Abstract—A major concern from potential white space device (WSD) users is the possibility of zero TVWS channel available at their locations. This resulted the regulator of Singapore to proposed a new type of TVWS channels called the high priority channels (HPCs). The HPCs are TVWS channels that will always be available to unlicensed WSDs although white space database (WSDB) provider may impose fees to WSDs for using HPCs. This new type of payable TVWS channels is of interest to the WSDB providers since it creates a new business opportunity for them. In this paper, we investigate the optimal fee for using HPCs such that WSDB provider achieves the maximum revenue. The solution of the optimal fee is provided in this paper. From this study, it is observed that WSDB provider should not always increase the fee when the total number of WSDs increases as our results show that sometimes lowering the fee to attract more WSDs to use HPCs actually increases the revenue.

I. Introduction

Television white spaces (TVWS) are the underutilized spectra that have been allocated for television (TV) broadcast. Regulators around the world have been proposing to allow unlicensed white space devices (WSDs) to use TVWS to increase the utilization [1], [2]. This includes the regulator of Singapore, Infocomm Development Authority (IDA), as previous study [3] has shown that most of the TV channels are underutilized in Singapore. To advance the TVWS technology developments in Singapore, IDA has finalized the regulatory framework for TVWS operations in June 2014 [4].

During the consultation phase of TVWS regulations in Singapore, one major concern from the industries is the possibility of zero TVWS channel available for WSDs in time or geographic locations as some applications may require more certainty in transmission. This may be a concern to other WSD users in other countries as well although most existing TVWS regulations worldwide have not address this concern yet. To address this concern, IDA proposed a new type of TVWS channels called the High-Priority Channels (HPCs) which are always available to the WSDs. However, white space database (WSDB) provider may impose fees to WSDs for using the HPCs. For a start, IDA has proposed to designate two HPCs within the TVWS channels for WSDs that are willing to pay for higher level of spectrum availability and access.

The fundamental task of the WSDB is to provide WSDs the information of available TVWS channels at their locations. With the introduction of HPCs, it created a new business opportunity for WSDB provider to earn revenue from WSDs that do not mind paying a fee for better quality-of-service (QoS). In future, pay-per-use unlicensed channels may also become a common feature for spectrum sharing if the demand for better QoS in unlicensed channels becomes high. In view of this development, we investigate how WSDB provider could maximize its revenue with the management of HPCs access.

A hybrid pricing scheme for WSDB provider is proposed in [5], where the WSDB provider will pay a fixed fee to regulator to reserve TVWS channels and then sublet the reserved channels back to WSDs for a fee. Each WSD in these reserved channels can occupy parts of the TV channels by itself without sharing with other WSDs. The rest of WSDs can use the non-reserved TVWS channels for a fee depending on the number of queries. A two dimensional exhaustive search is used to find the optimal fee for WSDs and the amount of TVWS channels that the WSDB provider needs to reserved. The operating model in [5] is different from HPCs and the criterion for WSDs to justify their rewards for a fee is different from this paper where they are based on channel capacity while we consider probability of successful packet transmission.

The rest of this paper is organized as follows. We will first introduce the system model of HPCs, their fee and reward function in Section II. In Section III, we provide the mathematical formulation of maximizing the revenue of WSDB provider and also the solution to the problem. Numerical results of the optimal fee and revenue are shown in Section IV. Finally, concluding remarks will be given in Section V.

II. System Model

In this paper, we consider that the WSDs coexist within TVWS channels via carrier sense multiple access with collision avoidance (CSMA/CA) protocol. Each WSD will transmit a packet in a slot time with a probability $\tau$ which is a function of the contention window in the CSMA/CA protocol [6]. In a constant backoff window problem, it was found that $\tau = \frac{W}{W+1}$ where $W$ is the minimum contention window [6]. Therefore, the average probability of a WSD transmitting a packet successfully in normal TVWS channels is given as

$$P_N = (1 - \tau)^{n_N/m_N-1}$$

where $n_N$ is the total number of WSDs in normal TVWS channels and $m_N$ is the total number of normal TVWS channels. Similarly, the average probability of a WSD transmitting a packet successfully in HPCs is given as

$$P_H = (1 - \tau)^{n_H/m_H-1}$$
where $n_H$ is the total number of WSDs in HPCs and $m_H$ is the total number of HPCs. Let the ratio of $P_H$ to $P_N$ be the reward function for a WSD to switch from normal TVWS channels to HPCs which is given as

$$R = (1 - \tau)^{n_H/m_H - n_N/m_N}.$$  

The fee that a WSD is willing to pay WSDB provider for using HPCs will be a function of the reward function $R$. When $R$ is less than or equal to 1, there is no benefit in switching from normal channels to HPCs since the probability of transmitting a packet successfully in normal channels is higher or the same as in HPCs. Hence, WSDs will not pay any fee to move to HPCs when $R \leq 1$. When $R > 1$, we assume that WSDs are willing to pay a fee proportional to $R$ until the fee reaches a maximum limit $f_M$. This is the price cap of the fee that a WSD is willing to pay regardless of how high $R$ is. In practice, WSDs may listen to the TVWS channels to estimate $R$, $N$ is the total number of WSDs. First, we determine $f_M$ based on the QoS constraint that a WSD is willing to pay a fee proportional to $R$ which is given as

$$R = f_M = \frac{m_H N + m_H m_N \alpha(\tilde{f})}{m_H + m_N}$$  

where

$$\alpha(\tilde{f}) = \frac{\ln \left( (R_T - 1) + f_M \right) - \ln(f_M)}{\ln(1 - \tau)}.$$  

Based on the QoS criterion that a WSD will join HPCs if $P_H \geq P_H$, we obtain

$$n_H \leq m_H \left( \frac{\ln(\bar{P}_H)}{\ln(1 - \tau)} + 1 \right) = \bar{n}_H.$$  

Therefore, from (7) and (9), the number of WSDs that will use HPCs at a given fee $\tilde{f}$ is given as

$$n_H(\tilde{f}) = \max \left\{ 0, \min \left\{ \left[ \frac{m_H N + m_H m_N \alpha(\tilde{f})}{m_H + m_N} \right], \bar{n}_H \right\} \right\} \text{ (10)}$$

where $[x]$ means rounding down $x$ to the nearest integer. The total fee collected by WSDB provider when it sets its fee to $\tilde{f}$ is given as

$$F(\tilde{f}) = n_H(\tilde{f}) \tilde{f}.$$  

From (10), $n_H(\tilde{f}) = \lceil \bar{n}_H \rceil$ when

$$f \leq f_M (1 - \tau)^{\frac{m_H + m_N}{m_H + m_N} \lceil \bar{n}_H \rceil - N/m_N - f_M}.$$  

This means that if a WSDB provider sets a fee lower than the right hand term in (12), it will not attract further more WSDs to use HPCs as the number of WSDs in HPCs is already saturated and could not provide the required QoS if more WSDs are to join in. Hence, the minimum fee that a WSDB provider should set based on the QoS constraint is given as

$$f_{QoS} = \max \left\{ 0, f_M (1 - \tau)^{\frac{m_H + m_N}{m_H + m_N} \lceil \bar{n}_H \rceil - N/m_N - f_M} \right\} \text{ (13)}$$

unless $f_{QoS} > f_M$ which is when

$$N > \frac{m_H + m_N}{m_H} \lceil \bar{n}_H \rceil - \frac{m_N \ln(R_T)}{\ln(1 - \tau)}.$$  

In this case, it is straightforward that the optimal fee $f^*$ is $f_M$ and the optimal revenue collected by the WSDB provider is $F^* = \lceil \bar{n}_H \rceil f_M$. This is because reducing the fee to be less than $f_M$ will not increase the number of WSDs in HPCs.

For $N \leq \frac{m_H + m_N}{m_H} \lceil \bar{n}_H \rceil - \frac{m_N \ln(R_T)}{\ln(1 - \tau)}$, the optimization problem becomes

$$\max_{\tilde{f}} \quad F(\tilde{f}) = \tilde{f} \left[ \frac{m_H N + m_H m_N \alpha(\tilde{f})}{m_H + m_N} \right] \text{ s.t. } f_{QoS} \leq \tilde{f} \leq f_M \text{ (15)}$$

To find the optimal $f^*$ that will maximize (15), we define a new function to approximate the total revenue collected by WSDB provider in (15) by

$$\hat{F}(\tilde{f}) = \tilde{f} \left[ \frac{m_H N + m_H m_N \alpha(\tilde{f})}{m_H + m_N} \right].$$  

III. MAXIMIZING REVENUE OF WSDB PROVIDER

The objective is for WSDB provider to set a fee that will maximize its revenue based on the number of WSDs. We assume that when WSDs decide to use normal TVWS channels after enquiring WSDB, they will inform the WSDB about their decisions. While the WSDB will also know the number of WSDs in HPCs since WSDs paid fees to use HPCs. Hence, the WSDB knows the total number of WSDs in TVWS. The mathematical formulation of this problem is given as

$$\max_{\tilde{f}} \quad F = n_H \tilde{f} \text{ s.t. \quad n_H + n_N = N, \quad 0 \leq \tilde{f} \leq f_M \text{ (15)}}$$

where $N$ is the total number of WSDs. First, we determine the number of WSDs that will use HPCs for a given fee $\tilde{f}$ between 0 and $f_M$. Based on the reward criterion that a WSD will join HPCs if

$$\frac{f_M}{R_T - 1} (R - 1) \geq \tilde{f},$$  

we obtain

$$n_H \leq \frac{m_H N + m_H m_N \alpha(\tilde{f})}{m_H + m_N} \text{ (7)}$$

where

$$\alpha(\tilde{f}) = \frac{\ln \left( (R_T - 1) + f_M \right) - \ln(f_M)}{\ln(1 - \tau)}.$$  

On the above criteria is given as

$$P_H = \tilde{f} \quad \text{s.t.} \quad n_H \leq \frac{m_H N + m_H m_N \alpha(\tilde{f})}{m_H + m_N}, \quad \text{max}$$

$$\tilde{f} \quad \text{s.t.} \quad n_H \leq m_H \left( \frac{\ln(\bar{P}_H)}{\ln(1 - \tau)} + 1 \right) = \bar{n}_H.$$  

Therefore, from (7) and (9), the number of WSDs that will use HPCs at a given fee $\tilde{f}$ is given as

$$n_H(\tilde{f}) = \max \left\{ 0, \min \left\{ \left[ \frac{m_H N + m_H m_N \alpha(\tilde{f})}{m_H + m_N} \right], \bar{n}_H \right\} \right\} \text{ (10)}$$

where $[x]$ means rounding down $x$ to the nearest integer. The total fee collected by WSDB provider when it sets its fee to $\tilde{f}$ is given as

$$F(\tilde{f}) = n_H(\tilde{f}) \tilde{f}.$$  

From (10), $n_H(\tilde{f}) = \lceil \bar{n}_H \rceil$ when

$$f \leq f_M (1 - \tau)^{\frac{m_H + m_N}{m_H + m_N} \lceil \bar{n}_H \rceil - N/m_N - f_M}.$$  

This means that if a WSDB provider sets a fee lower than the right hand term in (12), it will not attract further more WSDs to use HPCs as the number of WSDs in HPCs is already saturated and could not provide the required QoS if more WSDs are to join in. Hence, the minimum fee that a WSDB provider should set based on the QoS constraint is given as

$$f_{QoS} = \max \left\{ 0, f_M (1 - \tau)^{\frac{m_H + m_N}{m_H + m_N} \lceil \bar{n}_H \rceil - N/m_N - f_M} \right\} \text{ (13)}$$

unless $f_{QoS} > f_M$ which is when

$$N > \frac{m_H + m_N}{m_H} \lceil \bar{n}_H \rceil - \frac{m_N \ln(R_T)}{\ln(1 - \tau)}.$$  

In this case, it is straightforward that the optimal fee $f^*$ is $f_M$ and the optimal revenue collected by the WSDB provider is $F^* = \lceil \bar{n}_H \rceil f_M$. This is because reducing the fee to be less than $f_M$ will not increase the number of WSDs in HPCs.

For $N \leq \frac{m_H + m_N}{m_H} \lceil \bar{n}_H \rceil - \frac{m_N \ln(R_T)}{\ln(1 - \tau)}$, the optimization problem becomes

$$\max_{\tilde{f}} \quad F(\tilde{f}) = \tilde{f} \left[ \frac{m_H N + m_H m_N \alpha(\tilde{f})}{m_H + m_N} \right] \text{ s.t. } f_{QoS} \leq \tilde{f} \leq f_M \text{ (15)}$$

To find the optimal $f^*$ that will maximize (15), we define a new function to approximate the total revenue collected by WSDB provider in (15) by

$$\hat{F}(\tilde{f}) = \tilde{f} \left[ \frac{m_H N + m_H m_N \alpha(\tilde{f})}{m_H + m_N} \right].$$  


The equation \( \tilde{F}(f) \) is a concave function as its second derivative is always less than zero for all \( f \), given as
\[
\frac{\partial^2}{\partial f^2} \tilde{F}(f) = \left( \frac{m_H m_N (R_T - 1)}{(m_H + m_N) \ln(1 - \tau)} \right) \left( \frac{1}{f(R_T - 1) + f_M} + \frac{f_M}{(f(R_T - 1) + f_M)^2} \right) < 0 \forall f. \tag{17}
\]

The optimal \( \tilde{f}^* \) with (16) as the objective function occurs when \( \frac{\partial}{\partial f} \tilde{F}(f) = 0 \) which can be easily solved by basic descent algorithms [7]. The derivative of \( \tilde{F}(f) \) is given as
\[
\frac{\partial}{\partial f} \tilde{F}(f) = \left( \frac{m_H m_N}{m_H + m_N} \right) \left( \frac{N}{m_N} + \alpha(f) + \frac{f(R_T - 1)}{(f(R_T - 1) + f_M) \ln(1 - \tau)} \right). \tag{18}
\]

However, the value of \( \tilde{f}^* \) may not be the optimal solution for (15) as the optimal solution of (15) must either be \( f_M \) or a \( f \) value that makes \( \left( m_H + m_N \alpha(f) \right) / (m_H + m_N) \) an integer. The proof is provided in the Appendix.

Hence, we define
\[
\tilde{f}_L = f_M (1 - \tau) \left( \frac{m_H + m_N}{m_H m_N} \left[ \frac{m_H N + m_H m_N \alpha(f^*)}{m_N} \right] - \frac{N}{m_N} - f_M \right) / R_T - 1
\]
and
\[
\tilde{f}_U = \min \left\{ f_M \left( (1 - \tau) \left( \frac{m_H + m_N}{m_H m_N} \left[ \frac{m_H N + m_H m_N \alpha(f^*)}{m_N} \right] - \frac{N}{m_N} - 1 \right) / R_T - 1, f_M \right\}
\]
where \( \lceil x \rceil \) means rounding up \( x \) to the nearest integer. The optimal solution of (15) is therefore given as
\[
f^* = \left\{ \begin{array}{ll}
\tilde{f}_L & \text{if } F(\tilde{f}_L) > F(\tilde{f}_U) \\
\tilde{f}_U & \text{otherwise.}
\end{array} \right. \tag{21}
\]

This is because from (15) and (16), \( \tilde{F}(f) = F(f) \) when \( \frac{m_H N + m_H m_N \alpha(f)}{m_H m_N} \) is an integer which is also the points where the \( f \) values satisfy the optimal criterion of (15). Among all the \( f \) values between \( f_{\text{QoS}} \) and \( f^* \) that satisfy the optimal criterion, \( F(\tilde{f}_L) \) will obtain the largest revenue since we have proven \( \tilde{F}(f) \) is a concave function. Similarly, among all the \( f \) values between \( f^* \) and \( f_{\text{max}} \) that satisfy the optimal criterion, \( F(\tilde{f}_U) \) will obtain the largest revenue as \( \tilde{F}(f) \) is a concave function. Hence, the optimal solution of (15) is either \( \tilde{f}_L \) or \( \tilde{f}_U \) whichever that gives the higher revenue. Finally, the optimal solution of (5) is \( f_M \) when \( N > \frac{m_H + m_N}{m_H} \left[ \frac{n_H}{m_H} - \frac{m_N \ln(R_T)}{m_N \ln(1 - \tau)} \right] \) and \( f^* \) in (21) when \( N \leq \frac{m_H + m_N}{m_H} \left[ \frac{n_H}{m_H} - \frac{m_N \ln(R_T)}{m_N \ln(1 - \tau)} \right] \).

IV. NUMERICAL RESULTS

In this section, we present the numerical results of the proposed method to obtain the optimal fee that will maximize WSDB provider’s revenue. In the simulations, we set the number of normal TVWS channels and HPCs to be 3 and 2, respectively. The fee is normalized such that \( f_M = 1 \) and \( R_T \) is set to be 10 such that if \( P_H \) is more than 10 times better than \( P_N \), the WSDs will not increase their fee above \( f_M \) to use the HPCs. The minimum required probability to transmit a packet successfully in HPCs is set at \( P_H = 0.6 \).

The optimal revenues achieved by our proposed method under different values of \( \tau \) are shown in Fig. 1. The optimal revenue results obtained using exhaustive search are also plotted as comparison and it is shown that our proposed method matches the optimal solutions from exhaustive search. It is observed that there is a ceiling to the revenue for every \( \tau \) value. This is because of the QoS constraint which limits the number of WSDs in HPCs and this number multiply by \( f_M \) is the revenue ceiling. The revenue ceiling is higher for smaller \( \tau \) because since the probability of each WSD transmitting a packet is lower, it allows more WSDs to operate in HPCs with the given QoS constraint. It is observed that when the total number of WSDs is small, it is better for the WSDB provider that the WSDs are transmitting packets at a higher frequency since the normal TVWS channels will get congested more easily and WSDs are more willing to pay to move to HPCs. However, when the total number of WSDs is large, the reverse is true because of the QoS constraint.

The optimal normalized fees obtained using our proposed method and exhaustive search with different values of \( \tau \) are shown in Fig. 2. It is observed that our proposed method obtains the optimal solution. From the figure, it is shown that, in general, the WSDB provider should increase the fee as the number of WSDs increases. However, there are some exceptions, for example, when \( \tau = 0.2 \) and number of WSDs is 35, the optimal normalized fee is 0.6 but when the number of WSDs is 36, the optimal normalized fee falls to 0.53. This is because sometimes by lowering the fee, it will attract more WSDs to join HPCs and this will increase the overall revenue even though the fee from individual WSD is lower. In the above scenario, when the number of WSDs is 35 and with a
fee of 0.6, it attracts 4 WSDs to use the HPCs but when the number of WSDs is 36 and with a fee of 0.53, it will attract one more WSD and hence the total revenue actually increases. It is also observed that when \( \tau \) is high, WSDB provider should set a higher fee because the probability of successful packet transmission decreases at a faster rate when the number of WSDs increases and therefore, WSDs are more willing to pay higher fee to switch to HPCs to improve their probability of transmitting a packet successfully.

The number of WSDs in HPCs based on the optimal fee provided in Fig. 2 is shown in Fig. 3. It is observed that the number of WSDs in HPCs increases as the total number of WSDs increases although the fee is increasing as well in general. The number of WSDs in HPCs increases until it reaches the maximum number defined by the QoS constraint.

V. CONCLUSION

HPCs are new features proposed by IDA which provide WSDB provider a new business opportunity to increase revenue. In this paper, we optimize the fee of the HPCs to maximize the revenue of the WSDB provider based on the number of WSDs. We provided the optimal solution and proven that it is optimal. It is shown that WSDB provider should not always increase the fee when the number of WSDs increases as sometimes by reducing the fee to attract more WSDs to use HPCs actually increases the revenue. This study has also shown that when the total number of WSDs is low, it is better for the WSDB provider if the WSDs are transmitting more frequently. However, when the total number of WSDs is high, it becomes worse.

VI. APPENDICES

Let \( f^i \) be a value such that \( \frac{m_H N + m_H m_N \alpha(f^i)}{m_H + m_N} \) is an integer and \( f^{i+k} \) be a value such that

\[
\frac{m_H N + m_H m_N \alpha(f^{i+k})}{m_H + m_N} + k = \frac{m_H N + m_H m_N \alpha(f^{i+k})}{m_H + m_N}
\]

For a given \( \hat{f} \) that is \( f^i < \hat{f} < f^{i+1} \), we have \( F(\hat{f}) < F(f^{i+1}) \) since

\[
\left[ \frac{m_H N + m_H m_N \alpha(\hat{f})}{m_H + m_N} \right] = \left[ \frac{m_H N + m_H m_N \alpha(f^{i+1})}{m_H + m_N} \right].
\]

Hence any \( f \) value between \( f^i \) and \( f^{i+1} \) will not be optimal. Next, we let \( f^{i+k_{max}} \) be the largest value of \( f \) that makes \( \frac{m_H N + m_H m_N \alpha(f)}{m_H + m_N} \) an integer. For a given \( \hat{f} \) that is \( f^{i+k_{max}} < \hat{f} < f_{M} \), we have \( F(\hat{f}) < F(f_{M}) \) since

\[
\left[ \frac{m_H N + m_H m_N \alpha(\hat{f})}{m_H + m_N} \right] = \left[ \frac{m_H N + m_H m_N \alpha(f_{M})}{m_H + m_N} \right].
\]

Therefore, the optimal \( f^* \) for (15) must be either \( f_{M} \) or a \( f \) value that makes \( \frac{m_H N + m_H m_N \alpha(f)}{m_H + m_N} \) an integer.

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