Color Contrast-Preserving Decolorization

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Abstract—Decolorization is to convert a color image into a grayscale image while preserving image features like salient structure and chrominance contrast. The sign of the color contrast is crucial for the decolorization algorithm and is usually determined in existing works by giving a strict defined color order or two-mode weak order. In this paper, a fast computation on color order is achieved via a simple global mapping which is introduced in a linear parametric model using an extended structure transfer filter. The values of the parameters are obtained via an elegant approximation method. A local decolorization algorithm is finally designed on basis of the global linear mapping so that both color and spatial information are preserved robustly and accurately. Experimental results show that the proposed global decolorization algorithms obtain a good performance among existing quality metrics for the decolorization. In addition, the proposed global decolorization algorithm is friendly to mobile devices with limited computational resources.

Index Terms—Perceptual decolorization, color contrast, color fidelity, C2G-SSIM, linear mapping function.

I. INTRODUCTION

DECOLORIZATION refers to the process of converting color images into grayscale images. Although color imaging devices have become ubiquitous, there is a strong demand for grayscale images in specific applications due to cost-saving and resource. The typical applications include black-and-white printing, e-book devices, digital ink display and color-to-gray scanners as well as artistic purposes. Also, grayscale representation is important for fundamental image processing and computer vision tasks such as edge detection, feature extraction etc. The reason is that intensity often captures much of the visually important information presented in the color counterpart [1]. As decolorization reduces the dimensions of the input signal, it inevitably results in information loss. The goal of decolorization is to retain as much information about the original color image as possible, while simultaneously producing perceptually plausible grayscale results.

The widely-used color-to-gray conversion is an image processing to extract the luminance (in YUV space) or lightness (in CIELab space). A significant drawback of this conversion is the loss of color contrast in areas where color contrast dominates over luminance contrast. This problem reaches an extreme for iso-luminant regions, in which the lightness is the identical but the hue or chroma are quite different. Vision scientists found that the HVS perceives luminance perception are based upon relative assessments instead of absolute values, in part due to the center-surround organization of cells in the early stages of visual processing [2]. According to this hypothesis, many methods have been proposed to account for chromatic difference in color-to-grayscale conversion. Casting the decolorization problem in the form of a constrained and multi-variate optimization, Rasche et al [3] firstly derived a sequence of linear programming problems. The number of unknown variables is large if the intensities are selected as optimization variables as in [3]. That is because only the absolute gradient field is available for the gray scale image while their color directions are unknown, which is is very challenging to solve in this nonlinear decolorization problem. A possible way to reduce the complexity is to enforce color order for neighboring pixels based on the Euclidian color distance [4], [5]. Gooch et al. [4] proposed a novel saliency-preserving color removal algorithm using the Poisson equation [6] to define the cost function for the decolorization problem. Instead of only global optimization, Kuk et al. [7] considered both local and global contrasts. Using a similar energy optimization method in [4], Kim et al. proposed a minimum gradient energy function after determining a nonlinear global mapping [5]. Unlike the decolorization algorithm in [3], the number of unknown variables is reduced using the global mapping, whose complexity is significantly reduced.

The order of the color contrast is usually determined by the L component in the Lab color space. As pointed out in [8], enforcing this type of order for color pairs could cause the contrast-loss problem. Song et al. [9] considered multiple scales to simulate human preferences and mimicked human contrast perception based on bilateral filtering, whose algorithm is a searching-based method like [8]. The complexity of searching based algorithm depends heavily on the searching space. Thus, there are only three parameters in the [8], [9] and the values of the parameters are limited. Inspired by the phenomenon that the order of different colors cannot be uniquely determined by people, Lu et al. created a parametric decolorization model using a bimodal distribution to automatically select color orders and further produced a perception-based quality metrics for evaluating contrast preserving performance [10], [11]. To speed up computation, Zhao et al [12] used a highly parallel GPU-based color pair set construction with a multi-modal Gaussian distribution, which is a extended version of [11]. However, the complexity of these algorithms could still be an issue, especially for mobile devices. The searching-based algorithms in [10], [11] are quite simple, the large number of candidates is still a barrier for real time applications on mobile devices. Therefore, it is desired to design a simple and robust decolorization algorithm.

In this paper, a robust and fast decolorization approach is proposed with a global linear mapping and a refined local
decolorization. Inspired by the algorithms in [5], [11], a global linear mapping is estimated by considering color contrast and absolute luminance with a differentiable cost function. On top of the bimodal distribution, the estimation is formulated as an optimization problem. An elegant solution is proposed to solve the optimization problem. With the proposed solution, all the logarithm, exponent and division used in the bimodal distribution are not required they are required by the algorithms in [8], [10], [11]. All these operations are not friendly to mobile devices with limited computational resource. Therefore, the proposed global decolorization algorithm reduces the complexity of the algorithms in [8], [10], [11] significantly, especially for mobile devices.

As indicated in [8], [10], [11], a challenging problem in the decolorization is to determine the chroma direction in the color contrast automatically. Fortunately, this problem can be addressed by the proposed linear mapping via a sufficient sigmoid estimation in our model, which quickly determines the sign of color order. Combined with the image structure, a suitable color order is finally defined voluntarily. On the basis of the globally linear mapping, a local decolorization algorithm is put forward to recover local fine details using the global structure transfer filter in [13]. With the help of the globally linear mapping, the following proposed local decolorization algorithm obtain its inputs, an intermediate gray scale image and a vector field that generated from the color image. In our local method, this global optimal solution is refined with a new cost function in vector field in a similar structure, which is fast via a solution based on Fast Fourier Transform (FFT). In consequence, the proposed local algorithm is fast and can be adopted for PC based applications.

The main contributions of this work are listed as follows:

1) A novel two-stage global to local decolorization method is proposed. The local chromatic contrast is refined on basis of global optimal solution which determines a color order. Both subjective and objective quality of gray scale images are improved.

2) All the logarithm, exponent and division used in the bimodal distribution are not required by the proposed solution.

3) Our work provides not only perceptually satisfied results but also stays leading in objective performance evaluation.

The rest of this paper is organized as follows. The related works are reviewed in section II. The details on the proposed globally linear mapping and local decolorization algorithm are provided in sections III and IV, respectively. Extensive experimental results are given in section V to verify the efficiency of the proposed algorithm. Finally, conclusion remarks are presented in section VI.

II. RELATED WORKS

Conventional decolorization algorithms such as RGB-to-YUV, RGB-to-CIELab, are to find a fixed and input independent transform to carry out color-to-gray conversion. They can be classified as model-driven approaches. They are good choices when computational resources were limited. Unfortunately, color contrast loss could happen in these methods. As the development of computation platform, more powerful computation platforms are available. It is possible to develop a data-driven decolorization algorithm so as to achieve a better preservation of original color contrast and visual distinctiveness.

Since the achromatic induction in color contrast usually occurs due to its neighboring induction color [14] and influences on visual color perception according to models like Wallach’s ratio rule [15], color perception concerned color-to-gray conversion also draws attention. Bala et al. [16] firstly proposed to adopt the high-frequency chrominance information to put distinctions among spatial adjacent colors in the gray resultants. Inspired by multi-scale image fusion [17], Smith et al. [18] increased perceptually-based weighting on each decomposed band pass image separately and then manipulated contrasts in different scales directly. It is worth noting that Palma-Amestoy et al. [19] devised a set of basic requirements to be fulfilled by an energy to be considered as ‘perceptually inspired’, and they proposed to use a variational formulation of color contrast enhancement that is inspired by the basic phenomenology of color perception.

Effective measurements in the gradient domain, which was firstly proposed by Fattal et al. [20], are also incorporated into the color-to-gray compression. Liu et al. built a gradient correlation similarity model (Gcs) with a constrained least square problem [21] and then introduced a log-Euclidean metrics [22]. To show both surrounding saliency and prevent local contrast, a balanced solution was presented by Du et al. [23] with a region-based saliency model to guide the color-to-gray procedure using the guided image filter (GIF) [24], [25]. Ancuti et al. [26] also employed the saliency to guide and enhance the chromatic contrast. Based on the conventional filter theory, Ji et al. [27] chose the luminance filter, which is a variant of traditional Difference of Gaussian band-pass filter, to derive gray-scales from the luminance. Zhu et al. [28] introduced the channel saliency to measure the contrast levels among color channels. These approaches adopt data fidelity measures in gradient domain and clear represent visual distinctiveness on cost of pixel-wise or region-wise computation.

Same as these existing decolorization algorithms mentioned in the Introduction section, the complexity of all these algorithms could be an issue for devices with limited computational resources, such as, smart phones.

III. THE PROPOSED GLOBALLY LINEAR MAPPING

In this section, a globally linear mapping is proposed to convert a color image into a gray scale image by extending the global structure transfer filter in [13]. An advantage of the global structure filter with respect to the Poisson equation [6] is that the matrix associated with the global structure filter
is always non-singular while the matrix associated with the Poisson equation could be singular.

A. A Linear Mapping Function

For each input RGB vector \( I(p) = (I_r(p), I_g(p), I_b(p)) \) of a pixel \( p \) in an input image, the proposed decolorization function is defined as a global mapping function

\[
g(p) = f(I(p)). \tag{1}
\]

This implies that two pixels with the same color will have the same gray scale. A global finite multivariate polynomial function is adopted for mapping in [10] so as to reduce the complexity as follows:

\[
\Pi_n = \text{span}\{I_r^{d_1}(p)I_g^{d_2}(p)I_b^{d_3}(p) : d_i \geq 0, \sum_{i=1}^{3} d_i \leq 2\}. \tag{2}
\]

The mapping function is thus expressed as in [10]:

\[
f(I(p)) = \omega^T m_i(p), \tag{3}
\]

where \( m_i(p) \) is the \( i \)th polynomial basis of \( \Pi_n \). \( \omega_i \)'s are constants and their values will be determined according to the content of the input image. The mapping function is uniquely determined by weights \( \{\omega_i\} \). Clearly, the mapping function is a linear combination of elements in \( \{I_r, I_g, I_b, I_rI_g, I_rI_b, I_gI_b, I_r^2, I_g^2, I_b^2\} \). Instead of using the polynomial function (3), a linear mapping function is adopted in this paper and it is given by [8], [9]:

\[
f(I(p)) = \omega^T I(p), \tag{4}
\]

where \( \omega = \begin{bmatrix} \omega_r & \omega_g & \omega_b \end{bmatrix}^T \) is a vector in \( \mathbb{R}^3 \) to be determined. It is worth noting that there is no constraint on the parameter \( \omega \). This is different from the algorithms in [8], [9].

Since there are only three parameters in the linear mapping function (4) while there are nine parameters in the polynomial mapping function (3), the proposed decolorization algorithm is much simpler than the algorithm in [10].

B. The Proposed Cost Function

Let the corresponding Lab vector of the vector \( I(p) \) be denoted as \( (L(p), a(p), b(p)) \). There are two terms in the proposed cost function. One term is in the image domain and it is defined as in [10]:

\[
E_1 = \sum_p (\omega^T I(p) - L(p))^2. \tag{5}
\]

The other term is defined in the gradient domain and it is determined by the following color contrast which is an extension of the MSE contrast in CIELAB:

\[
\delta(p, p') = \sqrt{(L(p) - L(p'))^2 + |a(p) - a(p'')|^2 + |b(p) - b(p'')|^2}, \tag{6}
\]

where \( p' \) is a neighborhood pixel of \( p \).

Using the equation (4), it can be computed that

\[
g(p) - g(p') = \omega^T \Lambda(p, p'), \tag{7}
\]

where \( \Lambda(p, p') \) is \( (I(p) - I(p')) \) that indicates the luminance difference among pixel \( p \) and all the neighboring pixels.

The simplest way to define the second term is

\[
E_2 = \sum_{p, p'} ||\omega^T \Lambda(p, p')| - \delta(p, p')||, \tag{8}
\]

or

\[
E_2 = \sum_{p, p'} (||\omega^T \Lambda(p, p')| - \delta(p, p'))^2. \tag{9}
\]

However, neither of them is differentiable even though they are nice cost functions for a searching-based decolorization algorithm. Here, the idea in [7], [10] is borrowed to define the second term.

Let \( \sigma \) be a very small number and denote \( G(\delta(p, p'), \omega) \) as [10]

\[
G(\delta(p, p'), \omega) = \exp\left(\frac{\omega^T \Lambda(p, p') - \delta(p, p')}{2\sigma^2}\right), \tag{10}
\]

the second term is then defined as in [7]

\[
E_2 = -\sum_{p, p'} \log(G(\delta(p, p'), \omega) + G(-\delta(p, p'), \omega)). \tag{11}
\]

This Gaussian distribution illustrates the output grayscale appearance depends on the ratio of the grayscale differences to the color differences while perceived contrast relies on their perceived chromatic direction. This function was also suggested as the color appearance in [29]. It can be easily verified that the function \( E_2 \) in the equation (11) is differentiable. The value of \( E_2 \) approaches \( \frac{\omega^T \Lambda(p, p') - \delta(p, p')}{2\sigma^2} \) if the value of \( \omega^T \Lambda(p, p') \) is positive and \( \frac{\omega^T \Lambda(p, p') + \delta(p, p')}{2\sigma^2} \) if the value of \( \omega^T \Lambda(p, p') \) is negative. Therefore, the product of \( 2\sigma^2 \) and the term \( E_2 \) in the equation (11) is an approximation of the term \( E_2 \) in the equation (9).

The cost function \( E_2 \) in the equation (11) is differentiable and it is more complicated than the function \( E_2 \) in the equation (8) or (9). All of the cost functions above can be applied to achieve the same objective but are suitable for different methods. The cost function in the equation (8) or (9) is suitable for a searching based method while the cost function in (11) is suitable for a differentiation based method.

The overall cost function is defined as

\[
E = \lambda_1 E_1 + E_2, \tag{12}
\]

where \( \lambda_1 (\geq 0) \) is a constant. Its value is selected as 1/1024 if not specified. It is worth noting that the proposed cost function is the same as those in [4], [5], [10] if the value of \( \lambda_1 \) is zero. Since the value of \( \lambda_1 \) is very small, the second term dominates the proposed cost function. The optimal solution can be obtained by solving linear equations. As shown later, the main function of the first term is to guarantee the nonsingularity of the matrix associated with the linear equations in the proposed algorithm.
C. The Proposed Solution

For simplicity, denote \( \beta(p, p', \omega) \) as [10]

\[
\beta(p, p', \omega) = \frac{G(\delta(p, p'), \omega)}{G(\delta(p, p'), \omega) + G(-\delta(p, p'), \omega)}.
\]  

(13)

Define a function \( \Gamma(p, p', \omega) \) as

\[
\Gamma(p, p', \omega) = 2\beta(p, p', \omega) - 1,
\]

(14)

and define a matrix \( A \) and a vector \( b \) as

\[
A_{i,j} = \lambda_1 \sum_p I_i(p) J_j(p) + \sum_{p,p'} \Lambda_i(p, p') \Lambda_j(p, p'),
\]

(15)

\[
b_j(\omega) = \lambda_1 \sum_p L(p) I_j(p) + \sum_{p,p'} \Gamma(p, p', \omega) \Lambda_j(p, p') \delta(p, p').
\]

(16)

It can be easily verified that the matrix \( A \) is independent of \( \omega_c \in \{r, g, b\} \) and the vector \( b \) is a nonlinear function of \( \omega_c \in \{r, g, b\} \).

Using the equation

\[
\frac{\partial E}{\partial \omega_j} = 0,
\]

(17)

three equations can be derived as

\[
\sum_{i \in \{r, g, b\}} A_{i,j} w_i = b_j(\omega).
\]

(18)

It is worth noting that the matrix \( A \) and the vector \( b \) in [10] are

\[
A_{i,j} = \sum_{p,p'} \Lambda_i(p, p') \Lambda_j(p, p'),
\]

(19)

\[
b_j(\omega) = \sum_{p,p'} \Gamma(p, p', \omega) \Lambda_j(p, p') \delta(p, p').
\]

(20)

The matrix \( A \) in the equation (19) can be expressed as \( A^T \Lambda \).

When the matrix \( M \) is not full rank, the matrix \( A \) is singular. This implies that the matrix \( A \) in [11] could be singular. On the other hand, the matrix \( A \) of the proposed decolorization algorithm can be expressed as \( A^T \Lambda + \lambda_1 I \) and is always non-singular. This ensures that our cost function has solutions in any case. Thus, it is easier to solve the proposed decolorization problem than the problem in [10].

Borrowing the idea from [10], the values of \( w_i \)'s are then iteratively computed as

\[
\omega^{(k)} = A^{-1} b(\omega^{(k-1)}),
\]

(21)

and the value of \( \omega^{(0)} \) is \((1/3, 1/3, 1/3)\).

The proposed decolorization algorithm has a low computation complexity since both the inverse matrix \( A^{-1} \) and the first term of \( b_j \) in the equation (16) are only need to compute once for all the iterations. Then, a sub-sampling based method could be also adopted to simplify the computational cost of both the matrix \( A \) and the vector \( b \) as below.

It is worth noting that the algorithm in [10] is designed by directly solving the equation (18). The complexity of such a solution could be an issue if the algorithm in [10] runs on a mobile device. This is because that the computation of \( \Gamma(p, p', \omega) \) is a heavy burden for the mobile device. To further reduce the on-line computational complexity, a novel method is proposed to simplify the computation of the second term of the vector \( b \) as below. It can be derived from the equation (13) that

\[
\beta(p, p', \omega^{(k-1)}) = \frac{1}{1 + \exp \frac{\omega^{(k-1)} T \Lambda(p, p') \delta(p, p')}}{1 + \exp \frac{\delta(p, p')}{2}}.
\]

(22)

Since the value of \( \sigma \) is a very small, \( \beta(p, p', \omega^{(k-1)}) \) is actually a classical sigmoid function. The value of \( \beta(p, p', \omega^{(k-1)}) \) can be approximated as

\[
\beta(p, p', \omega^{(k-1)}) \approx \begin{cases} 
\frac{1}{2}; & \text{ if } \omega^{(k-1)} T \Lambda(p, p') \delta(p, p') = 0 \\
0; & \text{ if } \omega^{(k-1)} T \Lambda(p, p') \delta(p, p') < 0 \\
1; & \text{ otherwise }
\end{cases}
\]

(23)

Subsequently, the value of \( \Gamma(p, p', \omega^{(k-1)}) \) is approximated by

\[
\Gamma(p, p', \omega^{(k-1)}) \approx \begin{cases} 
0; & \text{ if } \omega^{(k-1)} T \Lambda(p, p') \delta(p, p') = 0 \\
-1; & \text{ if } \omega^{(k-1)} T \Lambda(p, p') \delta(p, p') < 0 \\
1; & \text{ otherwise }
\end{cases}
\]

(24)

It follows that

\[
\Gamma(p, p', \omega^{(k-1)}) \approx SGN(\omega^{(k-1)} T \Lambda(p, p')),
\]

(25)

where the function \( SGN(x) \) is defined as

\[
SGN(x) = \begin{cases} 
0; & \text{ if } x = 0 \\
-1; & \text{ if } x < 0 \\
1; & \text{ otherwise }
\end{cases}
\]

(26)

Clearly, the sign of the color order is not specified in prior but determined on-line to optimally find the suitable color order. As such, the color contrast can be preserved better.

Even though the proposed algorithm is based on the complex cost function in the equation (11), the proposed solution looks like to be derived from the simple cost function in the equation (8) or (9). In other words, the proposed solution preserves the simplicity of the decolorization algorithm based on the cost function in the equation (8) or (9).

With the above approximation, the computational cost of \( b(\omega^{(k-1)}) \) is significantly reduced. This is because all the logarithm, exponent and division are not required in the equation (25) while they are required in the equation (14). All these operations are not friendly to mobile devices such as smart phones. Thus, our intermediate parameters are calculated once for all iterations or approximated into a basic arithmetic. Furthermore, the default number of iterations in [10], [11] is 15 and it is reduced to 10 for the proposed algorithm. Thus, the proposed algorithm is much simpler than the algorithm in [10], [11] and it is suitable for mobile device with limited computational resource.

A possible issue of the global decolorization algorithm is detail on loss of fine details in low contrast regions. In the next section, a local decolorization algorithm will be provided to address this problem.
IV. THE PROPOSED LOCAL DECOLORIZATION ALGORITHM

In this section, a local decolorization algorithm is designed on top of the globally linear mapping in the previous section. The globally linear mapping is first applied to generate an intermediate gray scale image and a vector field. The global structure transfer filter in [13] is then adopted to design such a local decolorization algorithm.

The optimally global mapping function in the equation (4) is firstly used to determine the sign of \( \delta(p, p') \). Let \( \omega^* \) be the optimal solution for the proposed decolorization algorithm. The value of \( \Gamma(p, p', \omega^*) \) is computed

\[
\Gamma(p, p', \omega^*) = SGN(\omega^*, T \Lambda(p, p')).
\]

(27)

Let \( p_b \) and \( p_r \) be the bottom and right pixels of the pixel \( p \), respectively. A vector field \((V_x(p), V_y(p))\) is then constructed as

\[
\begin{align*}
V_x(p) &= \Gamma(p, p_r, \omega^*) \delta(p, p_r), \\
V_y(p) &= \Gamma(p, p_b, \omega^*) \delta(p, p_b).
\end{align*}
\]

(28) (29)

Once the vector field is available, a gray scale image can be obtained using the Poisson equation [6]. Unfortunately, experimental results show that the resultant decolorization algorithm cannot obtain a good trade-off among existing quality metrics for the decolorization algorithm. The global structure transfer filter in [13] is then adopted to transfer the structure of the vector field \((V_x(p), V_y(p))\) to an intermediate image that is generated using the globally linear mapping in the previous section. The final image is obtained by minimizing the following cost function:

\[
\sum_{p} (L(p) - L^* (p))^2 + \lambda_2 \left( \frac{\partial L(p)}{\partial x} - v_x(p) \right)^2 + \left( \frac{\partial L(p)}{\partial y} - v_y(p) \right)^2.
\]

(30)

where the value of \( \lambda_2 \) is 3/8 if not specified in this paper. \( L^*(p) \) is an intermediate gray scale image produced by the proposed globally linear mapping.

This is a quadratic optimization problem. It can be easily solved by using the method given in [13] or a Fast Fourier Transform (FFT) based method. Therefore, the proposed local decolorization algorithm is also friendly to mobile devices with limited computational resource. Overall, the proposed local decolorization algorithm is summarized as follows:

V. EXPERIMENTAL RESULTS

To quantitatively evaluate our decolorization algorithm in contrast-preserving, we have conducted comparison experiments with the other five latest decolorization algorithms: [9], [11], [21], [22], [23]. Both our global and local algorithms are included. The experiments are executed on Cadik’s dataset [30] and all three indices are shown with averaging results of all 24 images. The value of \( \lambda_2 \) is 3/8 in our methods and other four methods employ their default parameter settings.

Algorithm 1 Locally Perceptual Decolorization

Input: a color image
Output: a gray scale image

1. Compute the color contrast via the equation (6);
2. Estimate the linear global mapping using the equations (4)-(26);
3. Produce an intermediate gray scale image using the linear global mapping;
4. Generate the vector field using the equations (27)-(29);
5. Produce the final gray scale image using the equation (30).

Fig. 1: Global linear mapping results on different iterations. (a) original image and results from (b) iteration = 3, (c) iteration = 5, (d) iteration = 8 and (e) iteration = 10.

A. Performance of the proposed global linear mapping

The experiments of the global linear mapping on different times of iterations are shown in Fig. 1. The images in the first and the second row do not change since iteration 5 while the image in the third row keeps still from iteration 8. The results in the last row look the same regardless of iterations. These reflects the fast coverage of this global mapping method. As shown in [8], [10], the linear mapping model is not sensitive to the values of \( \omega_i (i = 1, 2, 3) \). Normally, 8 to 10 times of iterations are enough to produce the output gray scale image. Hence, 10-iteration is given as a default setting as it satisfies most conditions for a natural output.

The proposed solution (25) is much simpler than the solution (14). Here, it will be shown that the quality of the gray scale image is also slightly improved. As shown in the second row of Fig. 2, the deep blue and purples billiards show diacritical gray scales in proposed solution in the equation (25) but look too similar in results from the equation (14). In the fourth row, the pear gains rough texture so that looks darker than white tennis racket. In the last row, the number ‘2’ stands out with a larger contrast in result on the right side.

The results on Cadik’s dataset for visual qualitative evaluation are shown in Fig. 3. On the grounds of visual perceptions on all sets of results, the proposed algorithm produces results with smooth transitions and achieves most nature scenes in the fourth and fifth rows. In details, this algorithm captures
both distinct edges in the second row and gradually fading edges in the sixth row. This implies that our method deals with the gradually changing color and thus is capable to handle the Helmholtz-Kohlrausch effect as luminance increased by saturation can be perceived. However, there is still space to improve the proposed global linear mapping. In the first row, the letter 'S' on the right side does not stand out distinct as the original image and loses contrast. In the last row, the top side of color-wheel does not have distinguishable gray scales for the levels among dark regions which is made up of analogous colors. These failure cases will be fixed by our local approach as local details are enhanced.

The quantitative evaluation with color-to-gray quality indices of our global algorithm will be discussed in next section, compared to our local decolorization algorithm and other state-of-art decolorization algorithms.

B. Performance of the proposed local decolorization algor-ithm

1) CCPR evaluation: There are two widely used indices in color-to-gray evaluation: CCPR and CCFR. The color contrast preserving ratio (CCPR) measures the contrast loss in color-to-gray conversion and the color contrast fidelity ratio (CCFR) tells the similarity of structures of color input in statistics [11]. The CCPR is defined as follows:

$$CCPR = \frac{\#_\Omega \{ (x,y) \mid |g_x - g_y| \geq \tau \}}{\#\Omega},$$  \hspace{1cm} (31)

where \((x,y)\) is the sub-pixel pairs in \(\Omega\) that are still distinct after decolorization process. The CCFR is defined as

$$CCFR = \frac{\#_\Theta \{ (x,y) \mid |g_x - g_y| \geq \tau \}}{\#\Theta},$$  \hspace{1cm} (32)

where \((x,y)\) in \(\Theta\) measures structures with least contrast, which means unwanted artifacts in smooth areas. \(\tau\) is the threshold to indicate a visible gray scale difference.

Lu et al. [11] also gives an joint measure of CCPR and CCFR as E-score, which is an harmonic mean of two measures and shows an more reliable evaluation as it prevents high score due to artificial new edges.

$$E\text{-score} = \frac{2 \cdot CCPR \cdot CCFR}{CCFR + CCPR}.$$  \hspace{1cm} (33)

It is observed that our global and local method are in leading positions in both CCPR and E-score because our algorithms indeed preserve image contrast in the vector field in Fig. 4. Our local algorithm also demonstrates a competitive performance in the CCFR especially when \(\tau = 7\) to 20. Overall, our local decolorization algorithm can be applied to obtain a good trade-off among all these quality measurements.

2) SSIM evaluation: Color-to-gray (C2G) SSIM index [31], which measures both luminance and contrast similarities as supplementary to structure similarity, is an advanced version of popular SSIM index [32] in the decolorization area. A contrast similarity \(L(x_c)\) is expressed as a nonlinear mapping using the Euclidean distance. Both the contrast measure \(C(x_c)\) and the structure similarity \(S(x_c)\) follows the form of SSIM for differentials and variations separately. The overall C2G-SSIM index is an combinations of three components:

$$q(x_c) = L(x_c)^{\alpha} \cdot C(x_c)^{\beta} \cdot S(x_c)^{\gamma}.$$  \hspace{1cm} (34)

Where \(\alpha, \beta\) and \(\gamma\) are control parameters to adjust importance of three components. Here \(\alpha, \beta\) and \(\gamma\) are set to 1 in our evaluations.

According to the results in Table 1, Du’s method [23] and our local approach provides the best averaging performance for all color-2-gray SSIM results in the Cadik’s dataset. From the SSIM chart, the proposed local algorithm provides the best varying range for SSIM in 0.8 to 1. Lu’s [10], Du’s [23] and LeDecolor [22] have too broad range with a lower limit around 0.75. For the upper limit, Song’s [9], GcsDecolor [21] and LeDecolor [22] does not reach a high score that exceeds 0.98 in this test set.
Fig. 3: Visual comparisons on Cadik’s dataset. (from left to right) original color image and results produced by methods: Lu et al. [11], Du et al. [23], Song et al. [9], Liu’s GcsDecolor [21], Liu’s LeDecolor [22], our global and our local algorithm, respectively.
3) Parameter selection: In this part, we test different selections of $\lambda_2$, which is a user-controlled parameter, and find the best range for $\lambda_2$. Here we compare different choices of $\lambda_2$ using three images from nature.

It is obvious in Fig. 5 that resultant images of large $\lambda_2$ suffers a fat background because background increases in gray-scale values with a larger $\lambda_2$, which makes the background more difficult to be recognized from the foreground. A large $\lambda_2$ also produces unpleasant artifacts as too large weights are assigned to the contrast in the vector field. In the picture ‘Chinese Heritage Center’, there are artificial black dots inside flower shrubs in front. Meanwhile, the Chinese letters in electronic screen in background are coarsening with a blur in the picture ‘Happy Girl’. Overall, results from $\lambda_2 = 0$, $1/8$ and $3/8$ can produce visually pleasing gray scale images. Thus, $\lambda_2$ gives nature and color-distinguishable results in a range from 0 to 0.375.

The objective evaluation on these images provides additional evidences on this claim as shown in Table 2 and Fig. 6. There are only slight differences among different $\lambda_2$ for color-to-gray SSIM evaluation in Table 2 and $\lambda_2 = 0.375$ achieves the best. This shows that the framework of the proposed algorithm is robust in the structure similarity with the parameter $\lambda_2$. In the CCFR index of Fig. 6, there are negligible differences.
TABLE I: C2G-SSIM comparison on Cadik’s dataset

<table>
<thead>
<tr>
<th>Method</th>
<th>Average</th>
<th>Lu 2012</th>
<th>Du 2015</th>
<th>Song 2013</th>
<th>Liu 2015</th>
<th>Liu 2017</th>
<th>Our Global</th>
<th>Our Local</th>
</tr>
</thead>
<tbody>
<tr>
<td>C2G-SSIM</td>
<td>0.8768</td>
<td>0.9017</td>
<td>0.8967</td>
<td>0.8833</td>
<td>0.8702</td>
<td>0.8724</td>
<td>0.9003</td>
<td></td>
</tr>
</tbody>
</table>

among all the choices of $\lambda_2$. In the In CCPR and E-score indices, $\lambda_2=0$, 0.125 and 0.375 precede in coincidence with little difference. Moreover, the CCPR and E-score stay stable for varying $\lambda_2 = 0$ to 0.375 while other methods drops sharply in Fig. 6. Overall, $\lambda_2=0$ to 0.375 achieve best in objective measurements and $\lambda_2=0.375$ is rationally chosen as the optimal value in most cases for visual and indicative considerations.

The resultant images from Cadik’s dataset are shown in Fig.3. Our local algorithm is featured in decoloring color wheel images and performs better in this kind of images. The proposed local algorithm not only reveals low-contrast details but also keeps color orders: the original darkest color is preserved as black while other colors are reasonable sorted and assigned a much lighter luminance after decoloring. When there are only two colors in the image, the proposed algorithm represents one color to the other which transits smoothly from a high to a low luminance. In general, this new local algorithm surpasses other five state-of-art algorithms and our global linear mapping.

VI. CONCLUSION

We proposed a novel two-stage decolorization algorithm in this paper by considering color contrast in both image and gradient domain. A global linear mapping is firstly obtained via a simple and fast solution based on an elegant approximation. A local algorithm is then provided to refine the global linear mapping by using a cost function in the form of structure transfer filter. Experimental results show that our approach outperforms other latest algorithms in both subjective and objective comparisons. This proposed local algorithm has high robustness in various image contents. In addition, the proposed algorithm is suitable for mobile devices with limited computational resource.

Different users usually have different preference on the resultant gray scale images. It is desired to provide an interactive decolorization algorithm so as to provide freedom for the users. One more interesting problem is to consider the case that the color contrast is distorted. It is assumed by existing decolorization algorithms that there is no distortion in the color contrast. This is not usually true. For example, the color contrast of a haze image is distorted [13], [33]. These problems will be studied in our future research.

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REFERENCES

Fig. 5: Comparisons on varying values of $\lambda_2$ in equation (30). (from left to right) (a) original color images (Chinese Heritage Center, Succulents, Happy Girl); results from (b) $\lambda_2=0$, (c) $\lambda_2=1/8$, (d) $\lambda_2=3/8$, (e) $\lambda_2=2$, (f) $\lambda_2=16$.

**TABLE II: C2G-SSIM Comparisons on varying values of $\lambda_2$ in Fig. 3.**

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_2=0$</th>
<th>$\lambda_2=0.125$</th>
<th>$\lambda_2=0.375$</th>
<th>$\lambda_2=2$</th>
<th>$\lambda_2=16$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chinese Heritage Centre</td>
<td>0.8213</td>
<td>0.8266</td>
<td>0.8314</td>
<td>0.8198</td>
<td>0.8139</td>
</tr>
<tr>
<td>Succulents</td>
<td>0.9781</td>
<td>0.9787</td>
<td>0.9791</td>
<td>0.9779</td>
<td>0.9757</td>
</tr>
<tr>
<td>Happy Girl</td>
<td>0.9065</td>
<td>0.9591</td>
<td>0.9565</td>
<td>0.9505</td>
<td>0.9428</td>
</tr>
<tr>
<td>Average</td>
<td>0.9200</td>
<td>0.9215</td>
<td>0.9223</td>
<td>0.9161</td>
<td>0.9108</td>
</tr>
</tbody>
</table>


Fig. 6: Comparison of three measures obtained by seven methods on varying values of $\lambda_2$ in Fig. 3. (x-axis: contrast threshold; y-axis: CCPR, CCFR, and E-score, respectively.)


