**.

Table 7 The topologies, structural compliances (O), and iteration steps (I) for each method under various loads

Load (N)	Non-barrier	Log-barrier
10^2		
O - I	0.0007 - 45	0.0007 - 45
10^7		
O - I	6690070 - 43	6664544 - 58
10^8		
O - I	NaN	668484983 - 54
$2e8$	/	
O - I	/	2802099742 - 78
10^9	/	
O - I	/	116892762497 - 56
$1.4e9$	/	
O - I	/	NaN

The results indicate that the proposed barrier regularization approach is equally applicable to geometric nonlinear topology optimization within the finite element framework. Under small load conditions, the topologies, structural compliances, and iteration numbers of both methods are nearly identical, demonstrating that the barrier regularization of minor deformation does not compromise optimization accuracy. When the load is 10^7 N, the Log-barrier method outperforms the Non-barrier method in terms of structural configuration and performance, despite a slight increase in the number of iterations. As the load increases further to 10^8 N, the Non-barrier method exhibits numerical divergence. As the load continues to increase, the Log-barrier method remains capable of performing the corresponding optimization until numerical divergence occurs when the load reaches $1.4e9$ N.

1 In summary, the proposed Log-barrier method is not only applicable within the
2 IGA framework but also demonstrates good adaptability and numerical stability in
3 traditional FEM-based geometric nonlinear topology optimization. Its advantage
4 becomes particularly evident under high load conditions, and demonstrates practical
5 applicability.

6 **6. Conclusions**

7 Conventional FEM-based remedies for mesh distortion in topology optimization
8 under large deformations often rely on C^0 continuity at void boundaries. However, this
9 requirement poses significant challenges for isogeometric topology optimization due to
10 the non-zero support of NURBS basis function across multiple knot spans (elements).
11 This paper addresses the common issues of mesh distortion and numerical instability in
12 large-deformation ITO by introducing a barrier regularization method. It is
13 demonstrated that the method is broadly applicable to various types of basis functions.
14 It directly suppresses element distortion at the geometric level of voids, enabling zero-
15 stiffness regions to be modeled without making the stiffness matrix ill-conditioned, thus
16 enhancing the stability of both analysis and optimization. Within a nonlinear ITO
17 framework incorporating both material and geometric nonlinearities, we propose the
18 Log-barrier regularization function and derive its corresponding stiffness formulations
19 within a modified variational framework. Key findings from numerical studies are
20 summarized as follows:

21 1. For nonlinear IGA, the barrier regularization function significantly improves
22 mesh quality and convergence, demonstrating effective geometric control.

23 2. For nonlinear ITO, the barrier regularization function enables true void
24 modeling without introducing artificial stiffness, while substantially mitigating
25 numerical divergence under extreme deformations;

26 3. The approach shows strong topological convergence, notable computational
27 efficiency, and robust mesh control in benchmark cases such as compliant mechanism
28 designs;

29 4. When integrated into conventional FEM frameworks, the barrier regularization

also performs effectively, confirming its broad applicability across numerical platforms.

In summary, the proposed barrier regularization offers a simple, geometrically meaningful, and broadly applicable strategy for addressing mesh distortion in nonlinear topology optimization across both FEM and IGA frameworks. Moreover, since the barrier function is constructed at the energy level and depends only on the deformation gradient, it is inherently compatible with more general formulations. This makes the method extensible to three-dimensional (3D) problems, as well as nonlinear unstructured scenarios such as contact and fracture mechanics. Future work will focus on developing an adaptive barrier regularization strategy, in which the barrier penalty coefficient can be automatically adjusted based on local element deformation. Additionally, we aim to integrate the barrier regularization with non-conforming discretizations, interface tracking methods, or contact algorithms to further explore its applicability in complex multi-physics and multiscale optimization problems.

CRedit authorship contribution statement

Xinqing Li: Writing - original draft, Conceptualization, Methodology, Software, Validation. Yongzhen Mi: Writing - review & editing, Conceptualization, Methodology, Software. Zhenpei Wang: Writing, Conceptualization, Methodology, Software. David W. Rosen: Writing - review & editing, Methodology, Supervision. Yingjun Wang: Writing - review & editing, Methodology, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used ChatGPT in order to improve language and readability. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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