A two-dimensional probability model for evaluating reliability of piezoelectric micro-actuators

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Abstract

In this paper, a probabilistic approach is presented for the evaluation of the reliability of piezoelectric micro-actuators that takes into account the effects of both driving voltage and temperature. Based on the relationships between the lifetime and degradation mechanism of piezoelectric actuators and the electric field strength, as well as the actuator working temperature, a two-dimensional probability model for evaluating reliability of piezoelectric micro-actuators is described. The concept of two-dimensional strength is proposed to incorporate the electric driving voltage and the working temperature of the piezoelectric actuators at a specified lifetime. The lifetime (number of cycles to failure) of piezoelectric actuator, electric load and temperature are considered as the random variables and their probability distributions are discussed. A two-dimensional strength probability distribution function is derived. A two-dimensional interference model between the two-dimensional strength and load is also used to derive the reliability expression. Using this approach, the relationship among the reliability, lifetime, driving voltage and temperature can be described analytically. The relationship between reliability and the lifetime of piezoelectric actuator can be obtained. A case study of a piezoelectric micro-actuator used for head positioning in a disk drive head positioning system demonstrates the application of the approach. It is shown that the two-dimensional probability model can be used for reliability evaluation and lifetime estimation of piezoelectric micro-actuators according to design requirements.

Keywords: Piezoelectric actuators; Electric strength; Temperature; Reliability; Two-dimensional strength; Probability distribution; Fatigue life

1. Introduction

Piezoelectric actuators have been used as actuators, sensors or ultrasonic motors in a broad spectrum of fields such as optics, precision machining, fluid control, and disk drives because of their small size, simple structure and fast response [1–7]. In most cases, the piezoelectric actuator will experience repeated loads during their operation. Longtime usage of piezoelectric actuator will degrade the actuator performance and cause fatigue failure [8,9]. Therefore, the lifetime of piezoelectric actuators is an important factor to be considered for both researchers and engineers. Most of the research in the literature investigated the fatigue behavior and mechanism of the piezoelectric actuators and the improvement methods. In general, these can be categorized as performance degradation of piezoelectric material [10,11], actuator environmental stability [12], and fracture/crack growth of piezoelectric ceramics [13,14].

Based on earlier research [8,9,12], the performances and lifetime of piezoelectric actuators are sensitive to the operation environment and external mechanical and electric loads, experimental data shows the scatter and diversity of the material properties and the actuator performance. Therefore, there is a need to introduce probabilistic approaches and statistical tools in the reliability evaluation of piezoelectric actuators.

As a common practice in conventional fatigue design and life estimation for mechanical structure, the local strain
and stress of critical parts or locations of the structure are measured. Then the strain or stress is used incorporating the so-called measured. Then the strain or stress is used incorporating and stress of critical parts or locations of the structure are measured. Therefore, it is not convenient to obtain the local strain and stress of piezoelectric micro-actuators. However, for miniaturized piezoelectric actuators, although numerical methods can be used to obtain the stress and strain of piezoelectric material, experimentally it is not convenient to obtain the local strain and stress of the critical locations of piezoelectric micro-actuators. Therefore, \( P-S-N \) curve is not very useful for the fatigue life estimation of piezoelectric actuators.

In order to address the uncertainty factors and the performance diversities of piezoelectric micro-actuators, a probabilistic approach has been developed to evaluate the long term reliability of piezoelectric micro-actuators based on \( P-E \) curves [19]. A \( P-E \) curve describes the relationship among the probability, electric driving voltage and lifetime. An electric load–strength interference model is introduced to evaluate the reliability of piezoelectric micro-actuators. However in this model, the temperature effects are not accounted for. Temperature fluctuation is actually only considered as a noise factor in the model. In practical, temperature plays a significant role in fatigue failure of piezoelectric micro-actuators [8,9,12]. In general, an increase in temperature will reduce the lifetime of piezoelectric micro-actuators. Also, a temperature change shows dominant uncertainties in the daily operation of piezoelectric actuators.

Nomenclature

- \( N \) lifetime (life cycles) of piezoelectric actuator
- \( S \) nominal critical stress of material of a component or structure
- \( \varepsilon \) critical strain of material of a component or structure
- \( E \) driving voltage, electric strength
- \( e \) driving voltage, electric load
- \( T \) temperature strength
- \( t \) temperature load
- \( k \) Boltzmann’s constant
- \( (E, T) \) two-dimensional strength
- \( (e, t) \) two-dimensional load
- \( f(N/E, T) \) probability density function of lifetime at a specified two-dimensional strength
- \( P \) cumulative probability distribution function
- \( r \) coefficient of determination
- \( N-E-T \) plot of the relationship among the lifetime, surface electric strength, and temperature strength
- \( P-N-E-T \) plot of the relationship among the lifetime, surface electric strength and temperature with corresponding to reliability
- \( P(E, T) \) two-dimensional probability distribution of strength
- \( \mu(E, T) \) mean of logarithm of lifetime at a specified two-dimensional strength
- \( \sigma(E, T) \) standard deviation of logarithm of lifetime at a specified two-dimensional strength
- \( w_e(e) \) probability density function of electric load
- \( w_t(t) \) probability density function of temperature load
- \( w(e, t) \) probability density function of two-dimensional load
- \( N_d(E, T) \) minimum value of lifetime (Weibull distribution) at a specified two-dimensional strength
- \( N_c(E, T) \) characteristic value of lifetime (Weibull distribution) at a specified two-dimensional strength
- \( b(E, T) \) Weibull shape parameter at a specified two-dimensional strength
- \( R(N) \) reliability function, i.e., reliability as a function of lifetime cycles

For the study of the temperature effects in the reliability model of piezoelectric actuators, we consider both the electric driving voltage and the temperature as random variables in this paper. A two-dimensional reliability model is presented for evaluating the reliability of piezoelectric actuators. The \( P-N-E-T \) surface, which describes the relationship among the reliability, lifetime, electric driving voltage, and temperature, is presented. Based on that the probability distribution of lifetime usually follows a lognormal or Weibull distribution [20–23], the two-dimensional strength probability distribution function is derived. A two-dimensional load–strength interference model is proposed to calculate the reliability versus lifetime. In addition, a case study of a piezoelectric micro-actuator used for dual-stage head positioning in a disk drive demonstrates the application of the approach.

2. \( N-E-T \) surface and \( P-N-E-T \) surface

For multi-layer ceramic actuators, it is observed that the logarithm of the lifetime has a linear relationship with the logarithm of the electric field with a constant temperature [8], which is expressed as

\[
\log N = B \log E + C
\]  

where \( B \) and \( C \) are constants, \( E \) is the applied electric driving voltage, and \( N \) is the life cycles. Based on this relationship, He et al. [19] presented an \( E-N \) curve and a \( P-E-N \) curve which are used for the lifetime estimation and reliability evaluation of piezoelectric actuators. The
$E-N$ and $P-E-N$ curves can be used to determine the probability distributions of the lifetime and electric strength of piezoelectric actuators. With probability distributions of electric strength and electric load, the load–strength interference model can be set up to evaluate the reliability of piezoelectric actuators. However, in this load–strength interference model, only the electric driving voltage and working temperature are considered as the random variables. The effects of temperature variations are not taken into account. The model can be termed as one-dimensional reliability model.

In fact, for a constant driving voltage, it is also observed that a linear relationship exists between the logarithm of lifetime and the reciprocal of absolute temperature [8]. That is

$$N = A \exp \left(W/(k \times T) \right)$$

(2)

where $W$ is activation energy (J), $k$ is Boltzmann’s constant ($=1.38 \times 10^{-23}$ J/K), $T$ is absolute temperature (K) and $A$ is a constant. From experimental data, Nakamura et al. [9] derived an equation to formulate the relationship among the lifetime, electric driving voltage, and working temperature of piezoelectric micro-actuator as

$$\log N = B \log E + Q/(k \times T) + C$$

(3)

where $Q$ is a constant. The plot of Eq. (3) can be termed as $N-E-T$ surface, which corresponds to a probability of 50%.

Based on Eq. (3), considering the variation of material properties and uncertainties of the environment, we can consider $N$, $E$ and $T$ as random variables. The lifetime $N$ at a given electric driving voltage and a given temperature will then follow a certain probability distribution.

Similarly, the electric strength and temperature $(E, T)$ will also follow a certain probability distribution at a specified lifetime. Therefore, $(E, T)$ is a called a two-dimensional strength vector, which indicates a combination of the actuator driving voltage and working temperature corresponding to a specified lifetime. The $N-E-T$ surface with a certain probability $P$ is termed as $P-N-E-T$ surface. The $N-E-T$ surface and $P-N-E-T$ surface are used for the lifetime estimation and reliability evaluation of piezoelectric actuators by taking into account both electric driving voltage and working temperature, the most significant factors affecting the performance and lifetime of piezoelectric actuators.

3. Two-dimensional probability distribution of strength for piezoelectric micro-actuators

According to $N-E-T$ and $P-N-E-T$ surfaces, the two-dimensional strength for piezoelectric micro-actuator is expressed as $(E, T)$ and for an arbitrary $(E_0, T_0)$ on the $N-E-T$ surface, the probability distribution can be expressed as

$$P(E_0, T_0) = P(E_{\text{min}} \leq E \leq E_0, T_{\text{min}} \leq T \leq T_0)$$

(4)

where $E_{\text{min}}$ and $T_{\text{min}}$ are the minimum value of $E$ and $T$, respectively.

Based on the equivalence between the failure probability of electric strength at a specified lifetime and that of lifetime at a given electric strength [19], we have

$$P(E_{\text{min}} \leq E \leq E_0/T_0) = \int_{N_{\text{min}}}^{N_0} f(N/E_0) dN$$

$$= \int_{N_{\text{min}}}^{N_0} f(N/(E_0, T_0)) dN$$

(5)

where $f(N/(E_0, T_0))$ is the probability density function of lifetime $N$ at a given two-dimensional strength $(E_0, T_0)$.

Similar derivation applied to the temperature $T$ yields the following

$$P(T_{\text{min}} \leq T \leq T_0/E_0) = \int_{N_{\text{min}}}^{N_0} f(N/T_0) dN$$

$$= \int_{N_{\text{min}}}^{N_0} f(N/(E_0, T_0)) dN$$

(6)

Therefore, we have

$$P(E_{\text{min}} \leq E \leq E_0/T_0) = P(T_{\text{min}} \leq T \leq T_0/E_0)$$

(7)

If the lifetime of individual $(E, T)$ belonging to the population $(E_{\text{min}} \leq E \leq E_0, T_{\text{min}} \leq T \leq T_0)$ is $N$, there will certainly be an individual $(E', T_0) (E' \leq E_0)$ which corresponds to the lifetime of $N$. Hence, every individual $(E, T)$ which belongs to the population $(E_{\text{min}} \leq E \leq E_0, T_{\text{min}} \leq T \leq T_0)$ also belongs to the region $(E_{\text{min}} \leq E \leq E_0/T_0)$, on the condition $T = T_0$. Therefore, the occurrence of the event $(E_{\text{min}} \leq E \leq E_0, T_{\text{min}} \leq T \leq T_0)$ will cause the occurrence of the event $(E_{\text{min}} \leq E \leq E_0/T_0)$. Hence, we have

$$P(E_{\text{min}} \leq E \leq E_0, T_{\text{min}} \leq T \leq T_0) \geq P(E_{\text{min}} \leq E \leq E_0/T_0)$$

(9)

On the other hand, the region $(E_{\text{min}} \leq E \leq E_0/T_0)$ is a sub-assembly of the region $(E_{\text{min}} \leq E \leq E_0, T_{\text{min}} \leq T \leq T_0)$. Therefore, the occurrence of the event $(E_{\text{min}} \leq E \leq E_0/T_0)$ will cause the occurrence of the event $(E_{\text{min}} \leq E \leq E_0, T_{\text{min}} \leq T \leq T_0)$. Hence,

$$P(E_{\text{min}} \leq E \leq E_0, T_{\text{min}} \leq T \leq T_0) \geq P(E_{\text{min}} \leq E \leq E_0/T_0)$$

and we have,

$$P(E_{\text{min}} \leq E \leq E_0, T_{\text{min}} \leq T \leq T_0) = P(E_{\text{min}} \leq E \leq E_0/T_0) = P(T_{\text{min}} \leq T \leq T_0/E_0)$$

(10)

By Eqs. (5), (6) and (10),

$$P(E_0, T_0) = P(E_{\text{min}} \leq E \leq E_0, T_{\text{min}} \leq T \leq T_0)$$

$$= \int_{N_{\text{min}}}^{N_0} f(N/(E_0, T_0)) dN$$

(11)

Choosing an arbitrary point $(N, E, T)$ at the $N-E-T$ surface instead of $(N_0, E_0, T_0)$, we can obtain the probability distribution function of two-dimensional strength $(E, T)$ as

$$P(E, T) = \int_{N_{\text{min}}}^{N} f(N/(E, T)) dN$$

(12)
where \( f(N/E, T) \) is the probability density function of lifetime \( N \) at a given two-dimensional strength \((E, T)\). A lot of research has revealed that the lifetime of material failure follows a lognormal distribution or a Weibull distribution [20–23]. We will discuss the two cases in the followings.

3.1. Lognormal distribution

In this case, the logarithm of lifetime follows a normal distribution, i.e., the probability density function of the logarithm of lifetime can be written as

\[
f(\log N/E, T) = \frac{1}{\sqrt{2\pi}\sigma(E, T)} \exp\left\{ -\frac{[\log N - \mu(E, T)]^2}{2\sigma^2(E, T)} \right\}
\]

where \( \mu(E, T) \) and \( \sigma(E, T) \) are the mean value and standard deviation of lifetime at a given two-dimensional strength \((E, T)\). They are the functions of the electric driving voltage \( E \) and the temperature \( T \) and can be obtained by the regression of experimental data. By Eqs. (12) and (13), we have

\[
P(E, T) = \int_{\log N_{\min}}^{\log N} \frac{1}{\sqrt{2\pi}\sigma(E, T)} \exp\left\{ -\frac{[\log N - \mu(E, T)]^2}{2\sigma^2(E, T)} \right\} d(\log N)
\]

By introducing \( u \), where

\[
u = \frac{\log N - \mu(E, T)}{\sigma(E, T)}
\]

and let \( \log N_{\min} = -\infty \). Then, Eq. (14) becomes,

\[
P(E, T) = \int_{-\infty}^{\log N_{\min}} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du
\]

or

\[
u_p = \frac{\log N_p - \mu(E, T)}{\sigma(E, T)}
\]

and \( \nu_p \) is the standard normal deviate which can be obtained from the standard normal distribution table, i.e.,

\[
p = \int_{\nu_p}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du
\]

and, \( p = 1 - P(E, T) \). \( N_p \) is the corresponding value of the logarithm of lifetime.

3.2. Weibull distribution

When the lifetime follows a Weibull distribution, the probability density function can be written as

\[
f(N/E, T) = \frac{b(E, T)}{N_a(E, T) - N_0(E, T)} \left[ \frac{N - N_0(E, T)}{N_a(E, T) - N_0(E, T)} \right]^{b(E, T) - 1} \times \exp\left\{ -\left[ \frac{N - N_0(E, T)}{N_a(E, T) - N_0(E, T)} \right]^{b(E, T)} \right\}
\]

where \( N_0(E, T), N_a(E, T) \) and \( b(E, T) \) are the minimal lifetime, characteristic lifetime and Weibull shape parameter, respectively. They are the functions of the two-dimensional strength \((E, T)\) and can be obtained by the regression of experimental data.

According to Eqs. (12) and (20) and let \( N_{\min} = N_0(E, T) \), the probability distribution function can be written as

\[
P(E, T) = \int_{N_0(E, T)}^{N} \frac{b(E, T)}{N_a(E, T) - N_0(E, T)} \left[ \frac{N - N_0(E, T)}{N_a(E, T) - N_0(E, T)} \right]^{b(E, T) - 1} \times \exp\left\{ -\left[ \frac{N - N_0(E, T)}{N_a(E, T) - N_0(E, T)} \right]^{b(E, T)} \right\} dN
\]

By introducing \( Z \) as

\[
Z = \frac{N - N_0(E, T)}{N_a(E, T) - N_0(E, T)}
\]

Eq. (21) becomes

\[
P(E, T) = \int_{0}^{Z_{\infty}} e^{-Z^2} dZ
\]

4. Interference model for two-dimensional load \((e, t)\) and two-dimensional strength \((E, T)\)

If we let the probability density function of two-dimensional load, i.e, driving voltage and temperature \((e, t)\) as \( w(e, t) \), according to the probability distribution of two-dimensional strength, which is given in (12), the survival probability of piezoelectric micro-actuators can be written as

\[
dR = (1 - P(e, t/N)) \cdot w(e, t) \cdot de \cdot dt
\]

By integrating in the region of every possible \((e, t)\), the reliability can be written as

\[
R(N) = \int_{0}^{N} dR = \int_{0}^{N} (1 - P(e, t/N)) \cdot w(e, t) \cdot de \cdot dt
\]

i.e.,

\[
R(N) = 1 - \int_{0}^{N} P(e, t/N) \cdot w(e, t) \cdot de \cdot dt
\]

\[
= 1 - \int_{0}^{N} w(e, t) \cdot de \cdot dt \cdot \int_{N_{\min}}^{N} f(N/e, t) dN
\]

This is the interference model of two-dimensional load and two-dimensional strength, which gives the relationship between the reliability and lifetime.

When the logarithm of lifetime follows normal distribution, by Eqs. (17) and (26), the relationship between the reliability and lifetime can be written as

\[
R(N) = \int_{0}^{\infty} w(e, t) \cdot de \cdot \int_{\nu_p}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du
\]

(27)
When the lifetime follows Weibull distribution, by Eqs. (23) and (26), the relationship between the reliability and lifetime can be written as

$$ R(N) = \int \int_D w(e, t) \cdot \exp \left\{ - \frac{N - N_0(e, t)}{N_0(e, t) - N_0(e, t)} \right\} \cdot \frac{b(e)}{\Gamma(b(e))} \cdot t^{b(e) - 1} \cdot e^{-(t/N_0(e, t))^{b(e)}} \cdot \frac{1}{N_0(e, t)} \cdot \frac{d\theta}{\theta} \cdot dt $$

(28)

5. A case study

The case study in this section is the application of piezoelectric micro-actuator in magnetic recording head positioning system as described in [19]. In magnetic recording, hard disk drives (HDDs) are the most useful external storage devices for computer systems and home information equipment. Digital information is recorded in concentric tracks on the disk by means of miniaturized read/write (R/W) heads. These (R/W) heads are mounted on a flying slider. The slider is connected to a stainless steel suspension that in turn connected to an actuator arm, which is actuated by a voice coil motor. This setup enables cross-track seeking and track following operation of the R/W heads over the rotating disks. As the recording density of magnetic media continuously increases, the track density has also been increasing rapidly.

Due to the nonlinear friction in the pivot and structural resonance of the arm and suspension, it has become increasingly more difficult for the current servo system to satisfy the positioning requirement of high track density hard disk drives. An accepted solution to overcome the servo bandwidth limitations is a dual-stage servo control system, which uses the voice coil motor in combination with a micro-actuator. Piezoelectric micro-actuators are one of the options used as the secondary micro-actuator for their high resolution, fast response and high resonance frequencies [3–7].

Fig. 1 shows a disk drive assembly with a piggyback piezoelectric micro-actuator [9]. From the experimental results obtained by Nakamura et al. [9], we list the lifetime of the piezoelectric micro-actuator at different driving voltage and temperature in Table 1.


Based on Eq. (3) which describes the relationship among the lifetime, driving voltage and temperature, the least square method is used for the multiple linear regression [24]. The regression equation is obtained as

$$ \log N = -0.582 - 15.776 \log E + \frac{9.7704 \times 10^{-20}}{kT} $$

(29)

Furthermore, the difference of the lifetime between the experimental data and the same point at the fitted curve is also computed and shown in Table 2. The quantity \( y' \) is calculated as

$$ y' = -0.582 - 15.776 \log E + \frac{9.7704 \times 10^{-20}}{kT} - \log N_i $$

(30)

A probability plot of \( y' \) is shown in Fig. 2. We can see that \( y' \) can be considered following a normal distribution \( N(\mu, \sigma) \). The estimates of \( \mu, \sigma \) are

$$ \mu = \frac{1}{n} \sum_i y'_i = 0 $$

(31)

### Table 1

Lifetime of the piggyback piezoelectric micro-actuators in a hard disk drive dual-stage system

<table>
<thead>
<tr>
<th>Observations</th>
<th>Temperature (K)</th>
<th>Driving voltage, ( E_i ) (V)</th>
<th>Lifetime, ( N_i ) (cycles)</th>
<th>( \log E_i )</th>
<th>( \log N_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>298</td>
<td>6</td>
<td>1.9 \times 10^10</td>
<td>0.778</td>
<td>10.279</td>
</tr>
<tr>
<td>2</td>
<td>298</td>
<td>7</td>
<td>3.5 \times 10^6</td>
<td>0.845</td>
<td>9.544</td>
</tr>
<tr>
<td>3</td>
<td>298</td>
<td>9</td>
<td>9.6 \times 10^5</td>
<td>0.954</td>
<td>8.982</td>
</tr>
<tr>
<td>4</td>
<td>298</td>
<td>10</td>
<td>7.3 \times 10^5</td>
<td>1.000</td>
<td>7.865</td>
</tr>
<tr>
<td>5</td>
<td>298</td>
<td>11</td>
<td>1.1 \times 10^6</td>
<td>1.041</td>
<td>6.041</td>
</tr>
<tr>
<td>6</td>
<td>348</td>
<td>5</td>
<td>1.1 \times 10^6</td>
<td>0.699</td>
<td>9.041</td>
</tr>
<tr>
<td>7</td>
<td>348</td>
<td>6</td>
<td>9.0 \times 10^6</td>
<td>0.778</td>
<td>8.954</td>
</tr>
<tr>
<td>8</td>
<td>348</td>
<td>7</td>
<td>6.0 \times 10^6</td>
<td>0.845</td>
<td>5.778</td>
</tr>
<tr>
<td>9</td>
<td>373</td>
<td>5</td>
<td>1.1 \times 10^7</td>
<td>0.699</td>
<td>7.041</td>
</tr>
<tr>
<td>10</td>
<td>373</td>
<td>6</td>
<td>5.5 \times 10^3</td>
<td>0.778</td>
<td>5.740</td>
</tr>
<tr>
<td>11</td>
<td>373</td>
<td>7</td>
<td>1.0 \times 10^3</td>
<td>0.845</td>
<td>5.005</td>
</tr>
</tbody>
</table>

### Table 2

Lifetime difference between experimental data and estimated values

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>( \log E_i )</th>
<th>( \log N_i )</th>
<th>( -0.582 - 15.776 \log E_i + \frac{9.7704 \times 10^{-20}}{kT} ) (cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>298</td>
<td>0.778</td>
<td>10.279</td>
<td>10.9027</td>
</tr>
<tr>
<td>298</td>
<td>0.845</td>
<td>9.544</td>
<td>9.8457</td>
</tr>
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<td>298</td>
<td>0.954</td>
<td>8.982</td>
<td>8.1261</td>
</tr>
<tr>
<td>298</td>
<td>1.000</td>
<td>7.863</td>
<td>7.4004</td>
</tr>
<tr>
<td>298</td>
<td>1.041</td>
<td>6.041</td>
<td>6.7536</td>
</tr>
<tr>
<td>348</td>
<td>0.699</td>
<td>9.041</td>
<td>8.7425</td>
</tr>
<tr>
<td>348</td>
<td>0.778</td>
<td>8.954</td>
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<td>348</td>
<td>0.845</td>
<td>5.778</td>
<td>6.4392</td>
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<td>373</td>
<td>0.699</td>
<td>7.041</td>
<td>7.3718</td>
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<tr>
<td>373</td>
<td>0.778</td>
<td>5.740</td>
<td>6.1255</td>
</tr>
<tr>
<td>373</td>
<td>0.845</td>
<td>5.000</td>
<td>5.0685</td>
</tr>
</tbody>
</table>

Fig. 1. A push–pull multi-layer piggyback piezoelectric actuator for dual-stage servo in a hard disk drive [9].
Let \( x = (y_i - \bar{y}) / \sigma_y \), then \( x \) follows the standard normal distribution \( N(0, 1) \). Therefore, for a specified survival probability \( p \), we have

\[
\begin{align*}
\bar{y}_p &= \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n - 1} = 0.6963 \\
\end{align*}
\]  

For a survival probability, \( p = 99.9\% \), the normal standard deviate \( u_p = -3.09 \), then the \( P-E-T-N \) equation corresponding to 99.9\% survival probability can be written as

\[
\log N_{99.9} = -2.7336 - 15.776 \log E + \frac{9.7704 \times 10^{-20}}{kT} 
\]  

Fig. 3 shows the 50\% and 99.9\% probability \( E-T-N \) surface. Fig. 4 shows that \( P-E-T \) curve at a specified lifetime \( N = 10^{10} \) cycles. The \( E-T-N \) curve can be used to determine the driving voltage and working temperature according to specified reliability and lifetime requirements.

5.2. Determination of probability distribution function of two-dimensional strength

A probability plot of the logarithm of lifetime list in Table 1 is shown in Fig. 5. It can be seen that the logarithm of lifetime can be considered following a normal distribution.

Fig. 3. The \( N-E-T \) and \( P-N-E-T \) surface of the piggyback piezoelectric actuators.
According to Eq. (18)

\[
\begin{align*}
\upsilon_{50} &= 0 \\
\upsilon_{99.9} &= -3.09
\end{align*}
\]

We have

\[
\begin{align*}
\mu(E, T) &= -0.582 - 15.776 \log E + \frac{2.704 \times 10^{-20}}{4T} \\
\sigma(E, T) &= 0.6963
\end{align*}
\]

Therefore, the probability density function of the logarithm of lifetime, \(\log N\) can be written as

\[
\begin{align*}
f(\log N/E, T) &= \frac{1}{\sqrt{2\pi \times 0.6963}} \cdot \exp \left\{ \frac{[\log N + 0.582 + 15.776 \log E - 9.7704 \times 10^{-20}/(kT)^2]^2}{2 \times 0.6963^2} \right\}
\end{align*}
\]

The probability distribution function of the two-dimensional strength \((E, T)\) can be written as

\[
P(E, T) = \int_{\log N_{\min}}^{\log N} \frac{1}{\sqrt{2\pi \times 0.6963}} \cdot \exp \left\{ \frac{[\log N + 0.582 + 15.776 \log E - 9.7704 \times 10^{-20}/(kT)^2]^2}{2 \times 0.6963^2} \right\} d(\log N)
\]

5.3. Reliability evaluation

As reported in [19], the input voltage (sinusoidal AC) of a piggyback piezoelectric micro-actuator for the dual-stage control in a hard disk drive follows a Weibull distribution. The probability density function is

\[
w_e(e) = \frac{2.019 \left( e \right)^{1.019}}{2.67} \cdot \exp \left\{ -\left( \frac{e}{2.67} \right)^{2.019} \right\}
\]

In general practice, the design temperature for a hard disk drive is from 5 to 55 °C (278–328 K) [25], therefore, we assume the temperature follows a normal distribution with the mean \(\mu = 303 \text{ K}\) and six sigma \(\sigma = 328 - 278 = 50 \text{ K}\). The probability density function of temperature in a hard disk drive is written as

\[
w_t(t) = \frac{1}{\sqrt{2\pi \times 8.33}} \cdot \exp \left\{ -\left( t - 303 \right)^2 \right\}
\]

Further, the driving voltage \(e\), and temperature \(t\) are considered as independent random variables, the joint probability density function of \((e, t)\) can be written as

\[
w_e(t) = \frac{2.019 \left( e \right)^{1.019}}{2.67} \cdot \exp \left\{ -\left( \frac{e}{2.67} \right)^{2.019} \right\}
\]

By Eqs. (26), (39) and (42), the relationship between the reliability and the lifetime is expressed as

\[
R(N) = 1 - \int \int_{\Omega} 2.019 (e^{2.67})^{1.019} \exp \left[ -\left(\frac{e}{2.67}\right)^{2.019} \right] \cdot \frac{1}{\sqrt{2\pi} \times 8.33} \exp \left[ -\frac{(t - 303)^2}{2 \times 8.33^2} \right] \, de \cdot dt \\
\cdot \int_{\log N_{\min}}^{\log N} \frac{1}{\sqrt{2\pi} \times 0.6963} \cdot \exp \left\{ -\frac{\log N + 0.582 + 15.776 \log e - 9.7704 \times 10^{-20} / (kt)^2}{2 \times 0.6963^2} \right\} \, d(\log N)
\]

The integration range is selected as \(0 < e < \infty, 0 < t < \infty, \log(N_{\min}) = 0\). The plotting of Eq. (43) is shown in Fig. 6. We can see that with the increasing of lifetime, the reliability drops down. Generally the piezoelectric actuators are designed for 5 years use according to hard disk drive design requirement. It corresponds to \(4.7 \times 10^{11}\) cycles at 3 kHz driving. The calculated reliability is 96.3%.

Fig. 5. Probability plot of logarithm of lifetime log \(N\).

Fig. 6. Relationship between reliability and lifetime of piggyback piezoelectric micro-actuator.
Compared with the estimated reliability 96.7% obtained by one-dimensional model [19], the difference is less than 0.5%. In one-dimensional model, only the electric driving voltage is considered as the random variable, while the temperature is fixed at the room temperature. All the experimental data in one-dimensional model are selected when the temperature is fixed at 25 °C. Therefore, the temperature variation is well addressed by the two-dimensional model.

6. Summary

This paper describes a probability approach to evaluate the reliability of piezoelectric micro-actuators with consideration of temperature effects. A concept of two-dimensional strength (electric driving voltage and temperature) is proposed. According to the relationship among the electric driving voltage, the temperature, and the lifetime, the two-dimensional strength surface is presented. The probability distribution function of the two-dimensional strength is derived.

For the reliability analysis, a two-dimensional interference model using probabilistic approach is proposed here for the reliability evaluation of piezoelectric micro-actuators. With this interference model, the relationship between the reliability (survival probability) and the lifetime is established.

We also derived the probability distribution of the two-dimensional strength. It is shown that there exists equivalence between the failure probability of electric driving voltage, temperature at a specified lifetime and that of the lifetime at a given electric driving voltage or temperature. Based on this equivalence, the probability distribution of two-dimensional strength can be derived according to the probability distribution of lifetime. Generally in the practice of fatigue reliability evaluation, the probability distributions of the lifetime are determined by experiment and they follow lognormal distribution and Weibull distribution. The probability of two-dimensional load is characterized by stochastic process and statistical evaluation of a load spectrum of a certain period of time.

A case study of piezoelectric micro-actuator in magnetic disk drive dual-stage servo system is described to demonstrate the application of the probabilistic approach. In this case, the experimental data reveals that the lifetime of the selected piggyback piezoelectric actuator follows a lognormal distribution. The probability distribution of the two-dimensional strength of the piggyback piezoelectric actuator is obtained using the method described with the known probability distribution of the lifetime.

References