Modeling Price Elasticity of Electricity Demand using AIDS

Wenxian Yang, Xiaoming Bao, Rongshan Yu
Institute for Infocomm Research, A*STAR, Singapore
Email: {wyang, baoxm, ryu}@i2r.a-star.edu.sg

Abstract—In this paper, we utilise demand theory to analyse electricity consumption behaviour of household customers in the scenario of smart grid, where loads of smart home appliances are scheduled with home management systems to maximise individual user’s utility under Real-Time Price (RTP). An analytic model for the price responsive behaviours of electricity consumption is established using Almost Ideal Demand System (AIDS), from which both self- and cross-elasticities of electricity consumption can be estimated. The proposed model is evaluated using data populated from a Multi-Agent System (MAS) that simulates electricity consumption behaviours, and the results show the effectiveness of AIDS in modelling price responsive electricity consumption. The proposed model, together with MAS simulation, provides a useful framework for utility companies or policy makers to evaluate the effectiveness and benefits of RTP and demand response (DR) programs in particular during planning stage where real-life consumption data is unavailable.

Index Terms—AIDS, demand response, real-time pricing, price elasticity, multi-agent system.

I. INTRODUCTION

Early in the 1980s, economic measures as dynamic price were proposed for indirect load management in electricity consumption, such that customers respond to changing spot prices by adapting service requirements that are reschedulable or nonessential [1]. Nowadays, in the smart grid context, DR programs use RTP signals are proposed to incentivise customers to reschedule their electricity demands. High electricity price is set at peak demand to encourage customers to shed or delay some of their loads to a later time when price is lower. Ultimately, the objective of DR is to increase social welfare through a more flat aggregate load curve, which is beneficial to both generators and consumers. The success of a DR program largely depends on the customer’s capacity, including both subjective willingness and objective capability, to respond to dynamic price, which can be measured by the price elasticities for electricity consumption. To find the optimal RTP, utility companies, regulators and policy makers need to model the dynamics of consumer decisions to understand the efficiency gains relative to the costs induced by certain price structures. Evaluating the responsiveness of customers to price signals and measuring the welfare gains or losses are therefore crucial steps in optimal tariff and DR program design.

Despite much effort in engaging pilot DR programs in smart grid, there is still lacking of analytic tools for modeling consumer behaviours and their response to RTP. Traditional constant price-elasticity model [2] has shown to be insufficient unless the system is operated in a small region around its equilibrium conditions [3]. Statistic demand-price model was proposed in [3] for elasticity modelling in a DR framework for optimal RTP tariff design. However, it is nontrivial to obtain the probability model of marginal utility for electricity consumption. In this paper, we utilise demand theory to analyse the electricity consumption behaviour of household customers and estimate the price-elasticities.

In econometrics, demand models are used to analyse consumer behaviours for conventional commodities. Such demand models have been adopted in empirical studies [4], [5], [6], [7], [8] to analyse consumer behaviours, especially the price-elasticities, in household electricity consumption under dynamic pricing. In [4], a 6-rate tariff program is evaluated and the Frisch demand function of daily electricity consumption is estimated. In [5], log-linear econometric models are used to estimate the own-price elasticity of electricity demand. In [6], price elasticities are analyzed using log-log demand equations for peak and off-peak hours for both short-run and long-run. In [7], the AIDS is adopted to estimate the price and expenditure elasticities for a 2-rate TOU program for peak and off-peak electricity consumption. The work in [8] analyzed the direct link between RTP in retail market and the load by residential users. In the Energy-Smart Pricing Plan studied, some households were given dynamic electricity price base on the wholesale price of the previous day. Linear demand functions with indirect utility modelling are estimated. In short, most existing studies use linear or log-linear demand functions to model the price-elasticity of electricity demand based on data collected from small pilot DR programs. Although the estimated price-elasticity values from different sources are not very consistent, it is shown in general, electricity consumption is both self-elastic and cross-time elastic, especially in the long-run.

Our work on modelling price elasticity of electricity demand is based on AIDS in the context of smart grid. The contributions of this paper include the followings. Firstly, this paper verifies the capacity of AIDS in explaining the price responsive behaviour of aggregate loads from residential users under RTP in smart grid. It is demonstrated that AIDS can be effectively utilised to estimate the values of self- and cross-elasticities of household electricity consumption. Secondly, instead of relying on real-life consumer behaviour data which is very costly to obtain, an MAS is constructed to emulate the consumer behaviours under RTP, which helps to generate consumer data with great flexibility and can be adjusted to reflect the properties of a certain group of consumers. By using MAS,
the cost of collecting real-life consumption from DR programs can be avoided and the privacy concerns for consumers that subscribe to DR programs can be circumvented. Overall, this paper provides a flexible and general framework that can be used to evaluate the properties and welfare effects of RTP and DR programs even before they are actually implemented, and can be used for strategy planning for utility companies and policy makers.

In the rest of this paper, the proposed system model and how AIDS can be applied to model household electricity consumption behaviours are presented in Section II, followed by the details of the proposed multi-agent framework in Section III. Simulation results and discussions are given in Section IV, and conclusions are drawn in Section V.

II. AIDS FOR ELECTRICITY DEMAND MODELLING

A. Demand theory and electricity demand

This paper utilises demand theory to analyse the elasticity of household electricity consumption. The basis of demand modelling roots in the theory of consumer choice, which is concerned with how a rational consumer would make consumption decisions. The consumer theory allows the preference ordering to be represented by a utility function, which is an ordinal function defined up to a monotone increasing transformation. The consumer’s choice problem can then be transformed to the constrained maximisation of utility, i.e., to determine a combination of quantities of different goods and services from the feasible set of consumption bundles, that gives the maximum utility possible from a fixed amount of expenditure.

Many demand systems [9], [10], [11], [12] have been proposed based on direct or indirect utility functions. Once the demand models are estimated, the price-elasticities can be readily calculated from model parameters to measure the price-responsive behaviors of customers. Price-elasticities describe how sensitive the demand for a good is to changes in the price of itself or other related goods. Self-elasticity shows the percentage rise in the demand at a percentage rise in the price of the good itself, denoted as \( \varepsilon_{ii} \) for good \( i \). Self-elasticity is always negative which corresponds to a decline in the demand when the price increases. Cross-elasticity shows the percentage increase in demand for good \( i \) as a result of a percentage increase in the price of good \( j \), denoted as \( \varepsilon_{ij} \). If good \( i \) and good \( j \) are substitutes (e.g., electricity and natural gas), cross-elasticity \( \varepsilon_{ij} \) will be positive. If good \( i \) and \( j \) are complements (e.g., car and petrol), \( \varepsilon_{ij} \) will be negative. Insignificant value or zero of \( \varepsilon_{ij} \) means that the two goods are independent, or irrelevant. Note that there are other types of elasticity, e.g., income elasticity, which are not addressed in this work.

When dynamic electricity pricing is adopted in the retail market, consumption of electricity becomes akin to the consumption of conventional commodities. Consider household electricity consumption, which is embedded in the operation of electrical appliances to produce a series of final goods and services (light, hot water, prepare food, etc.). It is natural to define a set of virtual “goods” as the electricity consumed in different time slots, i.e., good \( i \) for electricity consumed in time slot \( t \), the price of good \( i \) being the electricity tariff at time slot \( t \) and the quantity being the amount of electricity consumed. The utility of good \( i \) is reflected by the benefits that the consumer perceived from goods and services produced by consuming the electricity. In response to variations in price, e.g., when price at time slot \( t \) increases, it is likely that consumer reduce electricity usage by shedding some non-important loads. The price responsiveness of consumption in good \( i \) could be measured by self-elasticity. Alternatively, consumer could delay some non-critical loads to a later time \( j \) when price is lower. In this case, good \( i \) and good \( j \) become substitutes and the consumer behaviour in load shifting could be measured by cross-elasticity.

B. System model and load rescheduling

Consider a smart grid where the retailer company purchases electricity from the wholesale market and sells it to a number of end users. The RTP of electricity in the retailer market is informed by the retailer company to end users over a digital communication infrastructure. Assume that the retailer company announces the tariff sufficiently ahead of time, such as the day-ahead dynamic pricing scheme where tariff for the next day at regular intervals is communicated to consumers in advance. We also assume that there is an Electricity Management Controller (EMC) unit embedded in each user’s smart meter. The role of the EMC is to schedule the operation of appliances of each user in response to the electricity price signal to achieve a trade-off between minimising the electricity payment and maximising the benefit from operation of those appliances for each user, base on the user’s own preferences.

Let the time period to be studied include \( n \) time slots. Consider a load \( i \in \mathcal{I} \) at time slot \( t \), where \( \mathcal{I} \) is the set of all loads originally scheduled at time \( t \). The initial utility of load \( i \) is given by

\[
 u_i = c_i \cdot q_i, \tag{1}
\]

which denotes the utility that the consumer perceived if load is performed at requested time. Here \( q_i \) is the electricity consumption of load \( i \) in KWh, and \( c_i \) is the unit utility denoting the general importance of the load per KWh.

If the load is delayed to time \( t' \), its utility will decay due to the inconvenience caused by the delay. Assuming the utility is decreasing exponentially, the time-utility function of load \( i \) if it is scheduled to some \( t' \geq t \) is given by:

\[
 u_i(t') = u_i \cdot \eta^{(t'-t)}, \tag{2}
\]

where \( 0 < \eta \leq 1 \). Therefore, considering the electricity cost, the welfare of conducting electricity load \( i \) at time \( t' \) becomes

\[
 W_i(t') = u_i(t') - p(t')q_i, \tag{3}
\]

where \( p(t') \) denotes the electricity tariff at \( t' \). Furthermore, without losing generality it can be assumed that a load can only be delayed within a maximum tolerable delay period \( t_d \), i.e., \( t \leq t' \leq (t + t_d) \).
The loads for each household members are grouped into four types, namely, critical loads, bi-mode loads, sheddable loads and deferrable loads depending on their time-utility functions. For different types of loads, the EMC determines the optimal schedule of the loads according to the welfare functions as follows.

1) Critical loads will not be rescheduled.
2) Bi-mode loads can be operated in either high-power or low-power mode depending on the welfare, but will not be cut off thoroughly. High-power mode is adopted if \( w_i(t) \geq 0 \) and low-power mode is adopted if \( w_i(t) < 0 \).
3) Sheddable loads will not be delayed but will be cut off if current price is too high such that the welfare becomes negative. Sheddable loads are conducted if \( w_i(t) \geq 0 \) and are cut off if \( w_i(t) < 0 \).
4) Deferrable loads can be delayed within the maximum tolerable delay period, but will not be shedded. For deferrable loads, EMC finds the time slot with maximum welfare among all feasible time slots, i.e., a load at time \( t \) will be rescheduled to time \( t' \) where

\[
\hat{t} = \arg \max_{t' \leq t' \leq (t + \tau)} w_i(t').
\]

C. AIDS modelling

In the following, AIDS is briefly introduced and adopted to model the aggregate household electricity demand under given RTP vectors. AIDS is first proposed for household consumption modelling in [12]. The model gives the share defined by

\[
\omega_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln(X/P)
\]

where \( p_i, \omega_i \) denote the price and budget shares of good \( i \) respectively, \( (\alpha, \beta, \gamma) \) are model parameters, \( X \) is the total expenditure defined by

\[
X = \sum_{i=1}^{n} p_i q_i
\]

where \( q_i \) is the quantity of good \( i \), \( P \) is the price index, where

\[
\ln P = \alpha_0 + \sum_{j=1}^{n} \alpha_j \ln p_j + \frac{1}{2} \sum_{k=1}^{n} \sum_{j=1}^{n} \gamma_{kj} \ln p_k \ln p_j
\]

in the nonlinear AIDS model, and can be approximated by

\[
\ln P = \sum_{i=1}^{n} \omega_i \ln p_i
\]

in the linear AIDS (LA-AIDS) model. \( \alpha_0 \) in (7) is a constant.

This model gives an arbitrary first-order approximation to any demand system and satisfies the following constraints:

\[
\sum_{i=1}^{n} \alpha_i = 1, \quad \sum_{j=1}^{n} \gamma_{ij} = 0, \quad \sum_{i=1}^{n} \beta_i = 0,
\]

\[
\sum_{j=1}^{n} \gamma_{ij} = 0, \quad \beta_i = 0, \quad \gamma_{ij} = \gamma_{ji}.
\]

The adding-up constraint (9) ensures that all shares sum up to 1, i.e., \( \sum_{i=1}^{n} \omega_i = 1 \). Constraint (10) ensures the homogeneity of the demand functions, and (11) ensures Slutsky symmetry. The negativity condition cannot be enforced but only be tested.

Note that the AIDS model permits exact aggregation over consumers. In modelling of electricity consumption, the model permits exact aggregation over different goods and services behind the consumption of electricity.

To map from traditional commodities to electricity, the electricity consumption at each time slot is considered as a "good". The price of different goods corresponds to the price of electricity at different time slots, i.e., \( p_i, (i = 1, \ldots, n) \) when \( n \) time slots are considered. The quantities of different goods correspond to the amount of electricity consumed within different time slots. Given a pair of price and load quantity \((p_i, q_i)(i = 1, \ldots, n)\), the total expenditure and the shares vector can be readily calculated by (6) and (12), respectively.

\[
\omega_i = \frac{\hat{p}_i \hat{q}_i}{X}
\]

The AIDS model parameters can be estimated from a sufficiently large set of price and quantity data pairs.

The self-elasticity of electricity consumption captures the user’s willingness and capability to shed or shift loads at a time slot when the current electricity price increases. The cross-elasticity captures the capacity for inter-temporal substitutes for loads between different time slots, reflected by load shifting when price changes. The Hicksian elasticity matrix can be calculated from the model parameters and an RTP vector as

\[
\varepsilon_{ij} = -\delta_{ij} + \left( \gamma_{ij} - \beta_i \frac{d \ln P}{d \ln p_j} / \omega_i + w_j \left( 1 + \frac{\beta_i}{\omega_i} \right) \right),
\]

where \( \delta_{ii} = 0 \) and \( \delta_{ij} = 1, i \neq j \).

III. MULTI-AGENT SIMULATION OF HOUSEHOLDS

A. Multi-agent simulation

The smart grid is a complex adaptive system consisting of generators, consumers and other participants. Mathematical tools such as pure theoretic game models cannot provide adequate insight into electricity market because too many important details, when omitted in the model, may give rise to different conclusions. On the other hand, grids in different countries and regions have their own technological and social features. Therefore the experiences and results of the grid in one region cannot be taken for granted to apply to others.

In [13], agent-based methods are introduced for handling dynamic and complex computational economics problems. The interactive MAS is the basis of agent-based computational economics, which has been widely used to explore complex phenomenon in many domains such as social sciences, ecology, biomedicine, business operations, etc. It is a system consisted of multiple interactive agents populated in a common environment. Instead of trying to model the entire complex system which usually requires a lot of simplifying assumptions, MAS aims to define each individual agent’s state, objectives and interaction spaces. Once the original conditions are defined, this “virtual world” evolves over time solely driven by
Household Agent

<table>
<thead>
<tr>
<th>Member</th>
<th>Air-con</th>
<th>Refrigerator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Member 1</td>
<td>t=1 sleeping</td>
<td>t=2 cleaning</td>
</tr>
<tr>
<td></td>
<td>0 KWh</td>
<td>500 KWh</td>
</tr>
<tr>
<td>Member m</td>
<td>t=(n-1) dining</td>
<td>t=n rest</td>
</tr>
<tr>
<td></td>
<td>Lighting, 80 KWh</td>
<td>TV, 120 KWh</td>
</tr>
</tbody>
</table>

Fig. 1. The structure of the household agent.

<table>
<thead>
<tr>
<th>Load Type</th>
<th>Load</th>
<th>Energy (Wh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>per household member</td>
<td>critical load</td>
<td>rest (TV)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>exercise (treadmill)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>dining (lighting)</td>
</tr>
<tr>
<td></td>
<td>bi-mode loads</td>
<td>cooking</td>
</tr>
<tr>
<td></td>
<td></td>
<td>high-power (oven, microwave, toaster, hood)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>low-power (microwave, hood)</td>
</tr>
<tr>
<td></td>
<td>study</td>
<td>high-power (computer, modem, ceiling fan, lights)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>low-power (light, laptop)</td>
</tr>
<tr>
<td></td>
<td>sheddable</td>
<td>entertainment (TV, game player, sound system, computer, lights)</td>
</tr>
<tr>
<td></td>
<td>loads</td>
<td>washing (washer, dryer)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>cleaning (vacuum)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>bath (towel, light, hairdryer)</td>
</tr>
<tr>
<td>per household sheddable load</td>
<td>air-con</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>refrigerator</td>
<td>540</td>
</tr>
</tbody>
</table>

Table I: Type of loads and their electricity consumption

Household loads that are considered in this paper are summarised in Table I. The household agent performs load rescheduling for the whole household. In details, for each household member and for this member’s load at each time slot, the agent determines the load type and performs rescheduling following the load rescheduling algorithm as described in Section II-B. For the household as a whole, the agent performs load rescheduling for air-con and refrigerator, following the same load rescheduling algorithm.

When the MAS runs in large scale, e.g., in thousands, the collective behaviour of all agents results in an optimised demand curve in responding to the day-ahead RTP. Given different price vectors, the MAS generates different aggregate load vectors, from which shares can be computed. The price vectors and corresponding shares vectors can then be effectively used to estimate the price elasticities of electricity demand using AIDS model.

IV. Simulation Results

A. MAS simulation

We set $n = 12$ time slots and simulated 1000 households, where the number of members in each household $m$ is randomised within the range of $[2, 6]$. Furthermore, the actual number in each household is modulated by non-electricity-consuming activities defined for each household members such as sleeping, working and shopping.

The types of household loads and their corresponding electricity consumption are listed in Table I. Equation (2) defines the time-utility function where utility decays exponentially with the decay rate is $\eta$, which is randomised for different consumers and different types of loads. The range of $\eta$ is set to $[0.9, 1]$ for deferrable loads and $[0.8, 1]$ for bi-mode and sheddable loads. The maximum tolerable delay period for deferrable loads is set to $t_d = 6$ time slots. The initial utility $u_i$ is determined as follows. First, since $c_i$ is more or less determined by the nature of the type of electricity load, its initial value is selected according to the type of the load. After that, it is randomised for different consumers by multiplying it with a random number $r$ ranged in $[0.5, 1]$ to represent varying preferences from different consumers.

In the MAS, all the customer parameters are randomised at the beginning and are fixed throughout the simulation with different input RTP vectors. By this we fix a group of households with members of certain requested activities and associated preferences. Consumption data is then generated by recording a set of input RTP vectors and their corresponding rescheduled aggregate load. An example output from MAS is shown in Fig. 2, which clearly demonstrates the influence of the electricity price on the electricity loads under the testing scenario. It particular, it clearly visualises the load shedding and shifting effect where the electricity loads originally scheduled during periods of peak electricity price ($t = 1, 3, 4, 6, \text{ and } 10$) have been significantly reduced while loads occurs at other time slots have been increased after the RTP rate is applied to MAS.
our simulation results demonstrate relatively higher elasticity that loads at different time slots are substitutes. In addition, in general smaller than self-elasticities in absolute values, of loads are sheddable or can be switched to low-power is partly due to the set-up in our MAS, where a number means that household electricity demand is self elastic. This has negative self-elasticity with large absolute values, which has inherent imposed in the AIDS model. Note that in the above three cases the sum-up restriction is much larger than without symmetry restriction. Results show that the average fitting error for with symmetry restriction is 3.6 3.8 4.0 4.2 4.4.

TABLE II

<table>
<thead>
<tr>
<th>ELASTICITIES OF HOUSEHOLD ELECTRICITY DEMAND.</th>
<th>min</th>
<th>max</th>
<th>median</th>
<th>mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>self-elasticity</td>
<td>-1.02</td>
<td>-17.97</td>
<td>-3.27</td>
<td>-6.52</td>
</tr>
<tr>
<td>cross-elasticity</td>
<td>0.59</td>
<td>12.26</td>
<td>0.08</td>
<td>0.59</td>
</tr>
</tbody>
</table>

B. AIDS modeling of electricity consumption

AIDS model is trained from the output data of MAS. The model parameters of AIDS can be computed using iterated linear least squares estimator (ILLE) [16]. In our simulations we adopt the implementations in micEconAids package of R (http://www.micecon.org/).

In our modelling, the symmetry restriction is not imposed. In electricity consumption, the effect of price $p_j$ on shares $\omega_j$ of electricity consumed at time $j$ may not equal to the effect of price $p_j$ on shares $\omega_i$. The main reason is that the time-utility model that we used only considers delaying activities within a maximum tolerable delay period but not shifting the activities to an earlier time slot. The following three cases are tested, and the mean values of absolute fitting error in shares are measured to evaluate the fitting of AIDS model to electricity consumption data.

1) AIDS with both homogeneity and symmetry constraints imposed. The mean absolute shares fitting error is $\omega_a = 4.88re - 3$.

2) AIDS with homogeneity restrictions imposed, and without symmetry restriction. Fitting error is $\omega_a = 1.11re - 3$.

3) AIDS without homogeneity and symmetry constraints.

Fitting error is $\omega_a = 1.05re - 3$.

Results show that the average fitting error for with symmetry restriction is much larger than without symmetry restriction. Note that in the above three cases the sum-up restriction is always (inherently) imposed in the AIDS model.

Both self- and cross-elasticities are calculated using the fitted model parameters. The results are summarised in Table II. Results show that electricity demand at all time slots has negative self-elasticity with large absolute values, which means that household electricity demand is self elastic. This is partly due to the set-up in our MAS, where a number of loads are sheddable or can be switched to low-power operation mode. Results also show that cross-elasticities are in smaller than self-elasticities in absolute values, while most cross-elasticity values are positive which means that loads at different time slots are substitutes. In addition, our simulation results demonstrate relatively higher elasticity values comparing to empirical studies, which is a direct result from the more aggressive load shedding/rescheduling using EMC in the MAS. This result also confirms the potential of RTP based DR program once EMC and smart home appliances are widely adopted for residential users.

V. Conclusions

This paper builds up a platform base on demand modelling and multi-agent simulations to analyse the price responsiveness of household consumers for electricity consumption in the context of smart grid. It is shown that demand models for conventional household commodities can be adopted to model aggregated household electricity consumption behaviours, thus provide a useful analytic tool for study the price responsive behaviour of electricity load. The theoretical result is evaluated using an MAS with 1000 households users. The proposed platform can also be used to quantify the performance and effectiveness of RTP and DR programs as early as in the planning stage, and therefore providing a useful analytic tool for utility companies and policy makers for planning, designing and implementing those programs.

REFERENCES


